# Marching-on-in-Time Method with Equivalent Dipole Moment Method for Time Domain Electric Field Integral Equation


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## 1. Introduction

For transient analysis of electromagnetic radiation and scattering, the time domain electric field integral equation (TD-EFIE) can be solved. For their numerical solution, the marching-on-in-time (MOT) [1] method can be applied, and TD-EFIE is solved in space by the method of moments (MoM). Although the MoM employing triangular surface patches is a versatile technique for variety EM problems, one of its major disadvantage is a high computational complexity which is proportional to the square of the number of surface unknowns, since the electromagnetic coupling between all discretized elements has to be considered. In recent years, several techniques have been proposed to decrease the computational complexity of the MOT method. The plane wave time domain (PWTD) algorithm [2] and time domain adaptive integral equation method (TD-AIM) [3] belong between the most popular ones. Although the computation complexity of these methods is lower than the classical MOT method, they are suitable mainly for modelling electrically large structures.

This paper is focused on the investigation of speeding up the classical MOT method for the TD-EFIE by the equivalent dipole moment (EDM) method. The EDM method has been applied for speeding up the MoM in the frequency domain [4], [5], however, no attempt has been done in the time domain. The own idea of the EDM method consists in computing the interaction between the source and testing function locations directly (the approximation of the radiated field by an infinitely small dipole with the equivalent moment) for a separation distance larger than the nominal value, without evaluating the double integral. It will be shown that the MOT method with the EDM method is faster than the classical one, even if it is not used for modelling electrically large structures. The investigation is limited to open perfectly electric conducting structures to avoid troubles with internal resonances.

## 2. MOT Method with EDM Method for TD-EFIE

Let's analyze the scattering of an open perfectly conducting structure illuminated by a transient electromagnetic wave. TD-EFIE is solved by the method of moments. The surface of the analyzed structure is approximated by planar triangular patches, and the RWG function [6] is used to expand the spatial variation of the electric current. In time, the TD-EFIE is approximated by central finite differences. The goal is to find the surface current due to the incident field. After several steps, the resultant implicit MOT scheme can be written in the following matrix form [1]

$$[\lambda_{mn}][I_m(t_i)] = [\beta_m(t_i)] = \Delta t[V_m(t_{i-1/2})] - \Delta t\left[\sum_{n=1}^{N_S} \chi_{mn}(t_i)\right], \quad (1)$$

where $[\lambda_{mn}]$ denotes a matrix of time invariant coefficients, $[I_m(t_i)]$ is a column vector of the unknown current coefficients at time $t_i$, $[\beta_m(t_i)]$ is a column vector related to the incident field $V_m(t_{i-1/2})$ located at $m$-the testing function, and the coefficient $\chi_{mn}(t_i)$ depending on the location of the $m$-th testing and $n$-th source function and the known current coefficients from time $t_0$ to $t_{i-1}$, $\Delta t$ is the length of the time step. The detailed derivation is given in [1].

If the size of triangles for approximating the analyzed structure is sufficiently small, the fields radiated due to the current on a triangle pair may be approximated by radiation of an infinitely small dipole with an equivalent moment, beyond the nominal value $R_0$ [4]. The radiation of the infinitely small dipole [6] after several steps and the transformation to the time domain can be described

$$\mathbf{E}(\mathbf{r},t)=\frac{\eta}{4\pi}\left[\frac{(\hat{\mathbf{r}}\cdot\mathbf{m}_n)\cdot\hat{\mathbf{r}}-\mathbf{m}_n}{cR}\frac{\partial I_0\left(t-\frac{R}{c}\right)}{\partial t}+\frac{3(\hat{\mathbf{r}}\cdot\mathbf{m}_n)\cdot\hat{\mathbf{r}}-\mathbf{m}_n}{R^2}I_0\left(t-\frac{R}{c}\right)+\frac{3(\hat{\mathbf{r}}\cdot\mathbf{m}_n)\cdot\hat{\mathbf{r}}-\mathbf{m}_n}{R^3}\int_0^{t-\frac{R}{c}}I_0(\tau)d\tau\right], (2)$$

where $I_0$ is the current on the infinitely small dipole, $\eta$ is the intrinsic impedance of the medium, and $c$ is the velocity of the wave propagation in that space, $R=|\mathbf{r}-\mathbf{r}'|$ is the distance between the observation point $\mathbf{r}$ and the position of the infinitely small dipole $\mathbf{r}'$, $\hat{\mathbf{r}}=(\mathbf{r}-\mathbf{r}')/R$ is the unit vector, and $\mathbf{m}_n$ is the equivalent moment which can be expressed for the RWG function

$$\mathbf{m}_n=l_n\left(\mathbf{r}_n^{c-}-\mathbf{r}_n^{c+}\right), (3)$$

where the $l_n$ is the length of the $n$-th common edge of the triangle pair $T_n^{\pm}$ and the $\mathbf{r}_n^{c\pm}$ is the position vector of the centroid of $T_n^{\pm}$. To speed up the scheme (1), let's approximate the contribution of the current at the $n$-th source function to the $m$-testing function by an infinitely small dipole (2) with the equivalent moment. After substituting $R=R_{mn}=|\mathbf{r}_m-\mathbf{r}_n|$, where the $\mathbf{r}_m$ and $\mathbf{r}_n$ are the position vectors of the centre of the $m$-the and $n$-th edge, respectively, and $\hat{\mathbf{r}}=\hat{\mathbf{r}}_{mn}=(\mathbf{r}_m-\mathbf{r}_n)/R_{mn}$ to (2), after the discretization of (2) in time in the same way as the scheme (1), and, finally, after the testing procedure we can write for the coefficient $\chi_{mn}(t_i)$

$$\chi_{mn}(t_i)=\frac{\eta}{4\pi}\mathbf{m}_m\cdot\left[\frac{(\hat{\mathbf{r}}_{mn}\cdot\mathbf{m}_n)\cdot\hat{\mathbf{r}}_{mn}-\mathbf{m}_n}{cR_{mn}\Delta t}\left(I_n\left(t_i-\frac{R_{mn}}{c}\right)-I_n\left(t_{i-1}-\frac{R_{mn}}{c}\right)\right)+\frac{3(\hat{\mathbf{r}}_{mn}\cdot\mathbf{m}_n)\cdot\hat{\mathbf{r}}_{mn}-\mathbf{m}_n}{2R_{mn}^2}\left(I_n\left(t_i-\frac{R_{mn}}{c}\right)\right.\right.$$
$$\left.\left.+I_n\left(t_{i-1}-\frac{R_{mn}}{c}\right)\right)+\frac{3(\hat{\mathbf{r}}_{mn}\cdot\mathbf{m}_n)\cdot\hat{\mathbf{r}}_{mn}-\mathbf{m}_n}{2R_{mn}^3}\left(Q_n\left(t_i-\frac{R_{mn}}{c}\right)+Q_n\left(t_{i-1}-\frac{R_{mn}}{c}\right)\right)\right], (4)$$

where

$$Q_n\left(t_i-\frac{R_{mn}}{c}\right)=\int_0^{t_i-\frac{R_{mn}}{c}}I_n(\tau)d\tau\,. (5)$$

Although the expression (2) for the radiation of the infinitely small dipole is valid at arbitrary distance from the dipole, the coefficient $\chi_{mn}(t_i)$ can be computed for the implicit MOT scheme (1) according to (4) only if the distance between the centre of the $m$-th and $n$-th edge is larger, than the nominal value $R_0$. If this condition is not met, the approximation (2) cannot be used, and the coefficient $\chi_{mn}(t_i)$ have to be computed in the classical way.

## 3. Investigation of MOT Method with EDM Method for TD-EFIE

To meet the requirement, that the size of triangles has to be sufficiently small for approximation of the body of the analyzed structure in the time domain, we have to compare the size of all triangles with the wavelength $\lambda(f_{max})$ at the maximum frequency $f_{max}$ of the important part of the spectrum of the excitation pulse. Thus, the requirement can be met if the length of all triangle edges much smaller than $\lambda(f_{max})$. Thus, for our investigation, we choose the average edge length of triangles comparable to $0.1\lambda(f_{max})$.

Let's firstly investigate how the MOT scheme (1) may be sensitive to small change of its coefficients. The equation (1) represents linear invariant discrete system, thus, we can transform it to the Z-domain and compute the condition number of this system for different structures, since it is known [7] that the desired sensitivity is proportional to the condition number of the system matrix. We have carried out extensive numerical experiments to compute the condition number for different structures (placed in free space) which were discretized to the frequency $f$= 300 MHz ($\lambda$= 1 m) with the maximum edge lengths given in the first paragraph. Here, the results for three structures, the strip (2 m x 0.08 m), the square plates (1 m x 1 m), and the rectangular plate (2 m x 1 m), are depicted in Fig. 1. It is seen that the scheme (1) is poorly conditioned at low frequencies, so it is very sensitive to change of the coefficient $\chi_{mn}(t_i)$. Thus, the MOT scheme (1) with EDM method cannot be used if the ratio of the maximum and minimum frequency of the important part of the spectrum of the excitation pulse is high. According to our investigation, the ratio should not exceed 4 for the discretization criterion.

Let's focus our attention on the accuracy of the approximation of the coefficient $\chi_{mn}(t_i)$ in (1) by (4), and than on the determination of the nominal value $R_0$. The accuracy investigation is carried out on the analysis of the strip with the dimensions 2 m x 0.08 m. The body of the strip is modelled with respect the frequency $f$= 300 MHz by 44 patches to meet the discretization criterion. After the transformation of $\chi_{mn}(t_i)$ to the Z-domain, firstly as it is defined for the scheme (1), and secondly by its approximation (4), the magnitude of a relative error can be obtained for the frequency $f$= 300 MHz depending on the distance between the centre of $m$-th and $n$-th edge for the different lengths of the time step (Fig. 2a). It is seen that for small distances, the relative error is high, however, from a certain distance it is small, but not negligible. Further, this error depends on the length of the time step ($R_{min}$ is the minimum distance between any two centres of triangular patches). Since this error is not negligible, and depends on the length of the time step, the nominal value $R_0$ cannot be determined from such kind of investigation. Thus, we have to proceed in a different way, directly in the time domain. We have carried out extensive numerical experiments on modelling different structures and compared the transient responses obtained by the MOT scheme (1), and by this scheme with the EDM approximation (4) to find a relative error of those responses lower than 3 %. It was observed, that the nominal value $R_0$ depends on the length of the time step, as it was expected, but even on the ratio of the maximum distance between any two centres of triangular patches $R_{max}$ and the wavelength $\lambda(f_{max})$. The normalized nominal values $R_0$ are depicted in Fig. 2b.

To demonstrate the efficiency of the scheme (1) with the EDM method, let's illuminate the strip (2 m x 0.08 m; 44 triangular patches; $R_{max}$= 1.94 m), the square plate (1 m x 1 m; 264 triangular patches; $R_{max}$= 1.33 m), and the rectangular plate (2 m x 1 m, 484 triangular patches; $R_{max}$= 2.15 m), all structures are placed in a $xy$ plane, by a harmonic plane wave modulated by Gaussian pulse [1] with the parameters: $\mathbf{E}_0 = 120\pi\hat{\mathbf{x}}, T = 7.2\,\text{LM}, ct_0 = 8\,\text{LM}, \hat{\mathbf{k}} = -\hat{\mathbf{z}}$, and the frequency of the harmonic signal is 187.5 MHz. The bandwidth of this wave is 225 MHz. The length of the time step is $\Delta t$=1.5*$R_{min}/c$. The nominal distances were chosen with the help of Fig. 2b: $R_0$= 1 m for the strip, $R_0$= 0.8 m for the square plate, and $R_0$= 1.05 m for the rectangular plate. Modelling these structures by the MOT scheme (1) with the EDM method saved 17 % of time for the strip, 10 % of time for the square plate, and 20 % of time for the rectangular plate in comparison to the classical MOT scheme. More time could be saved for a smaller length of the time step. The transient responses of the current at the centre of the rectangular plate obtained by both approaches are depicted in Fig. 3. The agreement is very good.

## 4. Conclusion

Although the presented scheme with the EDM approximation can save computational time in comparison to the classical one, its using is limited to the relative "narrow band" of the excitation pulse due to high sensitivity of the classical scheme to small changes of its coefficients. To decrease its sensitivity, a precondition procedure can be used. Now, we work on the MOT method with EDM method for the time domain combined field integral equation.


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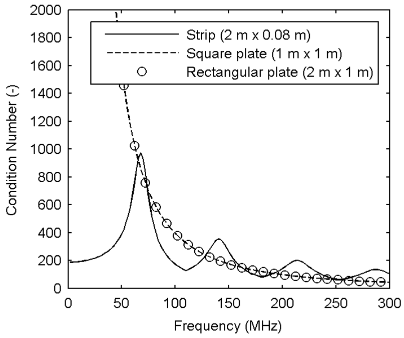


Figure 1: Dependence of condition number on frequency for different structures.

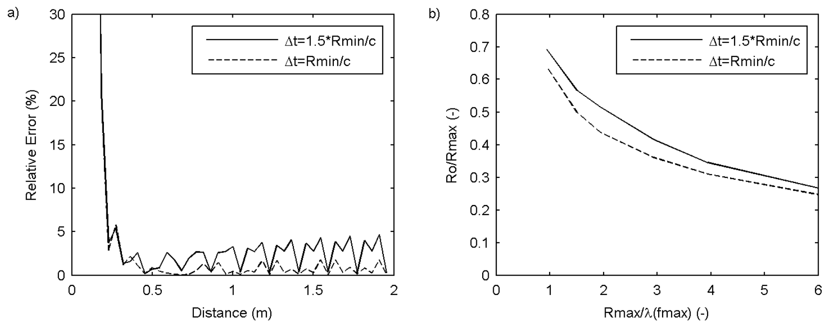


Figure 2: a) Relative error of approximation (4) depending on distance between centre of *m*-th and *n*-th edge, and b) normalized nominal value.

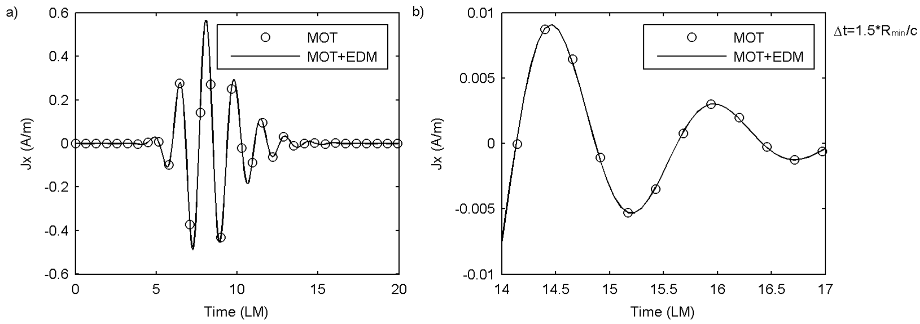


Figure 3: a) Current response at centre of rectangular plate (2 m x 1 m), and b) its enlarged detail.

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