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Study on Model of Optical CDMA System Using LED and APD Elements

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***Abstract*—In this paper, we propose a model of optical code-division multiple access (OCDMA) system using the light-emitting diode (LED) as an optical source and the avalanche photodiode (APD) as a light-receiving element. In addition, we compare the bit-error rate (BER) performance using the proposed model with that obtained by transmission experiments of the OCDMA system using the optical zero-correlation zone (ZCZ) sequence whose correlation property is completely orthogonal in a zone around shift 0 in order to evaluate the effectiveness of the proposed model. As a result, their BER performances are nearly equal, and it is clarified that the proposed model is well modeled on the actual system.**

I. INTRODUCTION

In recent years, products using light-emitting diodes (LEDs) have been put into practical use, and optical wireless communication in which data transfer is performed using the light of the products has been studied. Optical wireless communication can realize high-reliability communication where high-speed communication can be expected without being restricted by the Radio Law[1], [2].

In optical wireless communication, an optical code division multiple access (OCDMA) system has been studied as a multiple access method. This system realizes multiple access by using different spreading sequences according to the user, and the correlation function of the spreading sequence needs orthogonal property in order to suppress co-channel interference. As one of the orthogonal sequences, an optical zero-correlation zone (ZCZ) sequence has been proposed[3], [4], and its correlation property is completely orthogonal in a zone around shift 0 which is called a zero-correlation zone (ZCZ). The OCDMA system using the laser diode (LD) as an optical source and the avalanche photodiode (APD) as a light-receiving element has been modeled, and its bit-error rate (BER) performance has been evaluated. In addition, the visible light communication system using red, green and blue (RGB) multi-chip LED and OCDMA system using LED as the optical source have been proposed [5], [6]. However, those systems using LED as an optical source have not been modeled, therefore it is necessary to establish a new model.

In this paper, we propose a model of OCDMA system using LED and APD. In order to evaluate the effectiveness of the proposed model, we compare the BER performance using the proposed model with that obtained by transmission experiments of the OCDMA system using the optical ZCZ sequence.

II. OPTICAL ZCZ SEQUENCE SET

We describe the outline of the optical ZCZ sequence set and the construction method of $Zcz = 1$ and $Zcz = 2^{n_I}$. Let a_N^j be a bi-phase sequence consisting of 1 and -1 with a length N written as

$$\begin{aligned} a_N^j &= (a_{N,0}^j, \cdots, a_{N,i}^j, \cdots, a_{N,N-1}^j), \\ & \quad a_{N,i}^j \in \{1, -1\}, \end{aligned} \tag{1}$$

where j is a sequence number and i is an ordinal variable. Similarly, let $\hat{a}_N^{j,d}$ be a binary sequence $\hat{a}_N^{j,d}$ consisting of 1 and 0 written as

$$\begin{aligned} \hat{a}_N^{j,d} &= (\hat{a}_{N,0}^{j,d}, \cdots, \hat{a}_{N,i}^{j,d}, \cdots, \hat{a}_{N,N-1}^{j,d}), \\ & \quad \hat{a}_{N,i}^{j,d} \in \{1, 0\}, d \in \{1, 0\}. \end{aligned} \tag{2}$$

Let A be a set of pairs of a bi-phase sequence a_N^j and a binary sequence $\hat{a}_N^{j,d}$ written as

$$A = \{(a_N^1, \hat{a}_N^{1,d}), \cdots, (a_N^j, \hat{a}_N^{j,d}), \cdots, (a_N^M, \hat{a}_N^{M,d})\}, \tag{3}$$

where M is the number of sequences in a set and is called family size.

A periodic correlation function between sequences a_N^j and $\hat{a}_N^{j,d}$ at shift i' is defined by

$$\rho_{a_N^j, \hat{a}_N^{j',d}, i'} = \sum_{i=0}^{N-1} a_{N,i}^j \hat{a}_{N,(i+i') \bmod N}^{j',d}. \tag{4}$$

If the periodic correlation function satisfies

$$\rho_{a_N^j, \hat{a}_N^{j',d}, i'} = \begin{cases} w & ; i' = 0, j = j', d = 0, \\ -w & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \neq j', \\ 0 & ; 1 \leq |i'| \leq Zcz, \end{cases} \tag{5}$$

then the set A is called an optical zero-correlation zone (ZCZ) sequence set, where Zcz is a zero correlation zone, and w is the absolute value of the peak of auto-correlation function and is equal to the number of sequence values 1 in the binary sequence $\hat{a}_N^{j,d}$.

A. Construction of Optical ZCZ Sequence Set with $Zcz = 1$

Let H_{N_1} be the Sylvester-type Hadamard matrix of order $N_1 = 2^n, n \geq 2$ written as

$$H_{N_1} = [h_{N_1}^0, \cdots, h_{N_1}^j, \cdots, h_{N_1}^{N_1-1}]^T, \tag{6}$$

$$h_{N_1}^j = (h_{N_1,0}^j, \cdots, h_{N_1,i}^j, \cdots, h_{N_1,N_1-1}^j), \tag{7}$$

where the symbol T denotes the matrix transposition, which is defined by

$$H_{N_1} = H_{\frac{N_1}{2}} \otimes H_2, \tag{8}$$

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \tag{9}$$

where $\otimes$ denotes the Kronecker product, and $h^j_{N_1}$ is called the Sylvester-type Hadamard sequence.

An element of a bi-phase sequence $a^{j,0}_{N,i} \in \{1, -1\}$ of length $N = 2N_1$ is given by

$$a^j_{N,i} = \gamma_{N,i} \cdot h^j_{N_1, imodN_1}, \tag{10}$$

$$\gamma_{N,i} = \begin{cases} 1 & ; 0 \le i < \frac{N}{2}, \\ (-1)^{i+1} & ; \frac{N}{2} \le i < N. \end{cases} \tag{11}$$

On the other hand, an element of a binary sequence $\hat{a}^{j,d}_{N,i} \in \{1, 0\}$ of length N is given by

$$\hat{a}^{j,d}_N = \frac{1 + (-1)^d a^j_N}{2}. \tag{12}$$

The periodic correlation function between a^j_N and $\hat{a}^{j',d}_N$ except j, $j' \le 1$ is given by

$$\rho_{a^j_N, \hat{a}^{j',d}_N, i'} = \begin{cases} \frac{N}{2} & ; i' = 0, j = j', d = 0, \\ -\frac{N}{2} & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \ne j', \\ 0 & ; i' = \pm Zcz = \pm 1. \end{cases} \tag{13}$$

Therefore, the above set of M pairs consisting of a bi-phase sequence a^j_N and a binary sequence $\hat{a}^{j,d}_N$ is called optical ZCZ sequence set with $Zcz = 1$ and $M = \frac{N}{2} - 2 = \frac{N}{Zcz+1} - 2$.

B. Construction of Optical ZCZ Sequence Set with $Zcz = 2^{n_I}$

The optical ZCZ sequence of $Zcz = 2^1 = 2$ can be constructed by interleaving the optical ZCZ sequence of $Zcz = 1$. By repeating this operation, $Zcz = 2^{n_I}$ can also be composed from $Zcz = 2^{n_I - 1}$ optical ZCZ sequence set. Let $a^j_{N_2}, N_2 = 2^n \cdot 2^{n_I - 1} = 2^{n + n_I - 1}$ be the optical ZCZ sequence used for interleaving. Here, interleaving is performed as follows.

$$\begin{aligned} \alpha^j_N = \; & \big(a^{j-(j \bmod 2)}_{N_2,0}, (-1)^j a^{j+1-(j \bmod 2)}_{N_2,0}, \\ & a^{j-(j \bmod 2)}_{N_2,1}, (-1)^j a^{j+1-(j \bmod 2)}_{N_2,1}, \cdots, \\ & a^{j-(j \bmod 2)}_{N_2,i}, (-1)^j a^{j+1-(j \bmod 2)}_{N_2,i}, \cdots, \\ & a^{j-(j \bmod 2)}_{N_2,N_2-1}, (-1)^j a^{j+1-(j \bmod 2)}_{N_2,N_2-1}\big). \end{aligned} \tag{14}$$

Similarly, the binary sequence $\hat{\alpha}^{j,d}_N$ is obtained from (12) and (14). The periodic correlation function of this bi-phase sequence α^j_N and the binary sequence $\hat{\alpha}^{j,d}_N$ is expressed as

$$\rho_{a^j_N, \hat{a}^{j',d}_N, i'} = \begin{cases} \frac{N}{2} & ; i' = 0, j = j', d = 0, \\ -\frac{N}{2} & ; i' = 0, j = j', d = 1, \\ 0 & ; i' = 0, j \ne j', \\ 0 & ; i' = \pm Zcz = \pm 2^{n_I}. \end{cases} \tag{15}$$

The sequence set of α^j_N and $\hat{\alpha}^{j,d}_N$ is the zero-correlation zone $Zcz = 2^{n_I}$ and the optical ZCZ sequence set of $M = \frac{N}{2} - 2 = \frac{N}{2^{n_I + 1}} - 2 + (n_I \bmod 2)$ of sequences.

III. CONVENTIONAL AND PROPOSED MODELS

We describe the conventional model and the proposed model.

A. Conventional Model

In the conventional model using LD and APD, the average μ_{1c} of APD output for a mark and the average μ_{0c} of APD output for space is expressed as

$$\mu_{1c} = GT_c \left[\frac{\eta\lambda}{hc} P_c + \frac{\eta\lambda}{hc} P_b + \frac{I_b}{e} \right] + \frac{I_s T_c}{e}, \tag{16}$$

$$\mu_{0c} = GT_c \left[\frac{\eta\lambda}{hc} \frac{P_c}{M_e} + \frac{\eta\lambda}{hc} P_b + \frac{I_b}{e} \right] + \frac{I_s T_c}{e}, \tag{17}$$

where T_c is chip duration [s], η is quantum efficiency, λ is light wavelength [m], h is Planck's constant, c is the speed of light [m/s], P_c is the average received power per chip [W], and M_e is the modulation extinction ratio. On the other hand, the variance σ^2_{1c} of APD output for a mark and the variance σ^2_{0c} of APD output for space is expressed as

$$\sigma^2_{1c} = G^2 F_e T_c \left[\frac{\eta\lambda}{hc} P_c + \frac{\eta\lambda}{hc} P_b + \frac{I_b}{e} \right] + \frac{I_s T_c}{e} + \sigma^2_{th}, \tag{18}$$

$$\sigma^2_{0c} = G^2 F_e T_c \left[\frac{\eta\lambda}{hc} \frac{P_c}{M_e} + \frac{\eta\lambda}{hc} P_b + \frac{I_b}{e} \right] + \frac{I_s T_c}{e} + \sigma^2_{th}, \tag{19}$$

where F_e is the excess noise factor and σ^2_{th} is the thermal noise which is represented by

$$\sigma^2_{th} = \frac{2 k_B T_r T_c}{e^2 R}, \tag{20}$$

where k_B is Boltzmann's constant and T_r is receiver noise temperature [K].

B. Proposed Model

In general, the low-frequency component of APD output is cut in the amplifier , but it is not considered in the conventional model. Therefore, the problem is solved by formulating the difference value between a mark and space. The average difference value μ_p of the proposed model is expressed as

$$\mu_p = \frac{1}{2} S P_s G R, \tag{21}$$

where S is the light-receiving sensitivity and R is the load resistance [Ω]. The constant $1/2$ in (21) is derived from 50 [Ω] which is the oscilloscope termination resistance and the coaxial cable impedance. Since the difference value between the averages μ_{1c} and μ_{0c} is the subtraction of (17) from (16), the background noise P_b, the bulk leakage current I_b and the surface leakage current I_s are canceled each other, and are all zero, and the background noise and the surface leakage current are also called a dark current.

The variance of the proposed model is divided into two variances which are the variance σ^2_{1p} for the mark and the variance σ_{0p} for space. In the proposed model, the dark current is considered to be zero, so that no shot noise is generated. The

TABLE I
VALUES OF ALL PARAMETERS USED IN THE MODEL.

Light-receiving sensitivity S	0.7028
APD gain G	30
Load resistance R	9.1[kΩ]
Elementary charge e	1.6022×10^{-19}[C]
Background noise P_b	-82.604[dBm]
Receiver frequency bandwidth B	100[MHz]
Excess noise coefficient F_e	2.7742
Noise current of the amplifier circuit I_{an}	45.0[nA]
Noise current generated by the measuring system I_{on}	36.128[nA]

TABLE II
EXPERIMENTAL SPECIFICATIONS.

Spreading sequence	Optical ZCZ sequence
Sequence length N	8, 64
Zero-correlation zone Zcz	1, 4
Sequence number(desired station) j	2
Sequence number(Non-desired station) j	3
Transmission data	Repeat 1 and 0
Chip rate	10[Mcps]
Bit rate	1.25[Mbps]($N = 8$) 156.25[kbps]($N = 64$)
Number of trials	10^4

current term of the amplifier circuit and the noise current term of the measuring system are newly added, and thermal noise is included in the noise current of the amplifier circuit. In the proposed method, the variance σ_{1p}^2 of APD output for a mark and the variance σ_{0p}^2 of APD output for space is expressed as

$$\sigma_{1p}^2 = \frac{1}{4}R^2\{2eS(mP_s + P_b)BG^2F_e + I_{an}^2 + I_{on}^2\} \quad (22)$$

$$\sigma_{0p}^2 = \frac{1}{4}R^2(2eSP_bBG^2F_e + I_{an}^2 + I_{on}^2), \quad (23)$$

where e is the elementary charge, F_e is the excess noise coefficient, m is the number of users, B is the receiver frequency bandwidth, I_{an} is the noise current of the amplifier circuit, I_{on} is the noise current generated by the measuring system. Because the model equation for the average is multiplied by $1/2$, the equation for the variance is multiplied by $1/4$. Also, the dispersion does not depend on the chip rate within the bandwidth of the system. The values of the parameters used in the proposed model are shown in Tab. I.

IV. EXPERIMENTAL SYSTEM

Figure 1 shows the configuration of the experimental system. In this paper, in order to judge the validity of the proposed model by the bit-error rate (BER), an experiment of BER characteristic evaluation was conducted using this experimental system. Tab II shows the experimental specifications. The optical ZCZ sequence of $Zcz = 4$ was created by interleaving the optical ZCZ sequence set of $Zcz = 1$, $N = 16$ twice.

V. EVALUATION OF BER CHARACTERISTICS

A. BER Theoretical Formula

Let the average and variance after correlation processing of EWO modulation be μ and $\sigma^2(m)$, where m is the number of users. Using complementary error functions, the theoretical formula can be expressed as

$$P_B = \frac{1}{2}\mathrm{erfc}\left(\frac{\mu}{\sqrt{2}\sigma(m)}\right). \quad (24)$$

The theoretical formula can be expressed as follows by using the average $\mu_1, \mu_!$, and variance σ_1^2, σ_0^2 for a mark and space at communication by the optical ZCZ sequence set.

$$P_B = \frac{1}{2}\mathrm{erfc}\left(\frac{\omega(\mu_1 - \mu_0)}{\sqrt{2\omega(\sigma_1^2(m) + \sigma_0^2(m))}}\right). \quad (25)$$

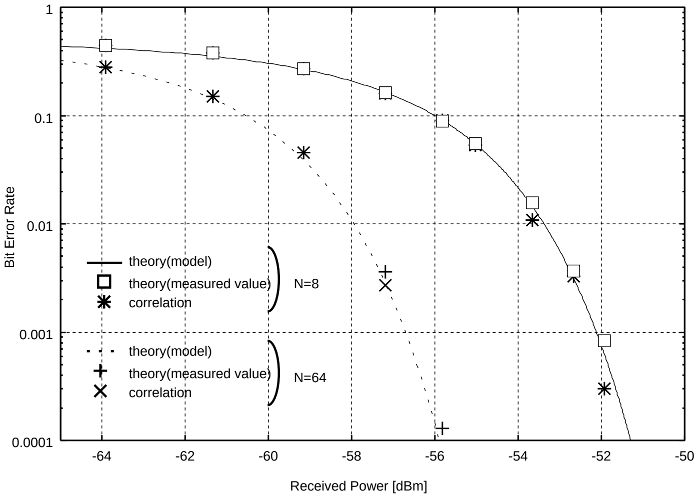

Fig. 2. BER performance of OCDMA system using optical ZCZ sequence for single-user.

B. Evaluation of BER Characteristics of Single Users

Compare BER characteristics for a single user. Figure 2 shows BER performance for a single-user. For single users, the theoretical BER values, the theoretical BER values by experimental values, and the results of the BER characteristics calculated by performing correlation output are shown. As shown in Figure 2, three types of BER characteristics almost overlap at $N = 8, 64$. Also, $N = 64$ has better BER because its peak value is larger.

C. Evaluation of BER Characteristics of Multi Users

Compare BER characteristics for multi-user case. Figure 3 shows BER performance for a single-user. The number of users to be multiplexed is $m = 2$. Figure 3 shows graphs of theoretical values of BER according to a model in the case of $N = 8, 64$ and $Zcz = 1$, theoretical values of BER by experimental values, and results of BER characteristics calculated by performing correlation processing. As shown in Figure 3, three types of BER characteristics almost overlap at $N = 8, 64$. It can be confirmed that the theoretical value of $m = 1$ and the theoretical value of $m = 2$ have almost no difference in each case of $N = 8, 64$.

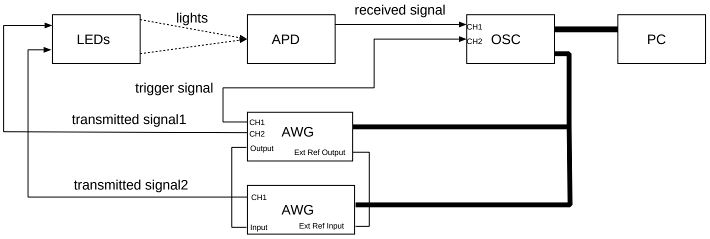

Fig. 1. Transmission experimental system for OCDMA system.

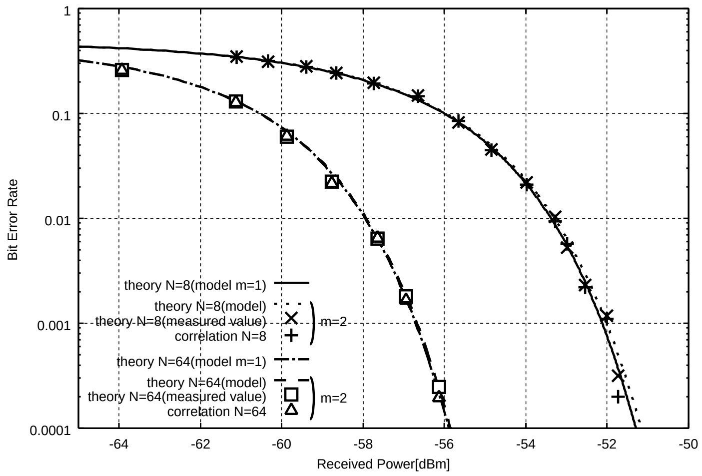

Fig. 3. BER performance of OCDMA system using optical ZCZ sequence for multi-user.

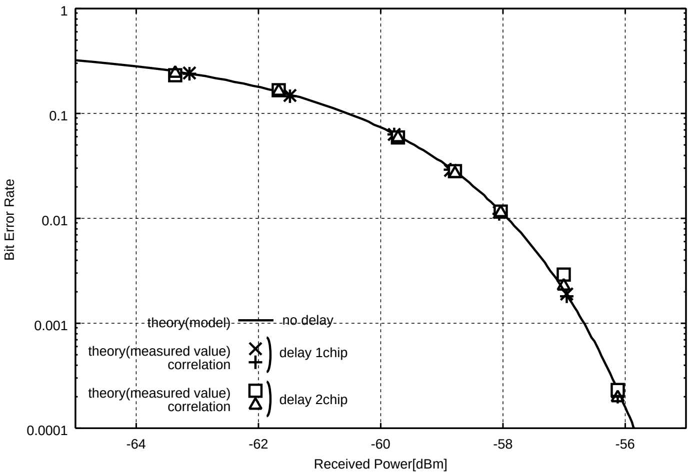

Fig. 4. BER performance of OCDMA system using optical ZCZ sequence with the synchronous deviation.

D. Evaluation of BER Characteristics Assuming Synchronization Deviation

Compare BER characteristics assuming synchronization deviation in multi-user case. The number of users is $m = 2$, and it is assumed that non-desired stations cause chip delay. Figure 4 shows the theoretical values of BER by the model for $N = 64$ and $Zcz = 4$, the theoretical valued of BER by experimental values and the graphs of the BER characteristics calculated by performing correlation processing. It can be seen that even if the chip delay occurs, there is almost no difference from the theoretical value. From these results, the proposed model is appropriate.

VI. CONCLUSION

In this paper, we have proposed a model of optical code-division multiple access (OCDMA) system using the light-emitting diode (LED) as an optical source and the avalanche photodiode (APD) as a light-receiving element. In addition, we have compared the bit-error rate (BER) performance using the proposed model with that obtained by transmission experiments of the OCDMA system using the optical zero-correlation zone (ZCZ) sequence in order to evaluate the effectiveness of the proposed model. As a result, their BER performances have been nearly equal, and it has clarified that the proposed model was well modeled on the actual system.

VII. ACKNOWLEDGMENT

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