Preliminary Uncertainty Analysis for Multi-Frequency Coherent Signal Measurement Technique

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***Abstract*— Preliminary uncertainty analysis for a multi-frequency coherent signal measurement technique is presented. An introductory description of a proposed new measurement signal, a multi-frequency coherent signal is given. This signal consists of a set of many independent analogue signals, which are coherent with a common clock signal. The multi-frequency coherent signal enables simultaneous, coherent reception of many analogue signals, appearing in a common measuring channel. The influence of noise, simultaneous presence of many coherent signals and A/D converter resolution on the measurement uncertainty is analyzed.**
Key words: coherent demodulation, multi-frequency coherent signal, measurement uncertainty

I. Introduction

A presentation of a preliminary uncertainty analysis for a multi-frequency coherent signal measurement technique needs an introductory description of the technique prior to this analysis. This is because of not so wide knowledge of the technique [1].

In measurement practice it is often necessary to carry out simultaneous measurement of many different weak signals (e.g. from many widely spread sensors) existing in a common transmission channel. Electromagnetic field strength measurement using MST (Modulated Scattering Technique), in which the measurement signals coming from many scattering sensors are present in a common measurement channel, is an example [2]. It is nearly a rule that a coherent demodulation is used for weak signals reception, making possible reception of signals much smaller than accompanying noise and disturbance.

The presence of reference signal, synchronous with a received signal is a necessary condition for applying a coherent demodulation. Therefore, in the case of many different simultaneously existing signals, it is necessary to distinguish them in order that the frequencies of these signals differ and are not mutual multiplies.

Moreover, these frequencies should be chosen in a way that secures that the frequencies of additional components, originated as a result of interference (caused by inevitable nonlinearity of the transmission channel), differ from frequencies of the useful signals. A particular signal measurement can be obviously made in turn using a single tuned coherent receiving apparatus.

However, a simultaneous coherent measurement of all the signals needs the use of separate coherent receiving apparatus for each signal. This solution can be acceptable in case of several signals but in the case of more than ten or a larger number of signals is unrealistic. It appears that choosing signal frequencies in a specific way it is possible to receive many signals simultaneously using only one coherent receiving system. The set of such signals the authors have called *a multi-frequency coherent signal.*

II. Multi-Frequency Coherent Signal and Multi-Frequency Coherent Demodulation

As mentioned above, the use of coherent demodulation is necessary for reception of very weak signals, often much lower than the level of accompanying noise. If many such signals exist in a common measurement channel then their simultaneous, coherent reception requires that all the signals are synchronous with one, common clock signal.

If a total sum of received signals drives an A/D converter with a sampling frequency equal to the frequency of the above-mentioned common clock signal, the sequence of numbers obtained at the converter output will enable, using a digital synchronous demodulation, separation of each received signal. The following condition is required for such an operation: *the number of samples falling into the period of each demodulated signal is an integer and is time-invariable.*

This condition can be fulfilled in the following ways:

- the common clock signal frequency is a multiple of each measured signal frequency
- each measured signal period is a multiple of a common clock signal period.

It appears that obtaining the above described 'frequency' coherence is difficult even for a small number of signals. The smallest common multiple can reach values several orders higher than the frequencies of measured signals. Getting the 'time' coherence is significantly easier.

If the frequencies of each signal are chosen in such a way that a period of any of them is a multiple of the clock signal period ΔT, the periods and frequencies of particular signals can be described as follows:

$$T_0 = k \cdot M \cdot \Delta T, \quad T_m = k(M+m) \cdot \Delta T \tag{1}$$

$$F_0 = \frac{1}{T_0} = \frac{1}{k \cdot M \cdot \Delta T}$$

$$F_m = \frac{1}{T_m} = \frac{1}{k \cdot (M+m) \cdot \Delta T} \tag{2}$$

$$F_S = \frac{1}{\Delta T}$$

where:

T_0 – signal period of the highest frequency (F_0)
T_m – m^{th}-signal period (frequency F_m)
$m = 0, 1, 2, \ldots M-1$ – following signal number
M - total number of signals
k – multiple factor (should be an even natural number)
F_S – clock signal frequency.

A frequency spectrum of signals described by equations (1) and (2) is shown in Fig. 1. The spectrum features unequal distances between particular spectrum lines. It has been shown in [1] that it is advantageous to choose particular frequencies in such a way that the ratio of the highest frequency F_0 to the lowest frequency F_{M-1} is less than 2.

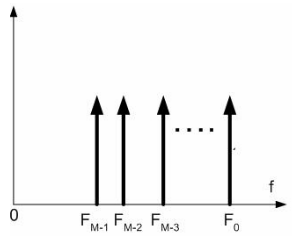

Fig.1 Example amplitude spectrum of the signal described by equations (1) and (2)

The signal appearing at the receiving channel output, containing components of frequencies described by equation (2), can be written as follows:

$$S(t) = \sum_{m=0}^{M-1} \left[S_m^{\mathrm{Re}}(t) + j \cdot S_m^{\mathrm{Im}}(t) \right] + N(t) \tag{3}$$

Where M – total number of signals, $S_m(t)$– particular useful signals, $N(t)$ – noise.

This signal, after pre-filtration and pre-amplification is converted with an A/D converter into a sequence of numbers. It can be described as follows:

$$S(n) \Leftrightarrow S(t) \cdot P(t) = S(t) \cdot \sum_{n=0}^{2 \cdot (2M-1) \cdot K} \delta(t - n \cdot \Delta T) \tag{4}$$

Where $P(t)$ – sampling function of the converter, K – the number of periods of the lowest frequency signal, $n = 0, 1, 2, 3 \ldots$ – the number of following sample.

If the frequencies of particular signals fulfill (2), then each input component can be separated by a digital coherent demodulation. The digital coherent demodulation is based on multiplying a train of registered signal samples by a digital reference function (rectangular or sinusoidal) and totalling the results of the following multiplications. A process of digital coherent demodulation can be described as follows:

$$S(m) = S(n) \otimes R_m(n) \tag{5}$$

Where $S(n)$ – output signal of A/D converter, $R_m(n)$ – digital reference function for m^{th}*- signal,* $n = 0, 1, 2, \ldots$

The form of the sinusoidal digital reference function for phases 0^o and 90^o is as follows:

$$R_m{}^{0}(n) = \sum_{n=0}^{k \bullet (M+m)} \sin \left[2 \cdot \pi \cdot \frac{n}{k \bullet (M+m)} \right] \tag{6a}$$

$$R_m{}^{90}(n) = \sum_{n=0}^{k \bullet (M+m)} \cos \left[2 \cdot \pi \cdot \frac{n}{k \bullet (M+m)} \right] \tag{6b}$$

where k – factor of proportionality.

For each m^{th}*- signal* the coherent demodulation process is carried out for both reference functions and then the modulus is calculated using the following dependence:

$$S_m = \sqrt{\left[S_m{}^{0} \right]^2 + \left[S_m{}^{90} \right]^2} \tag{7}$$

Two ways can be used to carry out the multi-frequency coherent demodulation:

1. The sequence of numerical values at the A/D output is at first recorded in a computer memory and then the coherent demodulation process is carried out for particular signals. The 'dead time' necessary to record an adequately long sequence of numbers is the shortcoming of this solution. However, the time of nothing but the coherent demodulation process depends on capacity of the computer.
2. The sequence of numerical values at the A/D output is directly subjected to the coherent demodulation process for all M signals at the same time.
   Obviously, this requires a dedicated digital circuit featuring adequately high capacity.

It is obvious that the second way, a real time coherent demodulation process carried on using a dedicated digital circuit , is more advantageous.

III. Measurement Uncertainty

Extraction of each m^{th}*-signal* from the set of signals described by equations (3) and (4) is an aim of the above-described coherent demodulation. One can indicate two basic sources of errors.

The first source of error is a simultaneous demodulation of many coherent signals. This error does not depend on the resolution of the A/D converter.

The second source of error is a limited resolution of the A/D converter. In this case the error depends on:

a. The ratio of a sum of all the coherent signals amplitudes to the amplitude of the smallest coherent signal at the A/D converter input.
b. The ratio of the noise level to the amplitude of the smallest coherent signal at the A/D converter input.

The biggest measurement uncertainty refers to the smallest measured coherent signal.

A. *Influence of the Ratio of a Sum of the Coherent Signals Amplitudes to the Smallest Signal Amplitude*

An analogue signal, converted into a sequence of numbers using an A/D converter, features a quantization error, depending on the A/D converter resolution.

The quantization error can be described by means of a quantization noise ratio (SNR) that for an A/D converter having b bits resolution is equal to:

$$SNR_{A/D} = (1.763 + 6.02 \cdot b)\ \mathrm{dB} \quad (8)$$

It was assumed that the maximum value of analogue input signal is equal to the input range of the A/D converter. For example, the quantization noise factor for 16 bit A/D converter according to (12) equals 98dB.

In the case of M coherent signals, it should be assumed that the sum of all signals amplitudes is contained in the input range of the A/D converter. The ratio of a sum of all the coherent signals amplitudes to the amplitude of the smallest coherent signal determines the measurement error for the smallest signal caused by the quantization noise occurrence.

If this ratio is D[dB], then the S/N ratio for the smallest signal falls to:

$$SNR_{LS} = SNR_{A/D} - D\ [\mathrm{dB}]$$

For example, for the ratio D=50dB we obtain SNR_{LS}=48dB and the measurement uncertainty of the smallest signal of about 0.4%.

For all the signals featuring similar amplitude (which means $D \approx 16/1$ or D[dB] = 24dB), we obtain:

$$SNR_{LS} = 98\text{-}24\ [\mathrm{dB}] = 74\mathrm{dB}$$

This corresponds with the measurement uncertainty of the smallest signal of about 0.02%.

B. *Influence of the Ratio of the A/D Converter Input Noise to the Smallest Coherent Signal Amplitude*

An effective resolution of the converter is influenced by the level of non-coherent signals, among them noise. Noise level is more often described by the input signal to noise ratio, defined by a ratio of rms value of wanted signal to rms value of noise.

Such a method of noise level definition cannot be directly used in the case of A/D conversion. It results from the demand that the sum of the useful signal and noise should not exceed the input range of the A/D converter. Therefore, the peak-to-peak value of the noise is more useful.

In the case of noise featuring normal distribution, we can assume (with probability $p = 0.999$) that peak-to-peak value $U_{p\text{-}p}$ will not exceed $U_{p\text{-}p} = 6.6\ U_{rms}$, where U_{rms} is rms value of noise.

In the case of sinusoidal signal, peak-to-peak value $U_{p\text{-}p}$ = 2.82 $\boldsymbol{U_{rms}}$. To include the crest factor of white noise, the signal/noise ratio for the A/D converter should be increased by factor D_{CF} = 6.6/2.82=2.34 or D_{CF} [dB] = 7.4dB. Lowering the frequency band, which is tantamount to the measurement time increasing, can decrease the influence of noise.

As mentioned earlier, the digital coherent demodulation is based on multiplying the train of registered signal samples by the digital reference function and summing the results of the following multiplications.

This procedure can be carried out for many periods of a chosen signal. In the process, the components coming from the coherent signals sum up linearly but the components coming from the noise sum up as mean square values.

As a result improvement of the signal to noise ratio (SNR) occurs. The improvement is $\sqrt{\boldsymbol{K}}$ times, where $\boldsymbol{K}$ is a number of periods subjected to the digital coherent demodulation. For example, for $K = 10000$, the SNR improvement is 100 times.

The avoidance of exceeding the input range of the A/D converter is a necessary condition of the utilization of the SNR improvement process. Therefore, the total input amplitude of signal and noise should be kept within the input range.

Theoretically, a suitably high number of periods subjected to the digital coherent demodulation allow decreasing the non-coherent signal influence to any extent.

Let us now assume that the total period of digital coherent demodulation is long enough for an effective elimination of additive noise. If the input noise level (amplitude) is bigger than the total level of the coherent signals then the ratio of the input noise level of the A/D converter to the amplitude of the smallest coherent signal will influence the measurement error of this smallest signal. It is caused by the presence of the quantization noise.

As we assumed large $\boldsymbol{K}$, the ratio mentioned above will determine the measurement error. For example, for SNR = -20dB or NSR=20dB, after including D_{CF} =7.4dB we have $(NSR)_{ampl}$ =27.4dB . Let us assume, as previously, that the ratio of a sum of all the coherent signals amplitudes to the amplitude of the smallest coherent signal is D=50dB. For 16 bit A/D converter we obtain:

$$SNR = SNR_{A/D} - (NSR)_{ampl} - D = 98\text{-}27.4\text{-}50\ [\mathrm{dB}] = 20.6\ \mathrm{dB}$$

This corresponds with the measurement uncertainty of the smallest signal of about 9.3%.

For all the signals featuring similar amplitude (D = 24dB), we obtain:

$$SNR = 98\text{-}27.4\text{-}24\ [\mathrm{dB}] = 46.6\mathrm{dB}$$

This corresponds with the measurement uncertainty of the smallest signal of about 0.47%.

C. *Influence of Simultaneous Demodulation of Many Coherent Signals*

The sequence of numerical values at the A/D output subjected to the coherent detection process contains components of the demodulated signal as also components of the other coherent signals (spectral leakage).

According to (5) and (6), the values of quadrature components of m^{th}*-signal* (with the amplitude equal to 1) for a single period $T_m = k\,(M+m)$ can be described as follows:

$$S_m^0 = \sum_{n=0}^{k\bullet(M+m)} \left\{ \sin\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+m)} + \phi_m \right] \right\} \times$$
$$\times \left\{ \sin\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+m)} \right] \right\}$$
$$S_m^{90} = \sum_{n=0}^{k\bullet(M+m)} \left\{ \sin\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+m)} + \phi_m \right] \right\} \times$$
$$\times \left\{ \cos\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+m)} \right] \right\} \qquad (9)$$

For a period number equal to K, (9) can be rewritten as:

$$S_{m,K}^0 = \sum_{n=0}^{K\bullet k\bullet(M+m)} \left\{ \sin\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+m)} + \phi_m \right] \right\} \times$$
$$\times \left\{ \sin\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+m)} \right] \right\}$$
$$S_{m,K}^{90} = \sum_{n=0}^{K\bullet k\bullet(M+m)} \left\{ \sin\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+m)} + \phi_m \right] \right\} \times$$
$$\times \left\{ \cos\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+m)} \right] \right\} \qquad (10)$$

Taking into account in (10) the presence of the other signals with frequencies different than that of m^{th}*-signal* (i.e. for $L \neq m$), (10) can be rewritten as:

$$S_{m,K}^0 = \sum_{n=0}^{K\bullet k\bullet(M+m)} \sum_{L=0}^{M-1} \left\{ \sin\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+L)} + \phi_L \right] \right\} \times$$
$$\times \left\{ \sin\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+m)} \right] \right\}$$
$$S_{m,K}^{90} = \sum_{n=0}^{K\bullet k\bullet(M+m)} \sum_{L=0}^{M-1} \left\{ \sin\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+L)} + \phi_L \right] \right\} \times$$
$$\times \left\{ \cos\left[2\cdot\pi\cdot\frac{n}{k\bullet(M+m)} \right] \right\} \qquad (11)$$

Assuming a rectangular window, a spectrum of each signal is of sinx/x type. For $2.4*10^5$ samples, summing up magnitudes of side lobe components, we obtain, for each chosen signal required and 15 other disturbing signals (for M=16 and k=2) with equal amplitude, a highest disturbance level equal to –32.1 dB.

The disturbance level can be also estimated using an envelope of sinx/x spectrum, equal to 1/x. The highest disturbance level obtained by summing up the envelopes of 15 disturbing signals, for similar assumptions as in the previous case, is equal to –30.4 dB. In this case the level is slightly overestimated and 2.3dB higher that in the previous case.

The error described in (11) resulting from spectral leakage can be reduced to a negligible level by adequately increasing the integration time (the disturbance level decreases in direct proportion to the number of samples) and using an efficient time-windowing procedure.

IV. Conclusion

Specific features of multi-frequency coherent signal means that when it is applied it enables the simultaneous measurement of many very weak analogue output signals at one port by their coherent demodulation, the only method that can be used in the case of signals several times smaller than the accompanying noise level.

In this paper, the main factors influencing measurement uncertainty of such signals have been presented.

It has been shown that:

- an extended integration time is of fundamental importance for the uncertainty caused by the simultaneous presence of many signals with different frequencies (spectral leakage),
- for an adequately long integration time, the A/D converter resolution is of fundamental importance for the weak signal measurement uncertainty in the presence of high level of other signals or high noise level at the A/D converter input.

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