Adaptive Beamforming of ESPAR Antenna with Hamiltonian Algorithm

Jun Ozawa[#1], Jun Cheng[#2], Yoichiro Watanabe[#3]

Department of Intelligent Information Eng. & Sci., Doshisha University

Kyoto 610-0321 Japan

[1] joza19@gmail.com, [2] jcheng@ieee.org, [3] watanabe@muc.doshisha.ac.jp

***Abstract*— Adaptive beamforming with Hamiltonian algorithm is proposed for ESPAR antenna. Hamilton's equations describing the motion of particles are used to solve optimization problems. In an application of Hamiltonian algorithm to ESPAR antenna, the cost function for beamforming is considered as the potential energy. Although the kinetic energy, acted by momentums, does not appear in the issue of the beamforming, the momentums help to provide a more possibility to search the global minimum of the cost function. It is shown that ESPAR antenna with Hamiltonian algorithm can steer its beam and null automatically.**

Key words: Hamiltonian algorithm, beamforming, array antenna

I. Introduction

In wireless communications systems, interference becomes a dominant factor in limiting quality and capacity. The adaptive array antenna has gained much attention over the last few years for its ability to increase the performance of wireless systems by effectively suppressing interferences.

It is well known that nearly all-existing antenna array techniques require one receiver chain per branch of antenna. Analog adaptive beamformers, e.g., electronically steerable parasitic array radiator (ESPAR) antenna [1] [2], has only a single-port output. It requires little hardware thus low power consumption, so has shown the potential for application to wireless communications systems.

The (M+1)-element ESPAR antenna has only an active radiator, the central element, connected to the receiver (see Fig. 1). The remaining M elements are parasitic, and are loaded with reactances. The antenna pattern is formed according to the values of the loaded reactances.

Because of the configuration of ESPAR antenna, we face the following three difficulties [1] [2] in the development of optimum algorithms: a) Signals on all elements cannot be observed. Only the single-port output can be observed. b) RF currents on the elements are not independent but mutually coupled with each other. c) The single-port output is a highly nonlinear function of the variable reactances that includes the admittance matrix inverse. In addition, unlike digital beamforming antennas, conventional criteria such as MMSE (Minimum Mean Square Error) are useless for the optimization of ESPAR antenna, since the amplitude of the antenna output is difficult to be adjusted [2].

There have been developed several algorithms for adaptive beamforming of ESPAR antenna. The gradient-based algorithm [2] [3], converges fast but sometimes unwillingly falls into a local minimum depending upon the initial values of reactances. On the other hand, randomized ones, such as the random search algorithm [4], tolerate local-minimum problems but are rather slow to reach the final goal. Ones based on the genetic concept seem to be deterministic at first glance, but they actually involve mutation in a random fashion to escape from local traps [5].

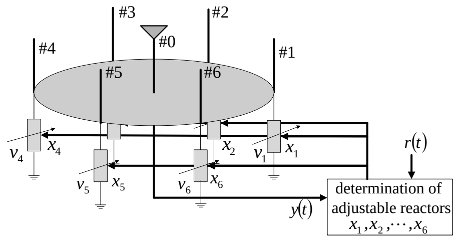


Fig. 1 (M+1)-element ESPAR antenna and its adaptive beamforming.

Hamiltonian algorithm [6] [7] intends to meet the two conflicting requirements, i.e. to be free from local problems, and to be deterministic. Hamiltonian algorithm originally stems from a heuristic idea with autonomous motion of mass in a friction-free potential space. The point of mass moves according to the law of energy conservation, i.e. kinetic energy plus potential energy keeps constant during the motion. In [8], Hamiltonian algorithm was applied to optimization design for ESPAR antenna to find optimal reactances.

In this paper, we further investigate the application of Hamiltonian algorithm to ESPAR antenna. In contrast to the antenna design in [8], we here apply Hamiltonian algorithm to adaptive beamforming for ESPAR antenna. The adaptive beamforming of ESPAR antenna is a non-linear optimization problem. The adaptive beamforming is carried out by finding an optimal reactance vector such that a cost function for ESPAR beamforming is minimal. In our work, the cost function, a normalized mean squared error, is considered as the potential energy in Hamiltonian algorithm. Although the kinetic energy, acted by momentums in Hamiltonian algorithm, does not appear in the issue of antenna beamforming, the momentums play an important role to find an optimal reactance vector. In Hamiltonian algorithm, one of the momentums becomes larger when the cost function towards its local minimum, meanwhile the corresponding reactance gets a larger increment. This helps the cost function escape from the local minimum, thus provides a more possibility to search the global minimum. The simulations show that ESPAR antenna, with Hamiltonian algorithm, can steer its beam and null automatically.

II. SIGNAL MODEL OF ESPAR ANTENNA

This section describes the structure and the signal model of ESPAR antenna [1] [2].

A. ESPAR Antenna

In an (M+1)-element ESPAR antenna (see Fig. 1), the 0-th element is an active radiator located at the centre of a circular ground plane. It is a λ/4-length monopole (where λ is the wavelength) and is excited from the bottom in a coaxial fashion. The remaining elements of λ/4-length monopoles are parasitic radiators surrounding the active radiator symmetrically, with the circle's radius λ/4. Each of these elements is terminated by a variable reactance x_m. The vector $\boldsymbol{x}=(x_1, x_2, ..., x_M)$ is called reactance vector.

The reactance loaded in each of the parasitic element electronically adjusts its element length and makes the monopole element appear as a director or a reflector, as in the Yagi-Uda array antenna [9], depending on the negative or positive value of the reactance. The element appears as an effectively 'shorter' monopole (director) if a negative reactance is loaded, while a positive reactance provides an effectively 'longer' monopole (reflector). The action of these loaded reactances causes a change in radiation pattern [10].

B. Signal Model

We now give a signal model of ESPAR antenna [1] [2]. Suppose there are a total number of Q signals $u_q(t)$ with DoAs ϕ_q, ($q=1,2,\ldots,Q$). The output of ESPAR antenna is

$$y(t)=\sum_{q=1}^{Q}\boldsymbol{w}^{\mathrm{T}}\boldsymbol{a}(\phi_q)u_q(t)+n(t)$$

where $\boldsymbol{a}(\phi_q)$ is the steering vector, $n(t)$ is noise, and the superscript $^{\mathrm{T}}$ is the transpose of vector or matrix. The RF current vector $\boldsymbol{w}$ in elements is called equivalent weight vector, since it acts as a weight vector like conventional array antenna.

The RF current vector is given by [1] [2]

$$\boldsymbol{w}=V_s(\boldsymbol{Z}+\boldsymbol{X})^{-1}\boldsymbol{u}_0$$

where V_s is a constant, and $\boldsymbol{u}_0=(1,0,\ldots,0)^{\mathrm{T}}$. The diagonal matrix $\boldsymbol{X}=\mathrm{diag}(Z_0, jx_1,\ldots, jx_M)$ is called the reactance matrix, and $\boldsymbol{Z}=(z_{kl})_{(M+1)\times(M+1)}$ is referred as to the impedance matrix, with z_{kl} expressing the mutual impedance between the elements k and l ($0\le k,l\le M$). Note that $\boldsymbol{w}$, thus $y(t)$, is a non-linear function of the reactance vector $\boldsymbol{x}$.

III. HAMILTONIAN ALGORITHM FOR OPTIMIZATION

In this section, after reviewing an M-particle problem, we describe Hamilton's equations and Hamiltonian algorithm for optimization.

A. M-Particle Problem and Newton's Equations of Motion

Conservative dynamical systems most often originate through application of Newton's second law which describes the motion of a particle in an applied force field. In a classical M-particle system, several point masses are involved and the force acting on any one particle arises from the presence of neighboring particles or some external fields.

Let the i-th particle be assigned a mass m_i, an instantaneous position $\boldsymbol{x}_i$ (with respect to some appropriate reference frame), and a velocity $\boldsymbol{v}_i$ ($i=1,2,\ldots,M$). Let $\boldsymbol{F}_i$ represent the force acting on the particle i (due, for example, to interactions with the other particles). For the motion in Euclidean space R^{ν}, ($\nu>0$), $\boldsymbol{x}_i$, $\boldsymbol{v}_i$, and $\boldsymbol{F}_i$ all in R^{ν}.

We assume that the force can be obtained as the negative gradient of a potential energy function $U(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_M)$ with respect to the i-th particle position, i.e.

$$\boldsymbol{F}_i=-\frac{\partial U}{\partial \boldsymbol{x}_i}.$$

The M particles move according to Newton's equations [11]

$$\frac{d\boldsymbol{x}_i}{dt}=\boldsymbol{v}_i \quad (1)$$

$$m_i\frac{d\boldsymbol{v}_i}{dt}=\boldsymbol{F}_i,\quad i=1,2,\ldots,M. \quad (2)$$

B. Hamilton's Equations

We now introduce a linear momentum defined by $\boldsymbol{p}_i=m_i\boldsymbol{v}_i$. Then the kinetic energy in this particular system is written as

$$K(\boldsymbol{p}_1,\boldsymbol{p}_2,\ldots,\boldsymbol{p}_M)=\frac{1}{2}\sum_{i=1}^{M}\frac{\boldsymbol{p}_i^2}{m_i}. \quad (3)$$

Hamiltonian function (or just the Hamiltonian) of the system is defined as the sum of potential energy U and kinetic energy

$$H(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_M,\boldsymbol{p}_1,\ldots,\boldsymbol{p}_M)=U(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_M)+K(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_M). \quad (4)$$

Appending M momentums $(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_M)$ extends the νM-dimensional positional space over $(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_M)$ to $2\nu M$-dimensional phase space over

$$(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_M,\boldsymbol{p}_1,\boldsymbol{p}_2,\ldots,\boldsymbol{p}_M).$$

Thus it is easy and useful to generalize Newton's equations (1) and (2) as [11]

$$\frac{d\boldsymbol{x}_i}{dt}=\frac{\partial H}{\partial \boldsymbol{p}_i}, \quad (5)$$

$$\frac{d\boldsymbol{p}_i}{dt}=-\frac{\partial H}{\partial \boldsymbol{x}_i},\qquad i=1,2,\ldots,M \quad (6)$$

which are called Hamilton's equations, and play the key role in dynamics.

Furthermore, we observe that

$$\frac{dH}{dt}=\sum_{i=1}^{M}\frac{\partial H}{\partial \boldsymbol{x}_i}\frac{d\boldsymbol{x}_i}{dt}+\sum_{i=1}^{M}\frac{\partial H}{\partial \boldsymbol{p}_i}\frac{d\boldsymbol{p}_i}{dt}=0.$$

The above observation tells us that the Hamiltonian is a constant E, which is called a total energy. It is obvious that Hamiltonian system is a conservation system of energy.

C. Hamiltonian Algorithm for Optimization

Let's consider optimization problem with Hamiltonian algorithm. In the statement above, the M-particle problem is described in the ν-dimensional Euclidean space R^{ν}. From now on, we consider the problem in a one-dimensional Euclidean space R^{ν}, ($\nu=1$). Thus $\boldsymbol{x}_i$ and $\boldsymbol{p}_i$ are rewritten as scalars x_i, and

p_i. Also, we assume that the mass of every particle is unity ($\boldsymbol{m}_i$ =1) for simplicity.

The purpose of an optimization problem is to find optimal values of augments such that a cost function is minimal. We will see below that Hamilton's equations give a tool to solve the optimization problem.

To map the *M*-dimensional optimization problem to an *M*-particle dynamics system, it is direct that the *M*-dimensional arguments are considered as the instantaneous positions $x_1,\ldots,x_M$, and the cost function as the potential energy $U(x_1,\ldots,x_M)$. By using (4) we rewrite (5) and (6) as

$$\frac{dx_i}{dt} = p_i \quad (7)$$

$$\frac{dp_i}{dt} = -\frac{\partial U}{\partial x_i}, \quad i = 1,2,\ldots,M. \quad (8)$$

Equation (7) means that the incremental value of the instantaneous position x_i is proportional to the value of p_i. Equation (8) shows that updating the momentum makes the cost function *U* towards to its local minimum. When *U* is minimal, the kinetic energy *K* of (3) thus p_i, is maximal since the Hamiltonian is a constant. The above observation tells us that the local minima of the cost function are given in the larger momentums, and but are easily escaped with the larger momentums since at the time the increments of positions are larger. This means that Hamiltonian algorithm is able to search a local minimum, but does not fall into the local minimum. In other word, the algorithm provides a more possibility to search the global minimum.

D. Numerical Method of Hamiltonian Algorithm

We now give the numerical method of Hamiltonian algorithm. Since the first-order derivative of a function can be approximated to a change of function, we have

$$x_i(t+\Delta t) \cong x_i(t) + \Delta t \cdot dx_i(t)/dt$$

$$p_i(t+\Delta t) \cong p_i(t) + \Delta t \cdot dp_i(t)/dt.$$

By inserting (7) and (8) into above equations, we obtain

$$x_i(t+\Delta t) \cong x_i(t) + \Delta t \cdot p_i(t) \quad (9)$$

$$p_i(t+\Delta t) \cong p_i(t) - \Delta t \cdot \nabla_{x_i}(t). \quad (10)$$

where $\nabla_{x_i}(t) \equiv \partial U(x_1(t),\ldots,x_M(t))/\partial x_i$ for compact reason.

A discrete version of above equations could be obtained by taking samples of the solution at equally spaced points in time $t_0+n\Delta t$, for $n = 1, 2, \ldots$. Assume that this is done by a sequential iterative procedure: starting at t_0, and given the initial value $x_i^0 = x_i(t_0)$, we compute the approximations of $x_i^n = x_i(t_0 + n\Delta t)$ and $p_i^n = p_i(t_0 + n\Delta t)$ via (9) and (10). Therefore, the numerical computation of Hamiltonian algorithm is summarized, for $n = 1, 2, \ldots$, as follows.

$$x_i^{n+1} = x_i^n + \Delta t \cdot p_i^n$$

$$p_i^{n+1} = p_i^n - \Delta t \cdot \nabla_{x_i}^n, \quad i = 1,2,\ldots,M$$

with the initial values x_i^0 and p_i^0. Note that p_i^0 is determined by (3) and (4) for a given constant *H*(=*E*) and x_i^0, $i = 1,\ldots,M$.

IV. Beamforming with Hamiltonian Algorithm

We are ready to describe an application of Hamiltonian algorithm to an adaptive beamforming of ESPAR antenna.

First, we give a cost function for ESPAR antenna beamforming. Let the error $\varepsilon(t) = y(t) - r(t)$ be defined as the difference between the actual response of ESPAR antenna *y*(*t*) and the reference signal *r*(*t*) (desired response). Let us turn to the measures as a mean squared error (MSE) or a normalized MSE (NMSE) of the output waveform *y*(*t*) relative to the desired waveform *r*(*t*)

$$MSN(y,r) = \mathrm{E}[\varepsilon(t)\varepsilon^*(t)] = \mathrm{E}\|y(t) - r(t)\|^2$$

$$NMSE(y,r) = NMSE(gy,r) = 1 - |\rho|^2.$$

Here E denotes a expectation operator, *g* is a complex scalar, and

$$\rho = \frac{\mathrm{E}[y(t)r^*(t)]}{\sqrt{\mathrm{E}[y(t)y^*(t)]}\sqrt{\mathrm{E}[r(t)r^*(t)]}}$$

is a cross-correlation coefficient.

Let the *P*-dimensional vector $\boldsymbol{y}(n)$ and $\boldsymbol{r}(n)$ to be the discrete-time samples of the output signal *y*(*t*) and reference signal *r*(*t*). The cost function employed as NMSE is thus

$$U(x_1, x_2, \ldots, x_M) = NMSE(\boldsymbol{y}(n), \boldsymbol{r}(n))$$

$$= 1 - |\rho^2(n)| = 1 - \frac{|\boldsymbol{y}^{\mathrm{H}}(n)\boldsymbol{r}(n)|^2}{\boldsymbol{y}^{\mathrm{H}}(n)\boldsymbol{y}(n) \cdot \boldsymbol{r}^{\mathrm{H}}(n)\boldsymbol{r}(n)} \quad (11)$$

where the superscript $^{\mathrm{H}}$ is the conjugate transpose of matrix. Notice that $y(t)$, thus *U*, is a non-linear function of reactance vector $\boldsymbol{x}=[x_1, x_2, \ldots, x_M]$. Also, for a given radio environment, the cross-correlation coefficient, thus *U*, varies from zero to unity as we control the reactance vector.

It is well known that the cross-correlation coefficient represents the similarity of two signals, while the error represents the difference. For a given radio environment, the normalized cross correlation coefficient ρ, between the output signal *y*(*t*) and the reference signal *r*(*t*), varies over the range [0, 1], as the reactance vector is controlled. The interference signals in the output signal *y*(*t*) are suppressed when *y*(*t*) becomes similar to the reference signal *r*(*t*), regardless of their difference in amplitude. Employing the cross-correlation coefficient avoids the need for an extra amplitude control (e.g., automatic gain control) on *y*(*t*). For ESPAR antenna, this provides an effective solution to the difficulty of adjusting the amplitude of the output signal such that it equals the amplitude of the reference signal.

Second, we describe the application of Hamiltonian algorithm to the beamforming of ESPAR antenna. We employ Hamiltonian algorithm to find the optimal reactance vector $\boldsymbol{x}$ such that the antenna can steer its beam for desired signal and its nulls to interference signals. A direct application of Hamiltonian algorithm to antenna beamforming is just to consider NMSE of (11) as the potential energy *U* in (4). We see that mapping an *M*-dimensional optimal problem to *M*-particle problem in dynamic is simple and direct.

Although the kinetic energy, acted by momentums, does not appear in antenna beamforming, the momentums help us to search the global minimum, as stated in Section III-C.

V. SIMULATIONS

In our simulations, a 7-element (M=6) ESPAR antenna is employed. The signals $u_q(t)$ and reference signal $r(t)$ are in binary phase shift keying (BPSK) modulation. The single-to-noise power ratio is 20 dB. The power of interference signal is the same as the desired signal’s. The data block size for each calculation of NMSE in (11) is taken to be P=50. In this simulation, the number of symbols used for training is

$$P(M+1)N = 50\times(6+1)\times 10000 = 3.5\times 10^{6}$$

because it is required (M+1) time to calculate the gradient values in (10) for i=1,2, …,M.

Let’s first consider the case where there are two signals from different directions 24° and 195°. At the initial step, the value of each reactance is set to zero. After N=10000 iterations, we find the lowest value of the cost function. With the corresponding reactance vector $\boldsymbol{x}$ = [-200 -119 236 67 57 -122] Ω, the beam is steered to 24° of the desired signal, while the deep null is formed towards the interference signal at 195° (see Fig. 2). The output signal-to-interference power ratio (SIR) of 40 dB is obtained. This verifies that Hamiltonian algorithm can steer null adaptively to interference.

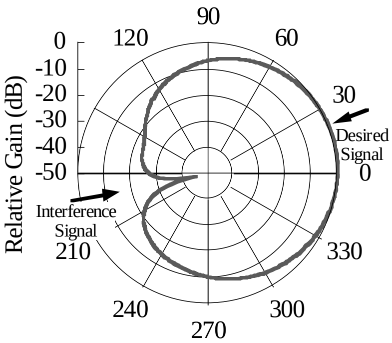
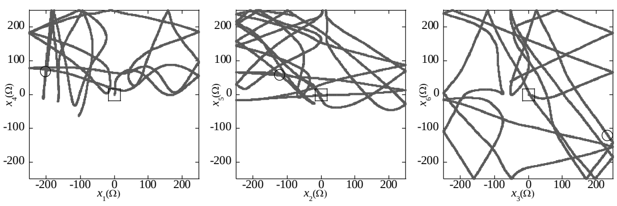


Fig. 2 Adaptive array pattern for SNR=20 dB. DoA: 24° (desired) and 195°.



(a) (b) (c)

Fig. 3 Loci of x_i-x_j during the iterations.
Initial points marked by squares and optimal points marked by cycles.

We now show in Fig. 3 the loci of x_i-x_j in above simulation. In the figures the initial points in iterations are marked by squares, and the optimal points are marked by cycles. For the desired signal at DoA 24° and the interference signal at DoA 195°, according to the action of the loaded reactance discussed in Section II-A, the 1st and 2nd elements should act as directors, loaded by negative reactances, to form beam to the desired signal. The 4th and 5th elements may act as reflectors, loaded by positive reactances. Figure 3 illustrates that loci spread on a larger area, but seem to be limited over a good solution range. For example, we see from Fig. 3(a) that in the search area, the reactance x_1 is frequently ranged over negative values while x_4 over positive values, but do not fall into a local minimum. The same phenomenon can be observed in Fig. 3(b), in which the reactance x_2 is frequently ranged over positive values, and the reactance x_5 is over positive values. In addition, x_3 and x_6 are scatted over all the possible area (see Fig. 3(c)), since the 3rd and 6th elements may not behave directly to steer beam and null at 24° and 195°. The discussion above shows that Hamiltonian algorithm is free from local minima, and provides a more possibility to search the global minimum deterministically. It is known that the gradient-based algorithm sometimes unwillingly falls into a local minimum. The loci given by the random search algorithm is uniformly and randomly distributed on all the reactance space.

VI. CONCLUSION

We have proposed an adaptive beamforming of ESPAR antenna with Hamiltonian algorithm. We employ Hamiltonian algorithm as a tool to solve the non-linear optimization problem. In an application to ESPAR antenna, the cost function, i.e. the normalized mean squared error, is considered as the potential energy. The momentums do not appear in antenna beamforming, but help to provide a more possibility to search the global minimum. We have showed that ESPAR antenna with Hamiltonian algorithm can steer its beam and null automatically.

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