# Analysis of Q-factor of Parallel Plate Resonance between BGA and PCB

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Abstract— The resonances of the electromagnetic field in the space between a BGA and a PCB has been analysed. The equations that give quality factors at resonant frequencies about Model A and Model B have been derived theoretically. In Model A, the space between the BGA and the PCB is shut by setting a lot of shield posts connecting the edges of the BGA to the PCB. Model B is without the shield posts. The equations have been examined in electromagnetic simulation.

Key words: printed circuit board, quality factor, simulation, parallel plate resonance, BGA, electromagnetic radiation

### I. INTRODUCTION

A BGA (ball grid array substrate) is mounted on a PCB (printed circuit board), and there are electric currents on some solder balls between the BGA and the PCB. The electric currents excite the electromagnetic field in the space between the BGA and PCB. The electromagnetic field resonates as parallel plate resonance in the space between the parallel plates that are composed of the BGA and PCB. It generates a coupling noise on the trace on the BGA and the PCB [1-5]. When the quality factor is high, the coupling noise at the resonant frequency is great.

This paper derives the equations that give the quality factors at the resonant frequencies between the BGA and PCB theoretically, and examines the equations by electromagnetic field simulation.

### II. MODELS

## A. Model A

Fig. 1 shows Model A (Model A-1). The BGA is simplified as a copper top plate. The PCB is simplified as an upper insulating layer and a copper ground plane. There is an air space between the top plate and the insulating layer of the PCB. The top plate and the ground plane construct a pair of parallel plates. The solder ball on the BGA is simplified as a conductive post. A lot of conductive posts (shield posts) connect the edges of the top plate to the ground plane, shutting the space between the parallel plates. Port 1 connects the top plate and the ground plane at the point near the edge of the top plate. The current of Port 1 excites the electromagnetic field in the space between the parallel plates. Port 3 connects the top plate and the ground plane at the point sensing the voltage between the parallel plates. Assume the copper top plate is thicker than the skin depth of the skin effect of copper. Assume the insulating layer has a relative dielectric constant  $(\varepsilon_r)$  of 4.5, and a dielectric loss (tan $\delta$ ) of 0.02. The thickness of the insulating layer is t, and the height of the top plate above the ground plane is h.

### B. Model B

Fig. 2 shows Model B (Model B-1). Model B is without the shield posts, and the space between the parallel plates is opened.

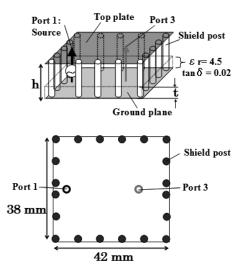


Fig. 1. Model A. BGA on PCB. The space between the top plate and ground plane is shut with shield posts.

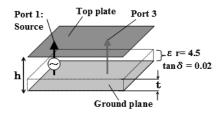


Fig. 2. Model B. The space between the top plate and ground plane is opened without shield posts.

### C. Resonant modes of the models

Fig. 3 shows the current distribution on the top plate at the resonant frequency of TM11 mode and TM21 mode in Model A. Fig. 4 shows the current distribution on the top plate at the

resonant frequency of TM10 mode and TM20 mode in Model B.

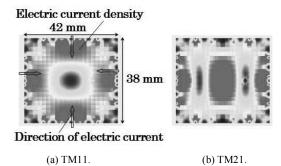


Fig. 3. Electric current distribution on the top plate in Model A.

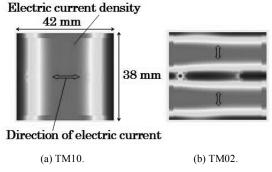


Fig. 4. Electric current distribution on the top plate in Model B.

Resonant frequency  $(f_{mn})$  of  $TM_{mn}$  mode in these models can be described by equation (1):

$$f_{mn} = \frac{c}{2 \bullet \sqrt{\varepsilon_a}} \bullet \sqrt{\left(\frac{m}{x}\right)^2 + \left(\frac{n}{y}\right)^2} \tag{1}$$

where c is the speed of light in vacuum  $3\times108\text{m/sec}$ , x and y are the length of the top plate in the direction of x and y,  $\epsilon_e$  effective relative dielectric constant of the space between the parallel plates. The effective relative dielectric constant  $\epsilon_e$  is obtained from equation (2):

$$\varepsilon_e = \frac{h}{h - t + \frac{t}{\varepsilon}} \tag{2}$$

where t is the thickness of the insulating layer, h height of the top plate above the ground plane, and  $\epsilon_r$  relative dielectric constant of the insulating layer.

According to equation (2),  $\epsilon_e$  changes when the height of the top plate (h) above the ground plane changes.

### III. THEORETICAL ESTIMATION OF QUALITY FACTOR

Unloaded Q-factor  $(Q_0)$  at the resonant frequency of the electromagnetic field between the parallel plates without the air space is expressed by the following equation (3) [6]:

$$\frac{1}{Q_0} = \tan \delta + \frac{1}{h\sqrt{\pi \bullet f \bullet \mu \sigma}}$$
 (3)

where  $\sigma$  is electric conductivity of the parallel plates ( $\sigma$ =5.8×107S/m for copper),  $\tan\delta$  dielectric loss of the insulating layer,  $\mu$  permeability of vacuum 1.26 $\mu$ V • sec/(A • m).

The first term of equation (3) represents the effect of the dielectric loss, and the second term the effect of the resistance of the parallel plates. The Q-factor does not vary with the resonant mode but with the resonant frequency.

# A. Derivation of the equation that gives Q-factor of Model A

Model A is different from the base model of equation (3) since Model A has the air space between the parallel plates. The space between the parallel plates consists of two layers. The first layer is the air layer in which thickness is (h-t) and tan  $\delta$  =0. The second layer is insulating layer in which thickness is t and tan $\delta$ >0 and relative dielectric constant  $\epsilon_r$ . Unloaded Q-factor (Q<sub>0</sub>) of Model A is described by the following equation (4):

$$\frac{1}{Q_0} = \frac{1}{Q_e} + \frac{1}{h\sqrt{\pi \bullet f \bullet \mu\sigma}} \tag{4}$$

In equation (4), the first term  $(1/Q_e)$  represents the effect of the dielectric loss of the insulating layer which is different from the base model of equation (3), and the second term represents the effect of the resistance of the parallel plates.

The first term  $(1/Q_e)$  is given by the following equation: (power loss in the dielectric substance)/ $\omega$ /(the electromagnetic field energy in the cavity) and described as follows:

$$\frac{1}{Q_e} = \frac{\omega \varepsilon_0 \varepsilon_r \tan \delta \bullet E_t^2 \bullet t}{\omega \bullet C_a \bullet V_p^2} = \frac{\varepsilon_0 \varepsilon_r \tan \delta (D/(\varepsilon_0 \varepsilon_r))^2 t}{C_a \bullet (h \bullet D/(\varepsilon_0 \varepsilon_e))^2}$$
(5)

where Et is an electric field density that is in the insulating layer, Ca is capacitance per unit area of the parallel plates, and  $Ca = \varepsilon_0 * \varepsilon_e / h$ ,  $V_p$  is voltage between the parallel plates, D is dielectric flux density between the parallel plates,  $\varepsilon_e$  is effective relative dielectric constant given by equation (2).

Since  $Ca = \varepsilon_0 * \varepsilon_e / h$ , the equation (5) is changed as follows:

$$\frac{1}{Q_e} = \tan \delta \bullet \frac{t \bullet \varepsilon_e}{h \bullet \varepsilon_r} \tag{6}$$

Equation (6) is substituted into the first term of equation (4).

# B. Derivation of the equation that gives Q-factor of Model B

The equation that gives the quality factor of Model B can be shown as follows:

$$\frac{1}{Q_0} = \frac{1}{Q_e} + \frac{1}{h\sqrt{\pi \bullet f \bullet \mu \sigma}} + \frac{1}{Q_r}$$
 (7)

In equation (7), the third term  $(1/Q_r)$  represents the effect of the loss of the electromagnetic radiation from the space between the parallel plates which is opened to outside. The third term  $(1/Q_r)$  is give by the following equation [8]:

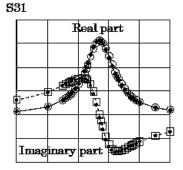
$$\frac{1}{Q_r} = \frac{4.8 \bullet h}{3 \varepsilon_e \lambda_0} \tag{8}$$

where  $\lambda 0$  is the wavelength of the electromagnetic field in the vacuum at the resonant frequency,  $\epsilon_e$  effective relative dielectric constant given by equation (2). The proportional factor in equation (8) was obtained by fitting equation (8) to simulation.

### IV. QUALITY FACTOR OBTAINED FROM SIMULATION

Electromagnetic field simulator Sonnet which is the method of moment was used for simulation. The quality factor  $(Q_0)$  and the resonant frequency  $f_0$  are obtained by fitting the S-parameters at Port 3 calculated using equation (9) [6-7] to the S-parameters obtained from simulation.

$$S_{31} = \frac{A}{1 + jQ_0 \left(\frac{f}{f_0} - \frac{f_0}{f}\right)} \tag{9}$$



4.92 4.94 4.96 4.98 5.00 5.02 f(GHz)

Fig. 5 S-parameters at Port 3 in Model A. Solid and broken lines; simulation.  $\bigcirc$ s,  $\square$ s; equation 9.

Fig. 5 shows the real part and imaginary part of S-parameters S31 at Port 3 for Model A as a function of frequency. In Fig. 5, the solid line and dotted line denote the real part and imaginary part of S31 obtained from simulation. The sign  $\bigcirc$  and  $\square$  denote the real part and imaginary part of S31 calculated using equation (9). The coefficients A,  $Q_0$  and  $f_0$  in equation (9) were adjusted to fit the S-parameters calculated using equation (9) to the S-parameters got by simulation. They fit very well. The quality factors of the simulation is the coefficient  $Q_0$  of equation (9).

Fig. 6 shows quality factors at various resonant frequencies for Model A and Model B. In Fig. 6, the solid lines denote the quality factors obtained from the simulation in Model A-1 and Model A-2 with the shield posts. The dotted line denotes the quality factors obtained from the simulation in Model B-1 without the shield posts.  $\bigcirc$  signs denote the quality factors calculated using equation (4) and (7).

In Model A-1 and Model B-1, there is the air space between the parallel plates, and the thickness t of the insulating layer is 50 microns, the height h of the top plate above the ground plane is 250 microns, the space between the top plate and the insulating layer of the PCB is filled with air.

In Model A-2, the space between the parallel plates is filled with the insulating material whose relative dielectric constant  $\varepsilon_r$  and  $\tan\delta$  is same as the insulating layer of the PCB.

The left point of the graph of Model A-1 (a) shows the quality factor at the resonant frequency of TM11 mode. The left point of the graph of Model B-1 (b) shows the quality factor at the resonant frequency of TM10 mode. The right point of the graph of Model A-1 shows the quality factor at the resonant frequency of TM21 mode. The right point of the graph of Model B-1 shows the quality factor at the resonant frequency of TM02 mode.

In Fig. 6, the quality factors obtained from the simulation agree well with the quality factors calculated using equation (4) and (7).

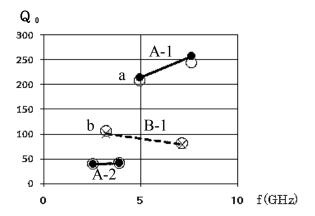


Fig. 6. Quality factors for Model A and Model B at various resonant frequencies. Solid line and broken line; quality factors obtained from simulation.  $\bigcirc$ s; quality factors calculated using equation 4 and 7.

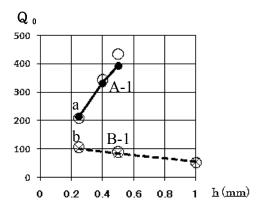


Fig. 7. Quality factor  $(Q_0)$  at a resonant mode in the model that is filled with air between the top plate and the insulating layer. Solid line; quality factor from simulation at TM11 mode in Model A-1. Broken line; quality factor from simulation at TM10 mode in Model B-1.  $\bigcirc$ s; quality factors calculated using equation 4 and 7.

Fig. 7 shows the quality factor at the resonant frequency of TM11 mode for Model A-1 and the quality factor at the resonant frequency of TM10 mode for Model B-1 as a function of the height h of the top plate above the ground plane. In both models, the spaces between the top plate and the insulating layer of the PCB are filled with air. The point (a) and (b) in Fig. 7 represent the point (a) and (b) in Fig. 6.

The solid line in Fig. 7 denotes the quality factor obtained from the simulation at the resonant frequency of TM11 mode

in Model A-1 with the shield posts. The broken line denotes the quality factor obtained from the simulation at the resonant frequency of TM10 mode in Model B-1 without the shield posts.  $\bigcirc$  s denote those quality factors calculated using equation (4) and (7).

Fig. 7 shows that the quality factor in Model A-1 with the shield posts grows as the height h rises. The quality factor in Model B-1 without the shield posts is much smaller than the quality factor in Model A-1. In Model B-1, the loss of the electromagnetic radiation is large and essential factor in determining the quality factor.

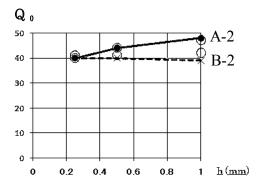


Fig. 8 Quality factor  $(Q_0)$  at a resonant mode in the model that is filled with insulating material between the top plate and the insulating layer. Solid line; quality factor from simulation at TM11 mode in Model A-2. Broken line; quality factor from simulation at TM10 mode in Model B-2.  $\bigcirc$ s; quality factors calculated using equation 4 and 7.

Fig. 8 shows the quality factor at the resonant frequency of TM11 mode for Model A-2 and the quality factor at the resonant frequency of TM10 mode for Model B-2 as a function of the height h of the top plate above the ground plane. In both models, the spaces between the top plate and the PCB are filled with insulating material.

The solid line in Fig. 8 denotes the quality factor obtained from the simulation at the resonant frequency of TM11 mode in Model A-2 with the shield posts. The broken line denotes the quality factor obtained from the simulation at the resonant frequency of TM10 mode in Model B-2 without the shield posts.  $\bigcirc$  s denote those quality factors calculated using equation (4) and (7).

In Model A-2 and Model B-2, the quality factors are much smaller than the quality factors in Model A-1 and Model B-1,

and the loss in the insulating material filled between the parallel plates is essential factor in determining the quality factor

### V. CONCLUSION

The quality factors in Model A with the shield posts are high when the space between the top plate and the PCB is filled with air. The quality factors in Model A become low when the space is filled with lossy insulating material and the loss in the insulating material is essential factor in determining the quality factors.

The quality factors in Model B without the shield posts are lower than the quality factors in Model A, and the loss of the electromagnetic radiation is essential factor in determining the quality factors when the space between the top plate and the PCB is filled with air.

The equations that give the quality factors at various resonant frequencies have been derived theoretically for Model A and Model B. The quality factors calculated using the equations agree well with the quality factors obtained from the electromagnetic field simulation.

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