Regime of Coupled Electromagnetic TE-TM Wave Propagation in a Plane Layer Waveguide with Kerr Nonlinearity

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***Abstract*—Coupled electromagnetic TE-TM wave propagation in a nonlinear dielectric layer filled with lossless, nonmagnetic, and isotropic medium is considered. The layer is located between two half-spaces with constant permittivities. The permittivity in the layer is described by Kerr law. The problem is reduced to the nonlinear two-parameter eigenvalue problem. We look for coupled eigenvalues of the problem and reduce the question to the analysis of the corresponding system of two dispersion equations. It is shown that the new waveguide regime for coupled TE and TM wave exists in a layer with Kerr nonlinearity.**

I. Introduction

Problems of electromagnetic wave propagation in nonlinear waveguide structures are intensively investigated during several decades. Propagation of electromagnetic waves in a layer and a circle cylindrical waveguide are among such problems.

Here we consider simultaneous electromagnetic TE-TM wave propagation in a layer with Kerr nonlinearity. It is well known that in a linear case two types of waves (TE and TM) propagate independently and do not affect each other. It turns out that in a layer with Kerr nonlinearity the situation is different. TE and TM waves can propagate in the layer with Kerr nonlinearity each with its own propagation constant (γ_E and γ_M respectively) and affect each other. However this nonlinear interaction between TE and TM waves does not destroy the waves and they continue to propagate. It should be noticed that this electromagnetic problem can be formulated as a nonlinear two-parameter eigenvalue problem. It is known that coupled TE-TM wave propagation in a Kerr layer can be considered (see [1] where physical treatment of the problem is given). However in [1] there are neither strict mathematical statement of the problem no strict mathematical results about coupled TE-TM wave existence.

There are a lot of studies where problems of propagation TE and TM waves in nonlinear layers are investigated separately. For TE waves and Kerr nonlinearity see, for example, [3], [4], [6]; general method for TE waves and arbitrary nonlinearity see in [6]; for TM waves and Kerr nonlinearity see, for example [6]; general method for TM waves and arbitrary nonlinearity see in [6], [8]. It has been shown that there are new eigenmodes in each of TE [6], [7] and TM [6], [5] cases and Kerr nonlinearity.

II. Statement of the Problem

Let us consider electromagnetic waves propagating through a nonlinear homogeneous isotropic nonmagnetic dielectric layer. The permittivity inside the layer is described by Kerr law. The layer is located between two half-spaces: $x < -h$ and $x > h$ in Cartesian coordinate system $Oxyz$. The half-spaces are filled with isotropic nonmagnetic media without any sources and characterized by permittivities $\varepsilon_1 \geq \varepsilon_0$ and $\varepsilon_3 \geq \varepsilon_0$, respectively, where ε_0 is the permittivity of free space. Everywhere below $\mu = \mu_0$ is the permeability of free space.

The electromagnetic field depends on time harmonically [2]

$$\tilde{\mathbf{E}}(x,y,z,t) = \mathbf{E}_+(x,y,z)\cos\omega t + \mathbf{E}_-(x,y,z)\sin\omega t,$$
$$\tilde{\mathbf{H}}(x,y,z,t) = \mathbf{H}_+(x,y,z)\cos\omega t + \mathbf{H}_-(x,y,z)\sin\omega t,$$

where ω is the circular frequency; $\mathbf{E}_+$, $\mathbf{E}_-$, $\mathbf{H}_+$, $\mathbf{H}_-$ are real functions.

Form complex amplitudes of the electromagnetic field [2]:

$$\mathbf{E} = \mathbf{E}_+ + i\mathbf{E}_-, \quad \mathbf{H} = \mathbf{H}_+ + i\mathbf{H}_-,$$

where $\mathbf{E} = (E_x, E_y, E_z)^T$, $\mathbf{H} = (H_x, H_y, H_z)^T$; $(\ \cdot\ )^T$ denotes the operation of transposition; and each component of the field is a function of three spatial variables.

Complex amplitudes $\mathbf{E}, \mathbf{H}$ satisfy Maxwell's equations

$$\begin{cases} \operatorname{rot}\mathbf{H} = -i\omega\varepsilon\mathbf{E} \\ \operatorname{rot}\mathbf{E} = i\omega\mu\mathbf{H}, \end{cases} \tag{1}$$

the continuity condition for the tangential field components on the media interfaces $x = -h$, $x = h$ and the radiation condition at infinity: the electromagnetic field exponentially decays as $|x| \to \infty$ in the domains $x < -h$, $x > h$.

The permittivity has the form

$$\varepsilon = \begin{cases} \varepsilon_3, & x > h \\ \varepsilon_2 + \alpha|\mathbf{E}|^2, & -h < x < h \\ \varepsilon_1, & x < -h, \end{cases}$$

where ε_1, ε_2, ε_3, and α are arbitrary real constants.

Solutions to Maxwell's equations are sought for in the entire space.

Let us give short background about purely TE and TM waves.

Complex amplitudes $\mathbf{E}, \mathbf{H}$ can be written in the following form

$$\begin{array}{rcccc} \mathbf{E} & = & (0, E_y, 0)^T & + & (E_x, 0, E_z)^T \\ \mathbf{H} & = & \underbrace{(H_x, 0, H_z)^T}_{\text{TE waves}} & + & \underbrace{(0, H_y, 0)^T}_{\text{TM waves}} \end{array}.$$

Let us consider TE and TM waves separately.

For TE waves propagating in the structure (Fig. 1) we have $\mathbf{E} = (0, E_y, 0)^T$, $\mathbf{H} = (H_x, 0, H_z)^T$, where $E_y \equiv E_y(x, y, z)$, $H_x \equiv H_x(x, y, z)$, $H_z \equiv H_z(x, y, z)$.

In this case it can be proved that the components E_y, H_x, H_z do not depend on y. In addition, waves propagating along the boundaries z depend harmonically on z. So we obtain that the components have the form

$$E_y = \mathrm{E}_y(x)e^{i\gamma_E z},\ H_x = \mathrm{H}_x(x)e^{i\gamma_E z},\ H_z = \mathrm{H}_z(x)e^{i\gamma_E z},$$

where γ_E is the spectral parameter of the problem (propagation constant).

The same conclusion can be made for TM case. Indeed for TM waves propagating in the structure (Fig. 1) we have $\mathbf{E} = (E_x, 0, E_z)^T$, $\mathbf{H} = (0, H_y, 0)^T$, where $E_x \equiv E_x(x, y, z)$, $E_z \equiv E_z(x, y, z)$, $H_y \equiv H_y(x, y, z)$.

In this case it can be proved that the components E_x, E_z, H_y do not depend on y. In addition, waves propagating along the boundaries z depend harmonically on z. So we obtain that the components have the form

$$E_x = \mathrm{E}_x(x)e^{i\gamma_M z},\ E_z = \mathrm{E}_z(x)e^{i\gamma_M z},\ H_y = \mathrm{H}_y(x)e^{i\gamma_M z},$$

where γ_M is the spectral parameter of the problem (propagation constant).

It is supposed that $\operatorname{Im}\gamma_E = 0$ and $\operatorname{Im}\gamma_M = 0$. This implies that $|\mathbf{E}|$ does not depend on z.

For each problem we look for surface waves propagating along boundaries of the layer. From mathematical standpoint the problem is to determine values of the propagation constant, which correspond to the surface waves (until now we considered TE and TM cases separately).

Keeping all the conclusions for the TE and TM cases we consider simultaneous TE-TM wave propagation in the nonlinear layer. There is no coupled regime if the permittivity in the layer is a constant. However the nonlinear permittivity leads to couple both types of waves.

Taking into account what we obtained for TE and TM cases consider electromagnetic field

$$\mathbf{E} = (E_x, E_y, E_z)^T, \quad \mathbf{H} = (H_x, H_y, H_z)^T, \tag{2}$$

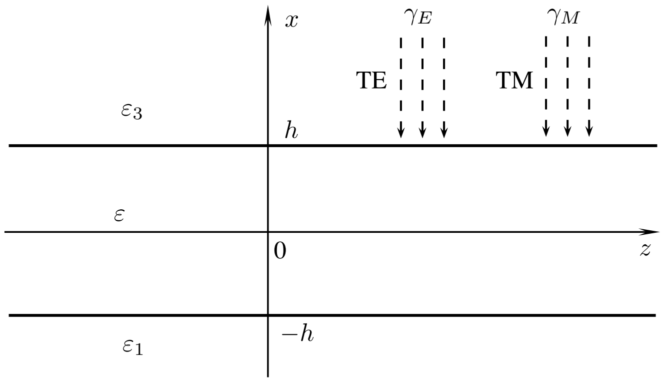

Fig. 1. Geometry of the problem

where

$$E_x = \mathrm{E}_x(x)e^{i\gamma_M z},\ E_y = \mathrm{E}_y(x)e^{i\gamma_E z},\ E_z = \mathrm{E}_z(x)e^{i\gamma_M z},$$
$$H_x = \mathrm{H}_x(x)e^{i\gamma_E z},\ H_y = \mathrm{H}_y(x)e^{i\gamma_M z},\ H_z = \mathrm{H}_z(x)e^{i\gamma_E z}. \tag{3}$$

It should be noticed that we consider $\gamma_E \neq \gamma_M$. Indeed, if we substitute fields (2) into Maxwell's equations (1) we can split the system into two subsystems and each subsystem depends only on γ_E or γ_M, respectively.

Thereby the problem is to determine coupled propagation constants (γ_E, γ_M) and corresponding functions, which describe electromagnetic field (2) in the waveguide shown in Fig. 1. It is supposed that fields (2) satisfy Maxwell's equations (1), transmission conditions at the interfaces $x = -h$ and $x = h$, and the radiation condition at infinity. The components of fields (2) have form (3).

III. Differential Equations of the Problem

Denote by $(\,\cdot\,)' \equiv \partial/\partial x$. Substituting fields (2) into system (1) we obtain

$$\left\{ \begin{array}{l} (i\omega\mu)^{-1}(i\gamma_M^2 \mathrm{E}_x - \gamma_M \mathrm{E}_z') = \omega\varepsilon \mathrm{E}_x \\ (i\omega\mu)^{-1}(-\gamma_E^2 \mathrm{E}_y + \mathrm{E}_y'') = i\omega\varepsilon \mathrm{E}_y \\ (i\omega\mu)^{-1}(i\gamma_M \mathrm{E}_x' - \mathrm{E}_z'') = -i\omega\varepsilon \mathrm{E}_z. \end{array} \right. \tag{4}$$

Normalizing system (4) according to the formulae $\tilde{x} = k_0 x$, $\frac{d}{dx} = k_0 \frac{d}{d\tilde{x}}$, $\tilde{\gamma}_E = \frac{\gamma_E}{k_0}$, $\tilde{\gamma}_M = \frac{\gamma_M}{k_0}$, $\tilde{\varepsilon}_j = \frac{\varepsilon_j}{\varepsilon_0}$, $\tilde{\alpha} = \frac{\alpha}{\varepsilon_0}$, where $k_0^2 = \omega^2 \varepsilon\mu$ we obtain

$$\left\{ \begin{array}{l} \tilde{\gamma}_M k_0^2 (\tilde{\gamma}_M (iE_x) - E_z') = \omega^2 \mu\varepsilon_0 \tilde{\varepsilon}(iE_x) \\ \tilde{\gamma}_E^2 k_0^2 E_y - k_0^2 E_y'' = \omega^2 \mu\varepsilon_0 \tilde{\varepsilon} E_y \\ \tilde{\gamma}_M k_0^2 (iE_x)' - k_0^2 E_z'' = \omega^2 \mu\varepsilon_0 \tilde{\varepsilon} E_z. \end{array} \right. \tag{5}$$

Introduce the notation $iE_x \equiv X$, $E_y \equiv Y$, $E_z \equiv Z$ and omitting the tilde we obtain

$$\left\{ \begin{array}{l} \gamma_M(\gamma_M X - Z') = \varepsilon X \\ \gamma_E^2 Y - Y'' = \varepsilon Y \\ \gamma_M X' - Z'' = \varepsilon Z, \end{array} \right. \tag{6}$$

where

$$\varepsilon = \begin{cases} \varepsilon_1, & x < -h \\ \varepsilon_2 + \alpha(X^2 + Y^2 + Z^2), & -h < x < h \\ \varepsilon_3, & x > h. \end{cases}$$

System (6) is the basic system, which we study.

Introduce the notation $k_{E1}^2 = \gamma_E^2 - \varepsilon_1$, $k_{E3}^2 = \gamma_E^2 - \varepsilon_3$, $k_{M1}^2 = \gamma_M^2 - \varepsilon_1$, $k_{M3}^2 = \gamma_M^2 - \varepsilon_3$.

System (6) for the half-spaces $x < -h$ and $x > h$ is linear. Its solutions have the form (according to the condition at infinity)

for $x < -h$:

$$\begin{cases} X(x) = C_1^{(-h)} e^{(x+h)k_{M1}} \\ Y(x) = C_2^{(-h)} e^{(x+h)k_{E1}} \\ Z(x) = \gamma_M^{-1} k_{M1} C_1^{(-h)} e^{(x+h)k_{M1}}; \end{cases} \tag{7}$$

for $x > h$:

$$\begin{cases} X(x) = C_1^{(h)} e^{-(x-h)k_{M3}} \\ Y(x) = C_2^{(h)} e^{-(x-h)k_{E3}} \\ Z(x) = -\gamma_M^{-1} k_{M3} C_1^{(h)} e^{-(x-h)k_{M3}}, \end{cases} \tag{8}$$

where $C_1^{(-h)}$, $C_2^{(-h)}$, $C_1^{(h)}$, and $C_2^{(h)}$ are constants of integration (for further details see the next section).

Inside the layer $-h < x < h$ system (6) takes the form

$$\begin{cases} \gamma_M(\gamma_M X - Z') = (\varepsilon_2 + \alpha(X^2+Y^2+Z^2))X, \\ \gamma_E^2 Y - Y'' = (\varepsilon_2 + \alpha(X^2+Y^2+Z^2))Y, \\ \gamma_M X' - Z'' = (\varepsilon_2 + \alpha(X^2+Y^2+Z^2))Z. \end{cases} \tag{9}$$

IV. Transmission Conditions and Mathematical Formulation of the Problem

Tangential components of electromagnetic field are known to be continuous at the interfaces. In the case tangential components are E_y, E_z, H_y, and H_z. It is easy to see from system (9) that continuity of H_y implies continuity of $Z' - \gamma_M X$ at the interfaces. It follows from above that the transmission conditions for the functions X, Y, Y', Z have the form

$$\begin{array}{ll} [Z' - \gamma_M X]|_{x=-h} = 0, & [Z' - \gamma_M X]|_{x=h} = 0, \\ [Y]|_{x=-h} = 0, & [Y]|_{x=h} = 0, \\ [Y']|_{x=-h} = 0, & [Y']|_{x=h} = 0, \\ [Z]|_{x=-h} = 0, & [Z]|_{x=h} = 0, \end{array} \tag{10}$$

where $[f]|_{x=x_0} = \lim\limits_{x\to x_0-0} f(x) - \lim\limits_{x\to x_0+0} f(x)$.

It should be noticed that the constants $C_1^{(h)}$, $C_2^{(h)}$ are supposed to be known (initial conditions). In this way we have 8 unknowns quantities: 2 constants $C_1^{(-h)}$, $C_2^{(-h)}$ in the half-space $x < -h$; 4 constants inside the layer (constants of integration of system (9)) and 2 propagation constants γ_E, γ_M. Transmission conditions (10) contain 8 equations also.

Definition 1: The pair (γ_E, γ_M) is called coupled eigenvalues if nontrivial functions X, Y, Z exist that are described by formulae (7), (8) in the half-spaces $h < -h$ and $x > h$, respectively; inside the layer they are solutions to equations (6); they also satisfy transmission conditions (10). The functions X, Y, Z are called eigenfunctions.

The main problem (*problem P*) is to prove existence of coupled eigenvalues.

Denote boundary values of the functions X, Y, Y', Z inside the layer by

$$\begin{array}{ll} X_{-h} := X(-h+0), & X_h := X(h-0) \\ Y_{-h} := Y(-h+0), & Y_h := Y(h-0), \\ Y'_{-h} := Y'(-h+0), & Y'_h := Y'(h-0) \\ Z_{-h} := Z(-h+0), & Z_h := Z(h-0). \end{array}$$

For the boundary values of the functions X, Y, Y', Z in the half-spaces $x < -h$, $x > h$ we obtain

$$\begin{array}{ll} X(-h-0) = C_1^{(-h)}, & X(h+0) = C_1^{(h)}, \\ Y(-h-0) = C_2^{(-h)}, & Y(h+0) = C_2^{(h)}, \\ Y'(-h-0) = k_{E1} C_2^{(-h)}, & Y'(h+0) = -k_{E3} C_2^{(h)}, \\ Z(-h-0) = \gamma_M^{-1} k_{M1} C_1^{(-h)}, & Z(h+0) = -\gamma_M^{-1} k_{M3} C_1^{(h)}. \end{array}$$

From the transmission conditions and the latter formulae we obtain

$$\begin{array}{lll} Y_{-h} = C_2^{(-h)}, & Y'_{-h} = k_{E1} C_2^{(-h)}, & Z_{-h} = \gamma_M^{-1} k_{M1} C_1^{(-h)}, \\ Y_h = C_2^{(h)}, & Y'_h = -k_{E3} C_2^{(h)}, & Z_h = -\gamma_M^{-1} k_{M3} C_1^{(h)}. \end{array} \tag{11}$$

V. Dispersion Equations

It can be proved that the DEs can be written in the form

$$C_2^{(h)} g_E(h, \gamma_E) = \alpha \frac{Q_E(h, \gamma_E, \gamma_M)}{\sin 2k_E h}, \tag{12}$$

$$C_1^{(h)} k_M g_M(h, \gamma_M) = \alpha \frac{Q_M(h, \gamma_M, \gamma_E)}{\sin 2k_M h}, \tag{13}$$

where

$$g_E(h, \gamma_E) = \\ = \left(k_E^2 - k_{E1}k_{E3}\right) \sin 2k_E h - k_E \left(k_{E1} + k_{E3}\right) \cos 2k_E h,$$

$$g_M(h, \gamma_M) = \\ = \left(\varepsilon_1\varepsilon_3 k_M^2 - \varepsilon_2^2 k_{M1}k_{M3}\right) \sin 2k_M h - \\ - \varepsilon_2 k_M \left(\varepsilon_1 k_{M3} + \varepsilon_3 k_{M1}\right) \cos 2k_M h,$$

and

$$Q_E(h, \gamma_E, \gamma_M) = \\ = (k_{E1} \cos 2k_E h - k_E \sin 2k_E h) \int_{-h}^{h} f_Y(x) \cos k_E(x+h) dx - \\ - k_{E1} \int_{-h}^{h} f_Y(x) \cos k_E(x-h) dx,$$

$$Q_M(h,\gamma_M,\gamma_E)=$$
$$=f_X(h-0)\sin 2k_M h\frac{\varepsilon_1 k_M\sin 2k_M h-\varepsilon_2 k_{M1}\cos 2k_M h}{(-2\gamma_M^2)^{-1}}+$$
$$+k_M\gamma_M(\varepsilon_1 k_M\sin 2k_M h-\varepsilon_2 k_{M1}\cos 2k_M h)\times$$
$$\times\int_{-h}^{h}\left[\frac{\sin k_M(x+h)}{k_M^{-1}}f_Z(x)-\frac{\cos k_M(x+h)}{\gamma_M^{-1}}f_X(x)\right]dx-$$
$$-2\gamma_M^2\varepsilon_2 k_{M1}f^X(-h+0)\sin 2k_M h+\varepsilon_2 k_{M1}k_M\gamma_M\times$$
$$\times\int_{-h}^{h}\left[\frac{\sin k_M(x-h)}{k_M^{-1}}f_Z(x)-\frac{\cos k_M(x-h)}{\gamma_M^{-1}}f_X(x)\right]dx,$$

where

$$f_X(x)=(X^2(x)+Y^2(x)+Z^2(x))X(x),$$
$$f_Y(x)=(X^2(x)+Y^2(x)+Z^2(x))Y(x),$$
$$f_Z(x)=(X^2(x)+Y^2(x)+Z^2(x))Z(x).$$

If we chose $\alpha=0$ we obtain equations $g_E(h,\gamma_E)=0$ and $g_M(h,\gamma_M)=0$ for linear TE and TM cases respectively.

Then the following theorem can be proved [9], [10].

Theorem 1: Let a pair $(\tilde{\gamma}_E,\tilde{\gamma}_M)$ be a solution of equations $g_E(h,\gamma_E)=0$ and $g_M(h,\gamma_M)=0$. It is possible to choose sufficiently small α such that in the vicinity of $(\tilde{\gamma}_E,\tilde{\gamma}_M)$ a pair $(\hat{\gamma}_E,\hat{\gamma}_M)$ exists and this pair is a solution of problem P.

Estimation for the value α is given in [9], [10].

Let us explain briefly in what way formulae (12), (13) were obtained. System (9) can be rewritten as

$$\begin{cases} X=-k_M^{-2}\left(\gamma_M Z'+\alpha\left(X^2+Y^2+Z^2\right)X\right),\\ Y''+k_E^2 Y=-\alpha f_1,\\ Z''+k_M^2 Z=-\alpha f_2.\end{cases}\qquad(14)$$

We are going to invert linear parts of the second and the third equations in (14). Let $L_1=\frac{d^2}{dx^2}+k_E^2$, $L_2=\frac{d^2}{dx^2}+k_M^2$. We construct Green's functions for the boundary problems

$$\begin{cases} L_1G_1=-\delta\left(x-s\right),\\ \partial_x G_1|_{x=-h}=0,\\ \partial_x G_1|_{x=h}=0;\end{cases}\quad\text{and}\quad\begin{cases} L_2G_2=-\delta\left(x-s\right),\\ G_2|_{x=-h}=0,\\ G_2|_{x=h}=0.\end{cases}$$

It can be proved that the Green functions have the forms

$$G_1\left(x,s\right)=\begin{cases}-\frac{\cos k_E(x+h)\cos k_E(s-h)}{k_E\sin 2k_E h},\ x<s\le h\\ -\frac{\cos k_E(x-h)\cos k_E(s+h)}{k_E\sin 2k_E h},\ s<x\le h;\end{cases}\qquad(15)$$

$$G_2\left(x,s\right)=\begin{cases}-\frac{\sin k_M(x+h)\sin k_M(s-h)}{k_M\sin 2k_M h},\ x<s\le h\\ -\frac{\sin k_M(x-h)\sin k_M(s+h)}{k_M\sin 2k_M h},\ s<x\le h.\end{cases}\qquad(16)$$

Then we can rewrite system (14) as a system of integral equations (nonlinear). Then we rewrite it as an operator equation. For this operator equation we can prove that it has unique solution for sufficiently small α. This solution is continuous w.r.t. both γ_E and γ_M. Then using conditions (10) we obtain DEs (12), (13).

VI. Conclusion

Results of the propagation of linear/nonlinear purely TE or TM waves in a layer are well known. For linear waves in the layer it is known that any electromagnetic guided wave can be represent as a superposition of TE and TM waves. It is also well known that in the linear case there is no interaction between TE and TM waves in the layer. As we know [1], [6] nonlinear purely TE or TM waves propagate in a layer with Kerr nonlinearity. However in this case there is senseless to consider a superposition of nonlinear waves (solutions to the Maxwell equations) in order to represent other nonlinear wave (other solution to the Maxwell equations).

After all, it is possible to look for new solutions to the Maxwell equations and new propagation regimes in nonlinear media. One of such regimes is coupled TE-TM wave propagation [1]. It should be noticed that coupled TE-TM waves exist in nonlinear media only. In this work we prove that coupled TE-TM wave exists in a layer with Kerr nonlinearity.

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