Electromagnetic interaction with exotic uniaxial media

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***Abstract*—The interface between an isotropic medium and a general uniaxial anisotropic medium is considered in different particular cases. A new boundary condition for the electromagnetic field is studied. It requires the cancellation of both the electric and the magnetic fields along a generic direction. The realization of this boundary condition is proposed by means of an exotic uniaxial medium: the properties of such medium are investigated. Possible applications of such media are analyzed.**

I. INTRODUCTION

In the last years many exotic boundary conditions have been proposed in the literature, e.g., the DB boundary condition, and the Perfect ElectroMagnetic Conductor (PEMC) [1]. Past years have witnessed a great activity in metamaterials research [2], which has provided several potential realizations for such type of boundary conditions. Two kinds of materials seem to be suitable for this purpose: the bi-anisotropic and the anisotropic materials, especially the so-called extreme-parameter materials, in other words such anisotropic materials for which some of the principal material parameter components are either very large or very small. Here, with the word "realization," we are not referring to the practical realization of the material or of the interface, but only to the possibility to obtain the wanted condition by a particular homogeneous medium. The boundary condition, in fact, is only a requirement on the electromagnetic field and it is not possible a priori to predict whether a physical medium exists which can mimic it. Once the characteristics of the medium capable to emulate the boundary condition are known, the next step is to face the practical realization of the material in terms of artificial composites. For studies focusing in this direction, see [3]–[6].

For instance, the DB boundary condition on a plane interface is defined as follows

$$\hat{\boldsymbol{u}}_z \cdot \boldsymbol{D} = 0 \qquad \text{and} \qquad \hat{\boldsymbol{u}}_z \cdot \boldsymbol{B} = 0 \tag{1}$$

where $\hat{\boldsymbol{u}}_z$ is the unit vector perpendicular to the interface. One possible realization of this boundary has been proposed in [1], with an anisotropic medium having the perpendicular components to the interface going to zero:

$$\hat{\boldsymbol{u}}_z \cdot \overline{\overline{\epsilon}} \cdot \hat{\boldsymbol{u}}_z \to 0 \qquad \hat{\boldsymbol{u}}_z \cdot \overline{\overline{\mu}} \cdot \hat{\boldsymbol{u}}_z \to 0 \tag{2}$$

Let us consider now the well known boundary conditions proposed in the literature called the Soft and Hard Surfaces (SHS) [7]. These boundary conditions are in such a way similar to the DB boundary conditions: in fact, on the SHS, the components of the electric and magnetic fields must be zero in a direction parallel to the interface. The conditions can be written as follows

$$\hat{\boldsymbol{u}}_y \cdot \boldsymbol{E} = 0 \qquad \text{and} \qquad \hat{\boldsymbol{u}}_y \cdot \boldsymbol{H} = 0 \tag{3}$$

where $\hat{\boldsymbol{u}}_y$ is a unit vector parallel to the interface. Both the SHS and the DB boundary conditions can be realized with a particular bi-anisotropic medium, i.e., a Skewon-Axion medium [8], proposed in [9]. These conditions read

$$T\omega\hat{\boldsymbol{u}}_z \cdot \boldsymbol{B} + k_0\hat{\boldsymbol{u}}_y \cdot \boldsymbol{E} = 0 \tag{4}$$

$$T\omega\hat{\boldsymbol{u}}_z \cdot \boldsymbol{D} - k_0\hat{\boldsymbol{u}}_y \cdot \boldsymbol{H} = 0 \tag{5}$$

where ω is the angular frequency and k_0 the vacuum wave number. It is easy to see that when $T \to 0$ the conditions approach that of the SHS; on the contrary, when $T \to \infty$, they approach the DB boundary conditions.

In the present paper, we investigate the behavior of the SH/DB boundary. Let us call $\hat{\boldsymbol{c}}$ a generic unit vector on the plane (y, z), and α the angle between this vector and the normal vector to the interface, $\hat{\boldsymbol{u}}_z$, see Fig. 1. The boundary conditions are:

$$\hat{\boldsymbol{c}} \cdot \boldsymbol{D} = 0 \qquad \text{and} \qquad \hat{\boldsymbol{c}} \cdot \boldsymbol{B} = 0 \tag{6}$$

We can easily see that for $\alpha = 0$ these conditions become the DB boundary conditions (1). However, if $\alpha = \pi/2$, the conditions become the following:

$$\hat{\boldsymbol{u}}_y \cdot \boldsymbol{D} = 0 \qquad \text{and} \qquad \hat{\boldsymbol{u}}_y \cdot \boldsymbol{B} = 0 \tag{7}$$

If we suppose that the medium of the incident wave, let us call it medium 1, is an isotropic medium, the conditions can be written as follows:

$$\epsilon_1\hat{\boldsymbol{u}}_y \cdot \boldsymbol{E} = 0 \qquad \text{and} \qquad \mu_1\hat{\boldsymbol{u}}_y \cdot \boldsymbol{H} = 0 \tag{8}$$

where ϵ_1 and μ_1 are the permittivity and permeability of the medium 1. The conditions in (8) for scalar values of ϵ_1 and

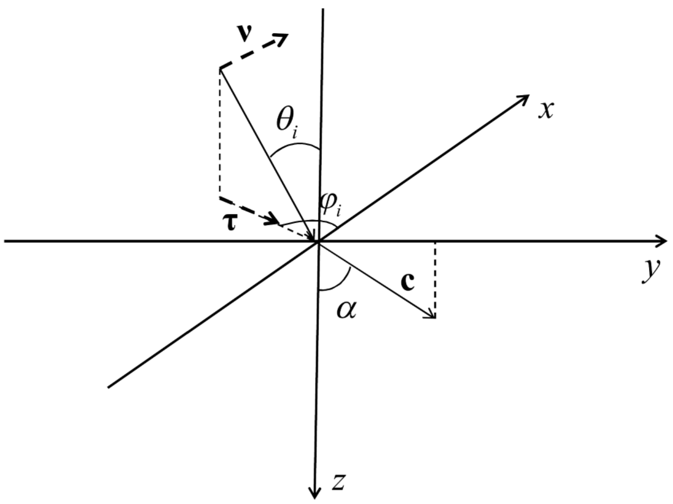

Fig. 1. Geometry of the problem

μ_1, are exactly the same as the SHS in (3). In this way, we can call the conditions in (6) as SH/DB conditions, because they can represent both the DB interface and the SHS, by varying the parameter α.

Let us note that the conditions (6) are intrinsically different from the conditions (4). In fact, they approach each other only in the case $T \to 0$ and $\alpha = \pi/2$ or in the case $T \to \infty$ and $\alpha = 0$. In all the other cases they radically differ from each other. This can be seen from the fact that a medium realizing conditions (4) is intrinsically bi-anisotropic, in the sense that the components of the electric and magnetic flux densities are connected to both the magnetic and the electric fields, respectively. On the other hand, the conditions in (6) can be realized by a simpler anisotropic medium, as we will see in the following. It can be demonstrated that there does not exist any affine transformation to convert the bi-anisotropic medium presented in [9], in an anisotropic medium, i.e., to make the cross parameters $\overline{\overline{\xi}}$ and $\overline{\overline{\zeta}}$ zero.

The SH/DB realization through an exotic uniaxial medium has recently been investigated [12]. In the following, we study the interaction of the electromagnetic radiation with these uniaxial media. Moreover, the reflection by an interface with a uniaxial medium with negative electromagnetic characteristics is analyzed.

In Section II, we present the theoretical formulation of the problem. In Section III, we consider the reflection at the interface between an isotropic medium and the uniaxial one in some special cases. In Section IV, the conclusions are drawn.

In the following, we assume a time dependence of the form $e^{-i\omega t}$. Moreover, the dyadics $\overline{\overline{\epsilon}}$ and $\overline{\overline{\mu}}$ are the dimensionless, relative electric permittivity and magnetic permeability, respectively. Furthermore, we indicate all the unit vectors by bold letters with a hat, e.g., $\hat{\boldsymbol{c}}$.

II. Theoretical Formulation

In this section, we define the problem of the reflection of a plane wave by a uniaxial medium.

Let us consider a plane wave propagating in an isotropic medium in the half-space $z < 0$. The half-space $z > 0$ is filled with a uniaxial medium with the following relative permittivity and permeability dyadics:

$$\overline{\overline{\epsilon}} = \epsilon_t \overline{\overline{I}}_t + \epsilon_c \hat{\boldsymbol{c}}\hat{\boldsymbol{c}} \qquad \text{and} \qquad \overline{\overline{\mu}} = \mu_t \overline{\overline{I}}_t + \mu_c \hat{\boldsymbol{c}}\hat{\boldsymbol{c}} \tag{9}$$

with $\overline{\overline{I}}_t = \overline{\overline{I}} - \hat{\boldsymbol{c}}\hat{\boldsymbol{c}}$ being the two-dimensional unit dyadic. When the longitudinal part of these dyadics is zero, the interface with this medium realizes the boundary conditions in (6). Let us note that this medium is quite different from an ordinary uniaxial medium, in fact, the ordinary uniaxial media have only the electric permittivity, i.e., the permittivity is a dyadic, but the permeability is a scalar quantity. The implementation of the SH/DB with this medium is easily obtained when the longitudinal characteristics approach zero: in fact, in this case, the components of the electric and magnetic field parallel to the optics axis approach zero, too.

The dispersion equation in this uniaxial medium is the following:

$$\left(\boldsymbol{k} \cdot \overline{\overline{\epsilon}} \cdot \boldsymbol{k} - \epsilon_c k_t^2\right)\left(\boldsymbol{k} \cdot \overline{\overline{\mu}} \cdot \boldsymbol{k} - \mu_c k_t^2\right) = 0 \tag{10}$$

where $\boldsymbol{k}$ is the propagation vector of the electromagnetic wave and $k_t = \omega\sqrt{\epsilon_t \mu_t}$ is the transverse wavenumber. From the dispersion equation the polarization vector of the two waves, allowed to propagate in the medium, can be obtained as a function of the optic axis $\hat{\boldsymbol{c}}$ and of the propagation vector $\boldsymbol{k}$:

$$\boldsymbol{e}_E = \hat{\boldsymbol{k}}_E \times \hat{\boldsymbol{c}} \qquad \boldsymbol{e}_H = \hat{\boldsymbol{c}} - \hat{k}_{H\|}\hat{\boldsymbol{k}}_H \tag{11}$$

$$\boldsymbol{h}_E = -\frac{1}{\zeta_t}\left(\hat{\boldsymbol{c}} - \hat{k}_{E\|}\hat{\boldsymbol{k}}_E\right) \qquad \boldsymbol{h}_H = \frac{1}{\zeta_t}\left(\hat{\boldsymbol{k}}_H \times \hat{\boldsymbol{c}}\right) \tag{12}$$

The polarization vectors of the electric field are dimensionless, on the other hand the vectors of the magnetic field have the dimensions of an admittance.

Starting from the expressions (11) and (12) and following a procedure similar to that developed in [10], the reflection and transmission coefficients of the interface with this medium can be evaluated. The explicit expression of such coefficients is presented in [12]. The reflection coefficients are in a matrix form, connecting the incident and the reflected waves in both the polarizations:

$$\begin{pmatrix} E_r^{TE} \\ E_r^{TM} \end{pmatrix} = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} \cdot \begin{pmatrix} E_i^{TE} \\ E_i^{TM} \end{pmatrix} \tag{13}$$

The obtained expressions are of particular interest because allow us to compute the electromagnetic wave reflected by a general uniaxial medium with any values of the parameters (9).

III. Reflection by exotic uniaxial media

In this section, the behavior of the coefficients developed in [12] is analyzed in some interesting cases, i.e., for some particular anisotropic media.

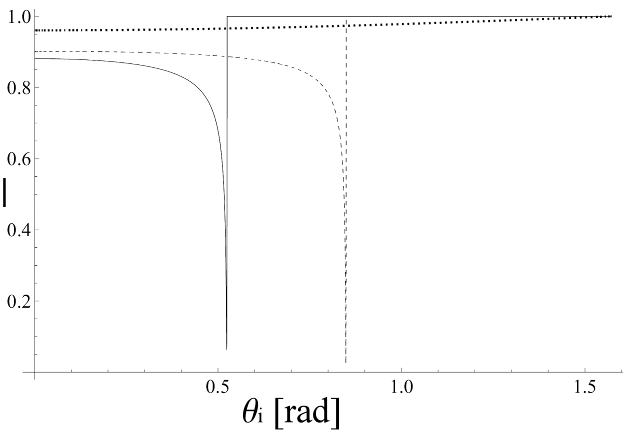

Fig. 2. Magnitude of the reflection coefficient Γ_{11} as a function of the azimuthal incident angle, when medium 1 is air and medium 2 has $\alpha = 30°$, longitudinal parameters $\epsilon_c = \mu_c = 0.001$ and for different values of the transverse parameters $\epsilon_t = \mu_t$: 1 (solid line), 1.5 (dashed line), and 10 (dotted line). The incident wave has the propagation vector on the (y, z) plane.

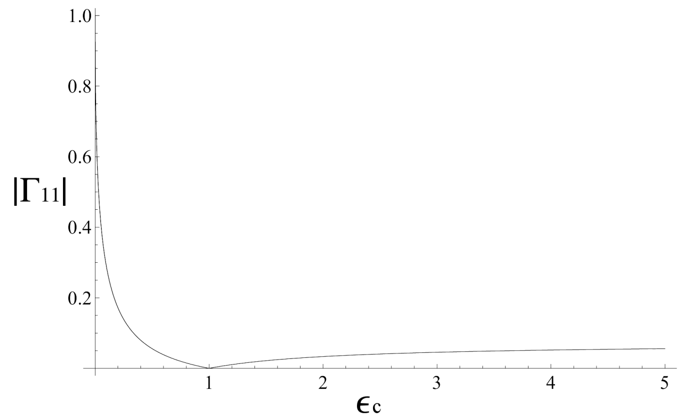

Fig. 3. Magnitude of the reflection coefficient Γ_{11} as a function of the longitudinal parameters of the second medium, when medium 1 is air and medium 2 has $\alpha = 30°$, transverse parameters $\epsilon_t = \mu_t = 1$. The incident wave has the propagation vector on the (y, z) plane, i.e., $\varphi_i = 90°$, with $\theta_i = 0°$.

Let us consider as a first example, a medium with $\overline{\overline{\epsilon}} = \overline{\overline{\mu}}$. In this case the reflection matrix becomes symmetric and the diagonal elements become equal to one another. Moreover, we suppose that the propagation vector of the incident wave, the optic axis and the vector perpendicular to the interface are coplanar. In this case the non diagonal coefficients are zero. The behavior of the diagonal coefficients is shown in Fig. 2 as a function of the azimuthal incident angle θ_i, when $\alpha = 30°$, $\epsilon_c = \mu_c = 0.001$ and for different values of the transverse parameters.

We can see that the reflection coefficient approaches zero in the cases $\epsilon_t = \mu_t = 1$ and $\epsilon_t = \mu_t = 1.5$, for two different values of the incident angle, let us call these values critical angles. In the first case the angle is $\theta_c = 30° = \alpha$: being the transverse parameters of the uniaxial medium equal to unity, when the incident wave is parallel to the optic axis, it does not "see" differences between the two media. Therefore, the reflection coefficient is zero at this angle. In the second case the total transmission occurs for a larger value of the incident angle: in fact, because of the difference of the refractive indexes, the transmission angle is parallel to the optic axis for an incident angle $\theta_i > \alpha$. In the third case, when $\epsilon_t = \mu_t = 10$, the transmitted wave vector is never parallel to the optic axis and the reflection coefficient is never zero. The amplitude of the reflection coefficient for angles greater than the critical angles is equal to unity because of the total-reflection effect. On the other hand, for values of the incident angle lower than the critical angle, the reflection coefficient is close to unity. It can be shown that, when the longitudinal parameters approach zero, the reflection coefficient around zero approaches unity. In Fig. 3 the magnitude of the reflection coefficient $|\Gamma_{11}|$ is shown at normal incident, $\theta_i = 0$, as a function of the anisotropy, i.e., as a function of the longitudinal properties, ϵ_c and μ_c, of the dyadics. The optics axis forms an angle $\alpha = 30°$ and the transverse properties of the dyadics have been considered equal to unity, $\epsilon_t = \mu_t = 1$. From the figure it is clear that the interface behaves as a perfect reflector when the longitudinal properties approach zero, i.e., when the interface behaves as a SH/DB boundary. The reflection coefficient approaches zero when the longitudinal characteristics become equal to unity, because the uniaxial medium becomes a vacuum. For greater values the reflection coefficient grows slowly.

At this point, we note a strange effect: we saw before that the interface act as a perfect reflector when $\epsilon_c \to 0$[1]. Let us consider now the case in which $\epsilon_t = \epsilon_c^{-1}$ and $\alpha = 0$. In this case the interface becomes totally transparent to the radiation for any incident angle. This fact was predicted in the Perfectly Matched Layer theory [13]. In Fig. 4, the behavior of the reflection coefficient Γ_{11} is shown as a function of the longitudinal properties of the dyadics. In this case we consider $\epsilon_t = \mu_t = 100$ and $\theta = 60°$, in a neighborhood of zero. As we predicted, the reflection coefficient becomes zero when $\epsilon_c = 0.01 = \epsilon_t^{-1}$. However, when ϵ_c becomes smaller, the interface behaves again as a perfect reflector. We can recognize two different zones: when $\epsilon_c > 0.01$, the reflection coefficient grows with the longitudinal characteristics, as usual, because the mismatch between the two media grows. When $0 < \epsilon_c < 0.01$, the reflection coefficient approaches unity: this is the total-reflection zone.

Let us consider the case when the incident propagation vector, the optic axis and the perpendicular to the interface are not coplanar. In this case, when the ratio $\epsilon_t/\epsilon_c \to \infty$, the interface behaves as a polarization inverter. In Fig. 5, the amplitude of the cross reflection coefficient Γ_{12} is shown as a function of $\epsilon_t = \mu_t$, when $\epsilon_c = \mu_c = \epsilon_t^{-1}$. The optic axis, on the plane (y, z), forms an angle $\alpha = 45°$ with the perpendicular to the interface and the incident plane wave is supposed at normal incidence. In this case the diagonal reflection coefficients $|\Gamma_{11}|$ and $|\Gamma_{22}|$ are both zero and the non

[1]Let us remind that, in the present example, the permittivity dyadic is equal to the permeability one. Therefore, $\epsilon_c = \mu_c$

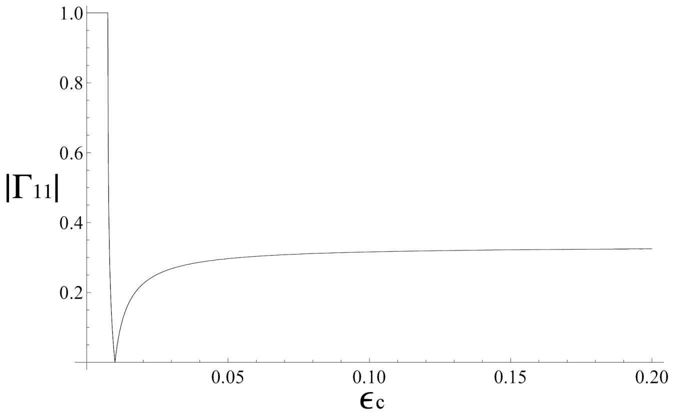

Fig. 4. Magnitude of the reflection coefficient Γ_{11} as a function of the longitudinal parameters of the second medium, when medium 1 is air and medium 2 has $\alpha = 0°$, transverse parameters $\epsilon_t = \mu_t = 100$. The incident wave has the propagation vector on the (y, z) plane, i.e., $\varphi_i = 90°$, with $\theta_i = 60°$.

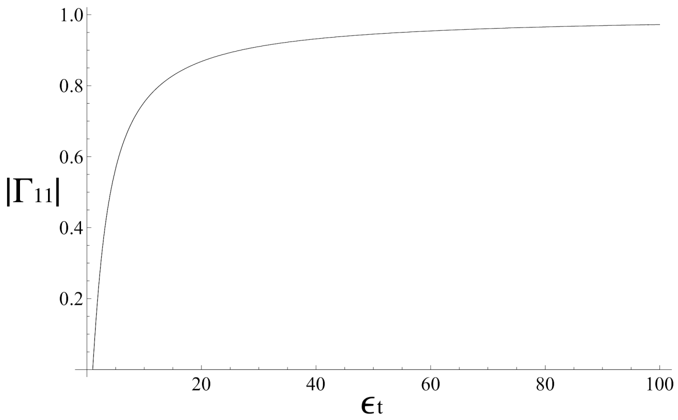

Fig. 5. Magnitude of the reflection coefficient Γ_{12}, when medium 1 is air and medium 2 has $\alpha = 45°$, longitudinal parameters $\epsilon_c = \mu_c = \epsilon_t^{-1}$, as a function of the transverse parameters. The incident wave is supposed at normal incidence.

diagonal coefficients are equal to each other in amplitude, i.e., $|\Gamma_{12}| = |\Gamma_{21}|$. We see that the cross coefficient tends to unity when the ratio ϵ_t/ϵ_c grows. As a consequence, an incident wave in the TE (or TM) polarization generates a reflected wave with TM (or TE) polarization. Therefore, this particular implementation of the SH/DB boundary has the property to invert the polarization of the incident wave. This behavior is similar in the case of oblique incidence, too. In this case the diagonal coefficients are not exactly zero, but they are close to zero. In Fig. 6, the coefficients Γ_{11} and Γ_{12} are shown as a function of ϵ_t in the same conditions of Fig. 4.

IV. Conclusion

In this paper the realization of a particular boundary condition, that imposes the cancellation of the electric and magnetic field along a generic direction, has been studied by means of a particular uniaxial medium. The interface of this medium with an isotropic medium has been analyzed.

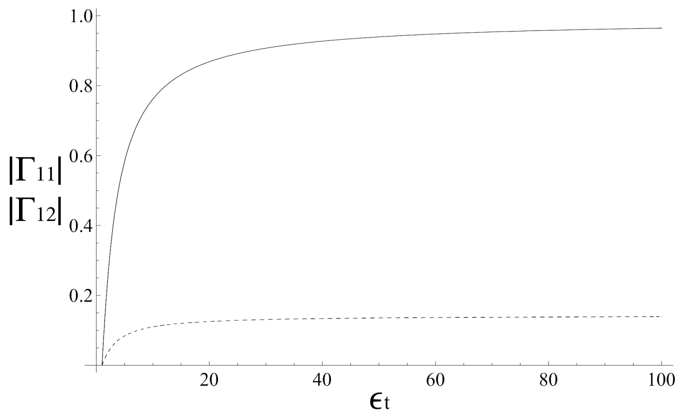

Fig. 6. Magnitude of the reflection coefficients Γ_{11} (dashed line) and Γ_{12} (solid line), in the same scenario of Fig. 4, but with an azimuthal incident angle $\theta_i = 30°$.

The reflection coefficients have been studied as a function of both the incident angle and the anisotropic properties of the uniaxial medium. The characteristics of the interface when the longitudinal parameters of the permittivity and permeability dyadics approach zero have been emphasized. These examples are of interest because, in such cases, the interface implements the SH/DB boundary condition. Possible applications of this boundary condition have been investigated.

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