Deterministic Synthesis of Non-Uniformly Spaced Isophoric Linear Antenna Arrays

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***Abstract*—A deterministic approach to the synthesis of linear aperiodic antenna arrays with uniform excitation is presented and thoroughly discussed in this paper. The proposed technique, based on the auxiliary array factor concept, is aimed at the deterministic determination of the optimal array element density useful to mimic a desired power pattern. In particular, the developed antenna placement method does not require any optimization procedure, resulting in a dramatic reduction of the antenna design times. Antenna mutual coupling is not accounted in the developed methodology and, hence, in order to assess the sensitivity of the design procedure to such non-ideality, a dedicated numerical investigation has been carried out by using a rigorous full-wave electromagnetic field prediction technique. The obtained results prove the effectiveness and the versatility of the proposed array synthesis approach even in operative scenarios where a significant deviation from desired antenna operation is observed.**

I. Introduction

Several approaches to the design of linear sparse arrays have been proposed in last years, mainly based on multidimensional optimization techniques [1], and evolutionary stochastic methods such as Genetic Algorithms (*GA*) [2]. Unfortunately, the synthesis procedures already available in the scientific literature either lack of generality, as the analytical techniques in [3]-[4], or they are very demanding in terms of computational resources. To overcome these limitations, a general and effective antenna placement technique for isophoric aperiodic linear arrays is developed, and here presented. Such technique, based on the Auxiliary Array Factor (*AAF*) concept [5]-[9], allows for the deterministic derivation of the element density distribution useful to mimic a given radiation pattern mask. In this way, the array sparseness can be usefully exploited as an additional degree of freedom in the design in order to meet possible requirements in terms of maximum aperture size.

The proposed antenna placement method is based on a fully analytical formulation, and does not require any iterative procedure. This is in turn useful to reduce synthesis times dramatically. In the presented formulation, the effect of mutual coupling between the antenna elements is neglected, although it is to be pointed out that, where the inter-element spacing is in the order of $0.25\,\lambda_0$, λ_0 denoting the operational wavelength, the overall performance of the radiating structure may be adversely affected. A detailed discussion of parasitic coupling-related issues in array design problems is beyond the scope of the presented research, although this antenna non-ideality may be handled by using the theory of Active Element Pattern (*AEP*), as already presented in [10].

II. Deterministic Antenna Placement Technique

In this section, emphasis is put on the analytical details of the proposed design technique for deterministic synthesis of isophoric aperiodic linear antenna arrays.

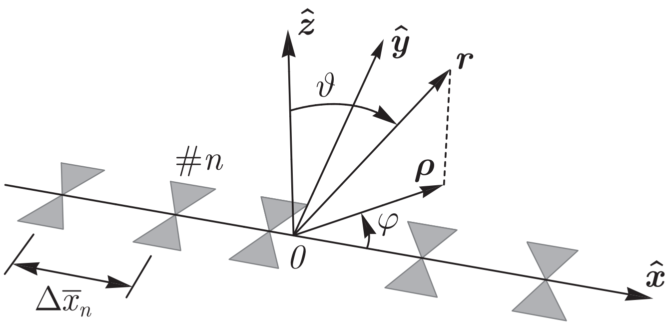

Fig. 1. Antenna element distribution in a general linear aperiodic array.

The developed methodology relies on the definition of *AAF* function depending on the continuous antenna element density which is determined, upon a suitable piecewise linearization procedure, by enforcing the equality to a given objective array factor mask in the domain of the Fourier transform with respect to the normalized direction parameter. In this way, one obtains an integral equation, involving the element taper distribution, which can be solved analytically or by means of computationally inexpensive numerical methods. Afterwards, the actual antenna positions are evaluated in a straightforward manner by uniform sampling.

A. Auxiliary Array Factor Function

Let us first consider the array factor relevant to a radiating structure consisting of N_a identical antennas deployed over a line (see Fig. 1), and excited with uniform amplitude and phase:

$$AF(\vartheta) = \sum_{n=1}^{N_a} e^{jk_0 \bar{x}_n \sin\vartheta}, \tag{1}$$

k_0 being the propagation constant in free space, and $x = \bar{x}_n$ the location of $n-$th array element. The expression in (1) may be regarded as the Riemann's sum approximating the *AAF* function defined below:

$$F_A(\vartheta) = \int_0^1 e^{j2\pi\xi(q)\sin\vartheta} dq, \tag{2}$$

where $\xi(q)$ denotes the continuous normalized positioning function which generalizes the discrete quantity $\frac{1}{2}k_0\bar{x}_n/\pi$ appearing in (1). Similarly, q is the continuous version of the index n relevant to the general antenna element forming the array.

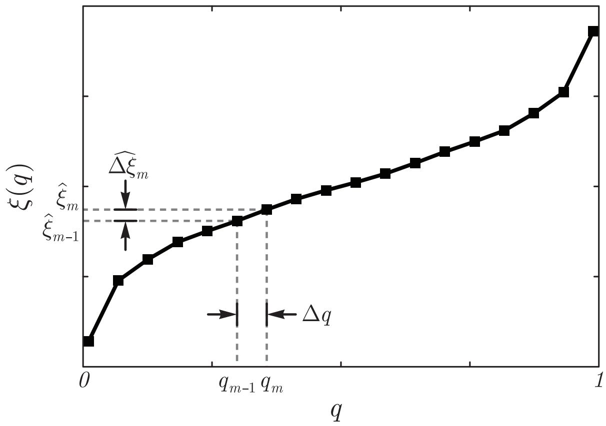

Fig. 2. Piecewise linear approximation of the element density distribution $\xi(q)$ as a function of the continuous index q relevant to the general antenna forming the array structure.

After setting for shortness $\psi = 2\pi\sin\vartheta$, the *AAF* function can be rewritten as:

$$F_A(\psi) = \sum_{m=1}^{M} F_{A_m}(\psi), \tag{3}$$

with:

$$F_{A_m}(\psi) = \int_{q_{m-1}}^{q_m} e^{j\psi\xi_m(q)} dq \tag{4}$$

being the contribution pertinent to the $m-$th interval $[q_{m-1}, q_m]$. In each interval, the positioning function is assumed to be linearized (see Fig. 2) according to the following expression:

$$\xi_m(q) = \hat{\xi}_{m-1} + \frac{q - q_{m-1}}{\Delta q}\widehat{\Delta\xi}_m, \tag{5}$$

where $\widehat{\Delta\xi}_m = \hat{\xi}_m - \hat{\xi}_{m-1}$ and $\Delta q = q_m - q_{m-1}$. In this way, the general term $F_{A_m}(\psi)$ in (4) can be evaluated in closed form as follows:

$$F_{A_m}(\psi) = \Delta q \ \mathrm{sinc}\frac{\psi\widehat{\Delta\xi}_m}{2} \ e^{j\psi\hat{\xi}_{m-\frac{1}{2}}}, \tag{6}$$

with $\mathrm{sinc}(\cdot)$ denoting the usual sine cardinal function, and $\hat{\xi}_{m-\frac{1}{2}} = \hat{\xi}_m + \widehat{\Delta\xi}_m/2$. In order to allow for a deterministic synthesis of the array aimed at mimicking a given power pattern mask $F_O(\psi)$, it is convenient to perform a Fourier transformation with respect to the normalized direction parameter ψ. In this way, one can readily obtain:

$$\begin{aligned}\tilde{F}_{A_m}(H) &= \int_{-\infty}^{+\infty} F_{A_m}(\psi) \ e^{j\psi H} d\psi = \\ &= \frac{2\pi\Delta q}{\widehat{\Delta\xi}_m} P_{\widehat{\Delta\xi}_m}\left(H + \hat{\xi}_{m-\frac{1}{2}}\right),\end{aligned} \tag{7}$$

$P_\Delta(H)$ being the pulse distribution having width Δ and centered at the origin $H = 0$. It is worth noting that, by virtue of the said piecewise linearization of the continuous normalized positioning function $\xi(q)$, each term $F_{A_m}(\psi)$ in (3) turns to be Fourier-transformed into a function with compact support centered at $H = -\hat{\xi}_{m-\frac{1}{2}}$ and having support-width $\widehat{\Delta\xi}_m$.

B. Deterministic Evaluation of the Array Element Density

The array synthesis is carried out by enforcing $\left|\tilde{F}_A(H)\right| = \left|\tilde{F}_O(H)\right|$ or, equivalently, $\left|\tilde{F}_{A_m}(H)\right| = \left|\tilde{F}_O(H)\right|$ within each interval $H \in \left[-\hat{\xi}_m, -\hat{\xi}_{m-1}\right]$. So, by making use of equation (7) it follows that:

$$\int_{\hat{\xi}_{m-1}}^{\hat{\xi}_m} \left|\tilde{F}_O(-H)\right| dH = \int_{\hat{\xi}_{m-1}}^{\hat{\xi}_m} \left|\tilde{F}_{A_m}(-H)\right| dH = 2\pi\Delta q, \tag{8}$$

where the following normalization condition on the magnitude of the Fourier-transformed pattern mask $\tilde{F}_O(H)$ is adopted:

$$\int_{\hat{\xi}_0}^{\hat{\xi}_N} \left|\tilde{F}_O(-H)\right| dH = 2\pi. \tag{9}$$

It is worth noting that, thanks to the outlined methodology, the proposed array synthesis technique turns to be fully deterministic without resorting to any optimization procedure. The solution of (8) can be carried out by using different numerical or analytical methods. In particular, a combined technique based on the Gauss-Kronrod quadrature formula [11], and the Newton-Raphson root-finding method [12] has been specifically developed within the proposed research. The considered approach is stable by construction and computationally efficient, resulting in extremely reduced solving times.

Let us now focus the attention on the selection of the parameter $\hat{\xi}_0$. To this end, a suitable energy criterion can be adopted by determing the value $\hat{\Xi}_0 > 0$ which satisfies:

$$\int_{-\hat{\Xi}_0}^{\hat{\Xi}_0} \left|\tilde{F}_O(H)\right|^2 dH = \kappa^2 \int_{-\infty}^{+\infty} \left|\tilde{F}_O(H)\right|^2 dH, \tag{10}$$

$\kappa < 1$ being a given threshold controlling the extension of the visibility region of the assigned radiation mask, and hence affecting the accuracy of the proposed antenna placement procedure. Under the assumption that the maximum array aperture is required to be $D_{\max}$, the mentioned parameter can be taken as:

$$\hat{\xi}_0 = -\min\left\{\hat{\Xi}_0, \frac{D_{\max}}{2}\right\}. \tag{11}$$

On the other hand, if no specification on the maximum array size is provided, $\hat{\xi}_0$ can be readily selected as $\hat{\Xi}_0$.

Finally, the antenna locations $\bar{x}_n$ (see 1) can be computed in a straightforward manner by uniformly sampling the normalized positioning function at the indicial barycenters:

$$\bar{q}_n = \frac{1}{N_a}\left(n - \frac{1}{2}\right), \tag{12}$$

with $n = 1, 2, \ldots, N_a$.

III. Numerical Example

The developed technique has been validated by application to the synthesis of a isophoric linear antenna array featuring the Dolph-Chebyshev pattern with side-lobe level $SLL = -20\,dB$ shown in Fig. 3 [13]. In doing so, the antenna aperture has been specified to be $D_{\max} = 9.725\,\lambda_0$ in such a way that most of the power ($> 95\,\%$) is radiated in the visible region. Subject to this constraint, the developed deterministic design method has been applied selecting the number of antennas as $N_a = 24$. The resulting element positions are plotted in Fig. 4. In particular, the minimum inter-element spacing is $\Delta\overline{x}_{\min} \simeq 0.341\,\lambda_0$, whereas the average antenna distance has been found to be about $0.425\,\lambda_0$. As it appears from Fig. 3, the array sparseness can be usefully exploited to accurately mimic the desired radiation pattern without using any excitation tapering. That in turn is important in order to allow all the amplifiers in the array feeding network to be operated under the same optimal condition, thus gaining efficiency and, at the same time, reducing the manufacturing costs significantly.

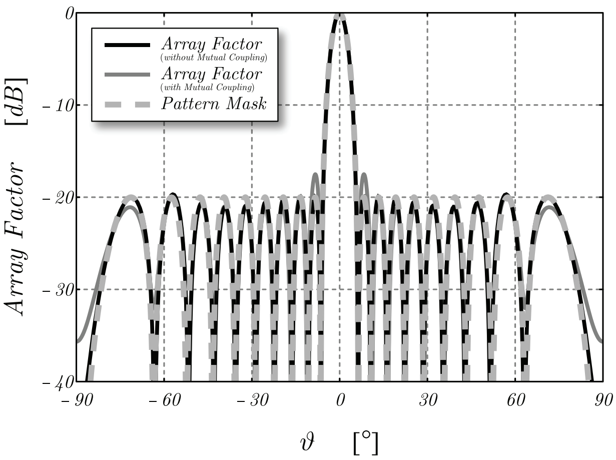

Fig. 3. Angular behavior of the synthesized array factor useful to mimic a Dolph-Chebyshev pattern mask with side-lobe level $SLL = -20\,dB$. The array aperture is specified to be $D_{\max} = 9.725\,\lambda_0$.

The selected example clearly shows the versatility of the developed technique which provides the antenna engineer with a useful array synthesis tool. An important feature of the considered methodology lies in the relevant fully deterministic formulation, which typically results in reduced computational times and negligible memory usage. In order to point out this important aspect, the problem of synthesizing the $24-$element isophoric aperiodic linear array with the Dolph-Chebyshev radiation pattern mask shown in Fig. 3 has been, also, addressed by means of the global optimization method discussed in [14], as well as the evolutionary *GA*-based procedure detailed in [2]. In doing so, the needed numerical computations have been performed in double-precision floating-point arithmetic on the same workstation equipped with a $2.99\,GHz$ *Intel Core Duo*® processor [15], and $3.25\,GByte$ memory. Notably, the considered approaches yield essentially the same results in terms of antenna positions although, as it can be noticed in Table II, the proposed analytical technique, featuring a nearly real-time computing capability, outperforms the alternative ones.The achieved performance is in full accordance with the Nemirovsky-Yudin theorem which states that the complexity and, therefore, the computational burden of numerical optimization algorithms increase dramatically (at exponential rate) with the number of problem unknowns [16].

TABLE I
Synthesis of a Dolph-Chebyshev Array with $SLL = -20\,dB$.

Array Element (n)	$\bar{x}_n/\lambda_0$
1	-4.86
2	-4.50
3	-3.85
4	-3.31
5	-2.80
6	-2.37
7	-1.97
8	-1.57
9	-1.21
10	-0.86
11	-0.51
12	-0.17
13	0.17
14	0.51
15	0.86
16	1.21
17	1.58
18	1.97
19	2.37
20	2.81
21	3.31
22	3.85
23	4.50
24	4.86

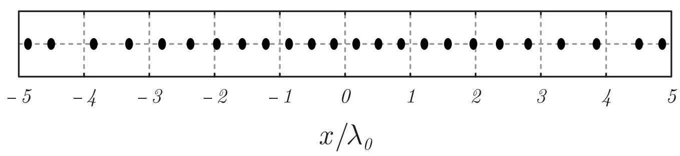

Fig. 4. Antenna element positions relevant to the $24-$element isophoric aperiodic linear array featuring the Dolph-Chebyshev radiation pattern shown in Fig. 3.

The degradation of the array characteristics due to the spurious mutual coupling is neglected in the presented research.

TABLE II
COMPUTATIONAL BURDEN OF DIFFERENT DESIGN METHODOLOGIES FOR SYNTHESIZING THE ISOPHORIC APERIODIC ARRAY SHOWN IN FIG. 4.

Array Synthesis Methodology	**Computational Time** (s)	**Memory Usage** ($Byte$)
Global Optimization Method [14]	7.55	21 151 744
Evolutionary *GA*-based Algorithm [2]	39 139.09	12 275 712
Proposed Deterministic Technique	3.28	1 563 608

However, in order to assess the sensitivity of the proposed design tool to such antenna non-ideality, a dedicated numerical investigation has been carried out. To this end, the radiation properties of the synthesized uniformly fed linear sparse array consisting of N_a identical perfectly conducting dipoles (see Fig. 4) have been analyzed in a rigorous way by using the full-wave electromagnetic field solver *CST Microwave Studio*® [17] based on the Finite Integration Technique (*FIT*). In particular, the length, radius, and feeding delta gap of the individual dipole have been selected to be $l_d = 0.4385\,\lambda_0$, $r_d = 0.01\,\lambda_0$, and $\delta_d = 0.02\,\lambda_0$, respectively, in such a way as to achieve a reasonably good performance in terms of return loss (with respect to the reference impedance $Z_0 = 50\,\Omega$) in the array configuration, namely $|S_{ii}| \lesssim -9.2\,dB$ ($i = 1, 2, \ldots, N_a$) at the central working frequency $f_0 = c_0/\lambda_0$, c_0 denoting the speed of light in free space. Under such assumption, the maximum parasitic coupling level between the radiating elements has been found to be $\max_{i \neq j} |S_{ij}(f_0)| \simeq -8.9\,dB$, hence resulting in a significant deviation from ideal antenna operation. Despite of that, as it can be noticed in Fig. 3, the angular behavior of the synthesized array factor is not very severely impacted, except for the first side-lobe level, showing that the presented antenna placement technique yields pretty accurate results in realistic operative scenarios too. It is, however, to be stressed out that the rigorous modeling of mutual coupling in sparse arrays is surely not trivial and strongly problem-dependent. This important point is to be carefully addressed in future research by exploring suitable extensions of the proposed analytical formulation based on the concept of approximate/average *AEP* [18]. In this respect, one can also readily infer that, in array design contexts adversely affected by a large parasitic coupling level and/or number of radiating elements, the developed algorithm may be usefully adopted as an effective preconditioner in advanced hybrid deterministic/metaheuristic antenna placement methodologies in order to derive a well-conditioned initial array configuration useful to enhance convergence in terms of the number of iterations within the metaheuristic procedure and, consequently, to reduce the total computational time required to obtain a converged solution of the problem.

IV. CONCLUSIONS

A general and computationally efficient antenna placement method for isophoric aperiodic linear antenna arrays has been presented. Such method, based on the *AAF* concept, allows for the deterministic derivation of the element density distribution useful to mimic a desired power pattern mask, without resorting to any iterative procedure. In this way, isophoric array synthesis problems can be addressed in a straightforward and computationally inexpensive way.

The developed technique has been successfully validated by application to a canonical problem regarding the synthesis of an array with a Dolph-Chebyshev radiation mask. In doing so, the impact of the parasitic antenna coupling has been investigated rigorously by carrying out a dedicated full-wave analysis. In this way, it has been found out that the proposed design methodology provides reasonably accurate results even in operative scenarios where a significant deviation from ideal antenna operation occurs.

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