Noise figure of broadband active metamaterials

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***Abstract*—Compared to passive metamaterials, active metamaterials made up of non-Fosters negative impedance converters (NICs) have been proposed with broadband and lossless negative-Re-μ (MNG) or μ-near-zero (MNZ) magnetic properties. However, the study of their noise performance still remains to be explored. Johnson noise will arise due to lossy conductors and nonlinear materials used for the design of the NICs. Based on the equivalent circuit model of the coupled elements, an analytical description is presented in this paper to predict noise performance of broadband active metamaterials based on NICs, and analyse its relation to effective constitutive material properties.**

I. Introduction

Although artificially engineered materials, or metamaterials have been theoretically demonstrated to possess extraordinary material properties, which may lead to many new breakthroughs in physics and engineering [1–3], there exist a number of challenging issues to enable practical demonstrations and applications to real-world problems. One of the main challenges is their inherent high dispersion, narrow-band characteristics and losses [4].

To overcome these problems, a new approach based on a non-Fosters impedance converter (NIC) has been proposed to enable broadband active metamaterials, which shows significant potential for advancing metamaterial-based applications [5, 6]. A limiting factor associated with active metamaterials is stability, as investigated by the authors [5, 7]. However, the study of their noise performance still remains unexplored. For passive metamaterials based on periodic and electrically-small resonant structures, only thermal noise from conductors needs to be taken into consideration [8], while nonlinear materials used for the design of active metamaterials based on, for example, switches, amplifiers and negative impedance converters (NICs) will give rise to thermal noise as well.

In order to accurately analyse noise performance of active metamaterials based on NICs, a circuit model is used followed by the analytic solution to the noise figure in 1D non-Foster loading metamaterials as well as the circuit simulations.

II. Effective medium model of broadband metamaterials under the stability condition

As proposed in [5], for a periodic medium of magnetically polarizable inclusions with arbitrary loading, an equivalent circuit of N loaded loops is established taking mutual coupling into account as shown in Fig. 1. Additionally, the effective relative magnetic permeability is obtained and verified by HFSS simulations [5]

$$\mu_r = 1 - \mu_b N \frac{j\omega S^2}{Z_0 + Z_L + j\omega M_p}, \tag{1}$$

where μ_b is the background permeability, S is the loop area, N is the volume density of loops, and M_p is the total mutual inductance term. In Fig. 2, the effective parameters of the non-Foster loading medium obtained from Eq. (1) are plotted with varying load inductances. The non-Foster load is defined as a parallel negative RLC circuit with $R_L = -1000\Omega$ and $C_L = -2pF$ for all elements. It is clear to see that the MNG band will be broadened with increasing load inductances whereas the real part of the permeability is decreased.

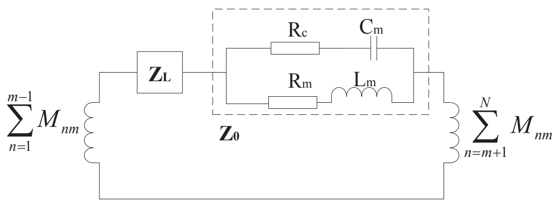

Fig. 1. Equivalent circuit of a loop and load in a system of N coupled loaded loops. Summations indicate the coupling factors between the loop m and the remaining $N-1$ loops.

III. Analysis of noise performance in broadband metamaterials loaded with NICs

In general, noise performance of broadband metamaterials loaded with NICs can be considered to be the sum of both loop resonators and NICs. Following the approach presented by Pierce [9], the noise produced by the loop itself can be described as a Johnson-Nyquist noise voltage source $V(\omega) = V_0 \exp(j\omega t)$ in series with a resistor R_w. The mean square value of V_0 in a small frequency range df is $V_0 V_0^* = 4KTR_w df$, where K is Boltzman constant and T is the absolute temperature (customarily taken to be room temperature, 290K). In order to analyse the noise effect of the active load, it is necessary to study in-depth the principle of non-Fosters circuits. The NIC is, by analogy, a form of an ideal transformer with a ratio of an impedance transformation of -1.

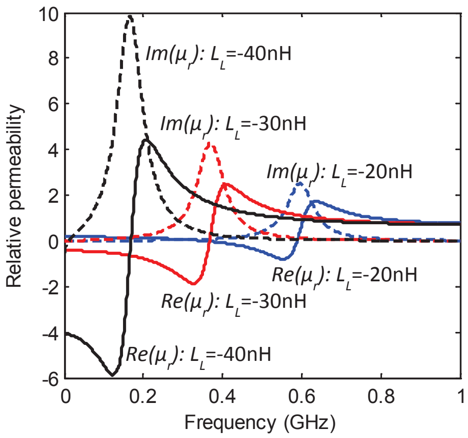

Fig. 2. Extracted parameters of effective medium comprising infinite loaded loops with varying load inductances.

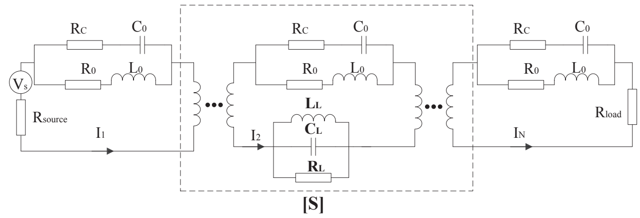

Fig. 4. 1D Non-Foster loaded loop array with a source and load at both ends.

This impedance transformation operates in both directions, and, as shown in Fig. 3, the non-Fosters load Z_L can be obtained as the NIC output with the parallel positive RLC load Z_N ($Z_L = -Z_N$). It is simple to see the output noise power should comprise the noise coming from the positive load Z_N as well as the NIC itself. Here, however we only take the thermal noise arise from the load resistor R_N into consideration by assumption of an ideal noise-free NIC which despite plays an important part of noise effect in case of actual NIC realization.

For an ideal NIC, we can present it by h parameters in the usual manner

$$\begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ v_2 \end{pmatrix}. \quad (2)$$

For Z_L to be the negative of Z_N, it is required that $h_{11} = h_{22} = 0$ and $h_{12} = h_{21} = \pm 1$. When $h_{12} = h_{21} = -1$, the circuit is called a voltage inversion NIC (VNIC). On the other hand $h_{12} = h_{21} = 1$, it is called a current inversion NIC (CNIC). For both situations, we can notice that the mean square value of input current of the NIC should be the same as the output. The Johnson-Nquist noise arise from the resistor R_N can be described as a mean square impressed current $I(\omega) = I_0 \exp(j\omega t)$ from an infinite-impedance source in shunt with the noise free conductance $G_N = 1/R_N$, where $R_N = -R_L > 0$ is the NIC load resistance. Thus we can obtain the mean square noise current produced by the active loads as $I_0 I_0^* = 4KTG_N df = 4KTdf/|R_L|$ due to the unity gain of ideal NIC.

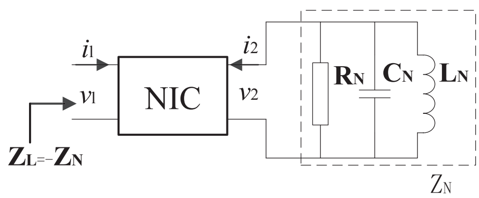

Fig. 3. Two port presentation of a negative impedance converter (NIC).

Now we will investigate the noise figure of active metamaterials with non-Fosters loading which is one of the most important factors for any active systems. Let us consider a system including a set of N-element non-Foster loaded loops as shown in Fig. 4 where the source loop and the load loop are located at both ends. As pointed out in [10], the available signal power at the input can be obtained as $S_i = {V_s}^2/(4R_{source})$ and the available noise power is given by $N_i = KTB$. In order to obtain the noise figure, the most straightforward way is to develop the impedance matrix of the equivalent circuit model and make independent analysis on the noise arising from the metallic conductors as well as the NIC load resistors respectively.

Assuming noise free conditions for all elements, it is not difficult to derive the relationship between the voltages and the related currents through the loop in the from of matrix as

$$\mathbf{V} = \mathbf{ZI}, \quad (3)$$

where $\mathbf{V} = (V_s, 0 \ldots, 0)$ and $\mathbf{I} = (I_1, I_1 \ldots I_N)$ are $N-$dimensional vectors. The output signal power is thereby obtained

$$S_o = I_N I_N^* R_{load}. \quad (4)$$

Similarly, replacing the voltage vector with $\mathbf{V}_s = (V_{n1}, 0 \ldots, 0)$ where $V_{n1}V_{n1}^* = 4KTR_{source}B$ is the mean square value of the voltage arising from the source resistor, it is simple to yield the noise current vector $\mathbf{I}_s = (I_{n1}, I_{n2} \ldots I_{nN})$. Furthermore, the output noise power due to the source resistance can be obtained

$$N_{o1} = I_{nN} I_{nN}^* R_{load}. \quad (5)$$

According to the properties of their generations, the noise sources of non-Foster loaded loop arrays can be classified into two groups apart from the source resistor R_{source}. The output noise current produced by the loop conductors can be obtained in a similar manner, using its respective symmetric impedance matrix. The $N-$dimensional current vector $\mathbf{I}_1^{(n)} = (i_1, i_2 \ldots i_N)$ due to the noise source at element n ($n = 1 \ldots N$) can be solved by applying the matrix transformation

$$\mathbf{I}_1^{(n)} = \mathbf{Z}_1^{-1} \mathbf{V}_1^{(n)}, \quad (6)$$

where $\mathbf{V}_1^{(n)} = (0, \ldots, 0, V_n, 0, \ldots, 0)$ is the corresponding voltage vector and $V_n V_n^* = 4KTR_w df$ is the mean square impressed voltage arising from the ohmic resistance. Since the noise voltage sources of different loops are un-correlated, then the output noise power produced by the loop conductors can be obtained by summing up the response for all the voltage vectors

$$N_{o2} = \sum_n (\int i_N i_N^* R_{load}). \tag{7}$$

In a similar manner, the relationship between the voltages across the loop and the related noise currents produced by the NIC loads can be written as

$$\mathbf{V}_2^{(n)} = \mathbf{Y}_2^{-1} \mathbf{I}_2^{(n)}, \tag{8}$$

where $\mathbf{V}_2^{(n)} = (v_1, v_2 \ldots v_N)$ is the $N-$dimensinal voltage vector and $\mathbf{I}_2^{(n)} = (0, \ldots, 0, I_n, 0, \ldots, 0)$ is the corresponding current vector which takes only the noise current source arising from the load resistance R_L at element *n* into consideration. Following the previous analysis, one may directly derive the output noise power produced by the NIC loads

$$N_{o3} = \sum_n (\int \frac{V_N V_N^*}{|Z_0 + R_{load}|^2} R_{load}). \tag{9}$$

Thus, the overall output noise power is given by $N_o = N_{o1} + N_{o2} + N_{o3}$. Now the noise figure for 1D case can be obtained.

$$\begin{aligned} F &= \frac{S_i/N_i}{S_0/N_0} = \frac{V_s^2}{4KTR_{source}B} \\ &\times \frac{I_{nN} I_{nN}^* + \sum(\int i_N i_N^*) + \sum(\int \frac{V_N V_N^*}{|Z_0+R_{load}|^2})}{I_N I_N^*}. \end{aligned} \tag{10}$$

As an example, the analytical results of a system made up a 1D array of 13 non-Foster loaded loops are plotted with the load inductances varying as shown in Fig. 5 It is clear that the frequency where the extreme of the noise figure occur will move upward for an increase of the load inductance ($L_L < 0$). Solving the derivative function of Eq. (10), the corresponding frequency point can be obtained

$$\omega_p = \omega\Big|_{\partial F/\partial \omega = 0} \approx \frac{1}{\sqrt{L_L C_L}}. \tag{11}$$

Comparing with the distribution of the effective parameters as shown in Fig. 2, it is not difficult to further extend to the relationship between the permeability and the noise figure with respect to the load inductance. In the case of a system made up of 1D non-Foster loading medium a less dispersive, albeit less negative MNG or MNZ band can be obtained with the decreased noise figure as the load inductance increases. Finally, it is possible to extend our analysis to 2D and 3D cases following the proposed approach.

IV. Conclusion

The noise performance of active broadband metamaterials consisting of loops with NIC loading are analysed based on the circuit model established in this paper. Starting from a zero-dimensional case, the analytical result of the noise figure

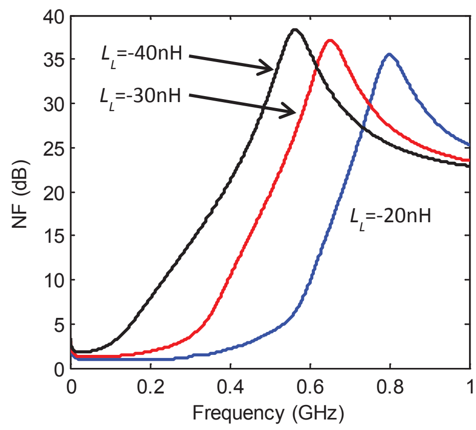

Fig. 5. The noise figure of one-dimensional non-Foster loaded loop arrays.

is given as well as the graphic description of the relationship between the noise figure and the NIC load inductance which plays an important part in stability control. Furthermore, we also give a solution to design a broadband MNG or MNZ with low noise figure by associating the distribution of the effective parameters.

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