Proceedings of the "2013 International Symposium on Electromagnetic Theory"

23PM1P-20

Electromagnetic Scattering from Multiple Objects by Superposition Solution Combined with Method of Moments

Masahiro Tanaka, Kazuo Tanaka

Faculty of Engineering, Gifu University
1-1 Yanagido, Gifu 501-1193, Japan
masahiro@info.gifu-u.ac.jp

***Abstract*—We apply the superposition solution combined with the method of moments into electromagnetic scattering problem from multiple objects. We compare our numerical results with those by the standard method of moments. We show that our results agree with the standard method of moments for some cases, but disagree for other cases.**

I. Introduction

The electromagnetic scattering problems from multiple objects include many important applications. Some examples of the applications are frequency selective surface, electromagnetic bandgap structure, photonic crystal structure, and metamaterial. They are composed of small objects periodically arranged. Another example is electromagnetic scattering by raindrop, and it is composed of small objects random-distributed.

When objects are spherical in shape, we can analytically evaluate electromagnetic scattering property by the superposition solution combined with Mie's theory [1]-[3].

In order to treat arbitrary shaped objects, we employ the method of moments (MoM) instead of Mie's theory. Namely, we apply the superposition solution combined with MoM into electromagnetic scattering problem from multiple objects in this paper.

When we solve scattering problem from multiple objects by the standard MoM, we have to treat all objects at same time, and have to solve large matrix equation. On the other hand, when we apply the superposition solution combined with MoM, we have to solve matrix equation for single object many times.

The iterative progressive numerical method (IPNM) [4]-[6] might be similar, but we consider that the superposition solution combined with MoM is different from IPNM.

In this paper, we consider two-dimensional structure, and a plane wave incident on a configuration of dielectric cylinders. The electric field is parallel to the axes of cylinders. A time dependence $\exp(j\omega t)$ is assumed and suppressed throughout this paper.

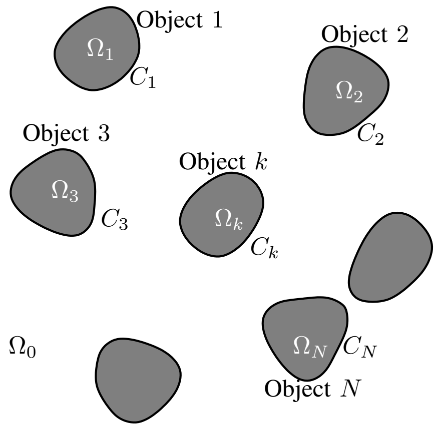

Fig. 1. Model of multiple objects

II. Model

Figure 1 shows a model of multiple dielectric objects, where the number of objects is N. Each object has arbitrary in shape and position. We consider only the case that each object is smaller than the wavelength of incident wave.

We define some symbols referring Fig. 1. The inside region of Object k is denoted as Ω_k, the boundary C_k, and the refractive index n_k. The outside region of all objects is denoted as Ω_0, and the refractive index n_0.

We assume that an plane wave incidents on the cylinders.

III. Formulation

A. Integral Equation Method for Single Object

We consider that Object k is only located in the medium with refractive index n_0. When Object k, whose boundary is C_k, is excited by electromagnetic wave with electric field $E_k^{imp}(\boldsymbol{x})$, the following integral equations are derived for the outside region whose refractive index is n_0, and the inside region whose refractive index is n_k.

$$\left.\begin{array}{cc} E_k(\boldsymbol{x}) & (\boldsymbol{x} \text{ out of } \Omega_k) \\ \frac{1}{2}E_k(\boldsymbol{x}) & (\boldsymbol{x} \text{ on } C_k) \\ 0 & (\boldsymbol{x} \text{ in } \Omega_k) \end{array}\right\} = -\int_{C_k}\left[G_0(\boldsymbol{x}|\boldsymbol{x}')\frac{\partial E_k(\boldsymbol{x}')}{\partial n'} - E_k(\boldsymbol{x}')\frac{\partial G_0(\boldsymbol{x}|\boldsymbol{x}')}{\partial n'}\right]dl' + E_k^{imp}(\boldsymbol{x}) \quad (1)$$

$$\left.\begin{array}{cc} E_k(\boldsymbol{x}) & (\boldsymbol{x} \text{ in } \Omega_k) \\ \frac{1}{2}E_k(\boldsymbol{x}) & (\boldsymbol{x} \text{ on } C_k) \\ 0 & (\boldsymbol{x} \text{ out of } \Omega_k) \end{array}\right\} = \int_{C_k}\left[G_k(\boldsymbol{x}|\boldsymbol{x}')\frac{\partial E_k(\boldsymbol{x}')}{\partial n'} - E_k(\boldsymbol{x}')\frac{\partial G_k(\boldsymbol{x}|\boldsymbol{x}')}{\partial n'}\right]dl' \quad (2)$$

where Green's function G_i is given by

$$G_i = -\frac{j}{4}H_0^{(2)}(k_0 n_i|\boldsymbol{x} - \boldsymbol{x}'|) \quad (i = 0, k) \quad (3)$$

and $H_0^{(2)}(x)$ indicates the 0-th order Hankel function of second kind.

Applying the method of moments into Eqs. (1), (2), we can obtain matrix equation

$$A_k \boldsymbol{x}_k = \boldsymbol{b}_k \quad (4)$$

where the observation $\boldsymbol{x}$ locates on the boundary C_k.

B. Superposition Solution

When an object is illuminated by an incident wave, the first order scattered field is generated. We denote the first order scattered electric field by Object k as $E_k^{sca,(1)}$. We note that $E_k^{sca,(1)}$ is calculated from Eq. (1) when $\partial E_k(\boldsymbol{x}')/\partial n'$ and $E_k(\boldsymbol{x}')$ is known.

We consider that the sum of the first order scattered electric field except for Object k is the second order impressed electric field $E_k^{imp,(2)}$ on Object k. When Object k is excited by the second order impressed wave, the second order scattered electric field $E_k^{sca,(2)}$ is generated.

In general, the n-th impressed electric field $E_k^{imp,(n)}$ which excites Object k is given by

$$E_k^{imp,(n)} = \left\{\begin{array}{cc} E^{inc} & (n = 1) \\ \sum_{i=1,i\neq k}^{N} E_i^{sca,(n-1)} & (n \neq 1) \end{array}\right. \quad (5)$$

When Object k is excited by the n-th impressed electric field $E_k^{imp,(n)}$, the n-th order scattered field $E_k^{sca,(n)}$ is generated.

C. Algorithm

We assume that the number of multiple objects is N, and the maximum order of scattered electric field is N_{itr}.

First, we execute two processes for each object once.

1) Calculate coefficient matrix A_k.
2) Perform LU decomposition into A_k.

Then, we execute two processes for each object N_{itr} times.

3) Calculate $\boldsymbol{b}_k^{(n)}$, that is a right-hand side vector of Eq. (4), by Eq. (5).
4) Solve matrix equation $A_k \boldsymbol{x}_k^{(n)} = \boldsymbol{b}_k^{(n)}$ by backward substitution.

We note that multiple scattering effect is included in Eq. (5).

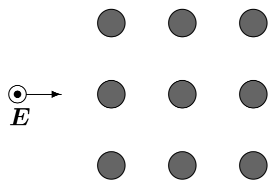

Fig. 2. Model of 3×3 dielectric circular cylinders

TABLE I
PARAMETERS USED IN SIMULATIONS

Refractive index of outside	1.0
Refractive index of cylinder	1.414, 1.5, 1.732, 2.0
Lattice constant	0.5λ
Radius of cylinder	0.1λ
Incident angle	180 [degrees]

IV. Numerical Results

We investigate the scattering problems composed of 3×3 dielectric circular cylinders shown in Fig. 2. The dielectric cylinders are arranged in square lattice. The parameters are shown in TABLE I.

Figures 3 and 4 show the comparisons of the coefficients of basis function obtained by the superposition solution combined with MOM and the standard MoM, where the refractive indices of cylinder are 1.414 and 2.0 respectively.

We see that our results agree with those by the standard MoM from Fig. 3, but disagree from Fig. 4. We note that our results agree when the refractive indices are 1.414 and 1.5, but disagree when the refractive indices are 1.732 and 2.0. The reason is under investigation.

V. Conclusion

We have applied the superposition solution combined with the method of moments into electromagnetic scattering from multiple objects. We have shown some numerical results for the case of dielectric circular cylinders arranged in square lattice. We found that our results agree with those by the standard method of moments when the refractive indices of cylinders are small, but disagree when they are large.

Acknowledgment

This work is partly supported by Grant-in-Aid for Scientific Research (C) (24510253).

References

[1] D. W. Mackowski, Analysis of radiative scattering for multiple sphere configurations, Proc. R. Soc. Lond. A, vol. 433, pp. 599-614, 1991.
[2] Y. L. Xu, Electromagnetic scattering by an aggregate of spheres, Appl. Opt., vol. 34, no. 21, pp. 4573-4588, 1995.
[3] N. T. Dong, M. Tanaka, K. Tanaka, Improved algorithms for calculating addition coefficients in electromagnetic scattering by multi-sphere systems, IEEE Trans. Electron. vol. E95-C, no. 1, pp. 27-35, 2012.

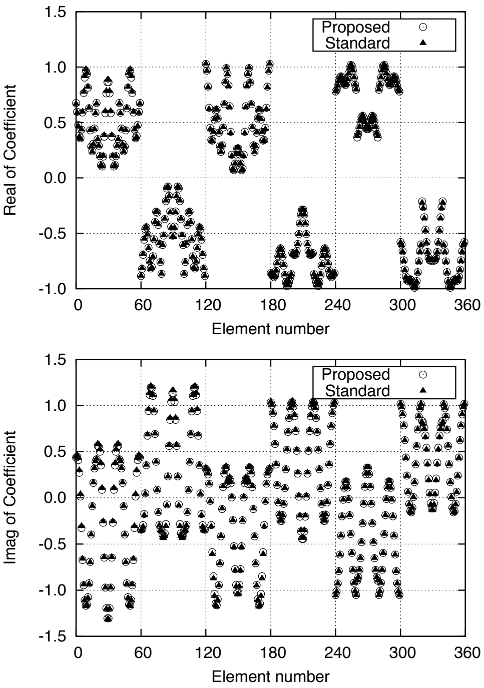

Fig. 3. Comparison of coefficients of basis functions for refractive indices of cylinders 1.414.

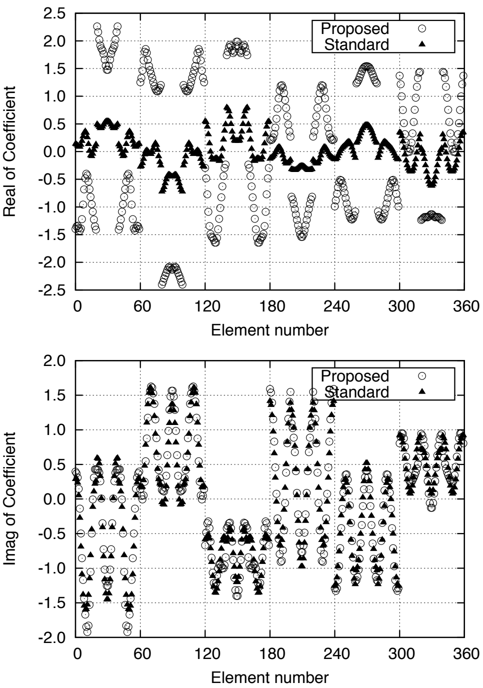

Fig. 4. Comparison of coefficients of basis functions for refractive indices of cylinders 2.0.

[4] L. Shafai: "A progressive numerical method and its application to large field problems in antennas and electromagnetic scattering," Can. Elec. Eng. J., vol.2, no.4, pp.17-23, 1977.

[5] N. Nakashima, M. Tateiba: "A comparative study on iterative progressive numerical methods for boundary element analysis of electromagnetic multiple scattering," IEICE Trans. Electron., vol.E94-C, no.5, pp.865-873, 2011.

[6] N. Nakashima, S. Fujino, M. Tateiba: "The IDR-based IPNMs for the fast boundary element analysis of electromagnetic wave multiple scattering," IEICE Trans. Electron., vol.E95-C, no.1, pp.63-70, 2012.

[7] R.F. Harrington, Field computation by moment methods, IEEE Press, 1993.