Formulation of Electromagnetic Scattering from Surface-Relief Grating with Period Modulation

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***Abstract*—This paper considers the electromagnetic scattering problem of surface-relief grating with period modulation, and presents a formulation based on the coordinate transformation method (C-method). The C-method is originally developed to analyze the plane-wave scattering from perfectly periodic structures, and uses the pseudo-periodic property of the fields. Since the structure under consideration is not perfectly periodic, the fields are not pseudo-periodic and the C-method cannot be directly applied. This paper introduces the pseudo-periodic Fourier transform to convert the fields in imperfectly periodic structures to pseudo-periodic ones, and the C-method becomes then applicable.**

I. Introduction

When a plane wave illuminates a perfectly periodic structure, the Floquet theorem asserts that the scattered fields are pseudo-periodic (namely, each field component is a product of a periodic function and an exponential phase factor) and the analysis region can be reduced to one periodicity cell. However, in case of structure in which the periodicity is collapsed even if locally, the Floquet theorem is no longer applicable and the computation has been mainly performed with the finite difference time-domain method, the finite element method, the time-domain beam propagation method, the method of the fictitious sources, etc., in which the analysis region has to cover whole scatterers under consideration.

This paper deals with the electromagnetic scattering from a surface-relief grating with period modulation and shows a spectral-domain formulation based on the differential method of Chandezon et al. (C-method) [1] and the pseudo-periodic Fourier transform (PPFT) [2]. The PPFT converts an arbitrary function into a pseudo-periodic one and the transformed function can be expressed in the generalized Fourier series expansion [3]. The generalized Fourier coefficients, which are functions of the transform parameter, are approximated by introducing a discretization, and then the conventional formulations for perfectly periodic structures based on the Floquet theorem (such as the C-method) can be applied to analyze the scattering problem of imperfectly periodic structures. The transformed function has also a periodic property in terms of the transform parameter, which is related to the wavenumber, and the analysis region in the spectral domain is reduced to the Brillouin zone. Therefore, the discretization scheme in terms of the transform parameter can be considered in the Brillouin zone.

II. Settings of the Problem

The present paper considers the scattering problem of electromagnetic fields with a time-dependence $\exp(-i\,\omega\,t)$ from surface-relief grating, in which structural period is locally changed. Figure 1 shows an example of the structures under consideration. The structure is uniform in the z-direction and the y-axis is perpendicular to the periodicity direction though the periodicity is not perfect. The equation of the corrugated surface is expressed by $y = g(x)$. Let $g_p(x)$ be a smooth periodic function with the period d and $\Delta(x)$ be a function that has nonzero value only at $a_1 < x < a_2$. Then the surface profile function $g(x)$ is given by

$$g(x) = g_p\left(\int_0^x \frac{d}{d+\Delta(\eta)}\,d\eta\right). \tag{1}$$

The local period of $g(x)$ is $d+\Delta(x)$, and changes from the original period d in $a_1 < x < a_2$. The surrounding region $y > g(x)$ is filled with a homogeneous and isotropic medium with the permittivity ε_s and the permeability μ_s, and the substrate region $y < g(x)$ is also filled with a homogeneous and isotropic medium described by the permittivity ε_c and the permeability μ_c. The regions $y > g(x)$ and $y < g(x)$ are specified by s and c, respectively, and the wavenumber in each region is denoted by $k_r = \omega\sqrt{\varepsilon_r\,\mu_r}$ for $r = s, c$. The electromagnetic fields are uniform in the z-direction and two-dimensional scattering problem is considered here. Two fundamental polarizations are expressed by TE and TM, in

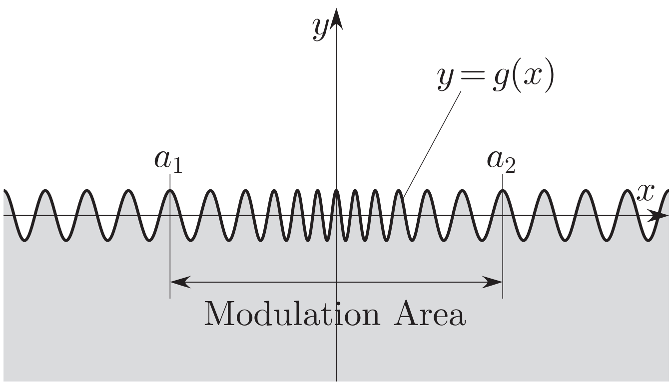

Fig. 1. Surface-relief grating with period modulation

which the electric and the magnetic fields are respectively parallel to the z-axis. The incident field is supposed to illuminate the corrugated surface from the region s and there exists no source inside the region c.

III. Field Expressions in Homogeneous Regions

First, we consider the field in the TE-polarization. From Maxwell's curl equations and the constitutive relations, the filed in the region r ($r = s, c$) satisfy the following relations:

$$\frac{\partial}{\partial y} E_z(x,y) = i\,\omega\,\mu_r\,H_x(x,y) \tag{2}$$

$$\frac{\partial}{\partial x} E_z(x,y) = -i\,\omega\,\mu_r\,H_y(x,y) \tag{3}$$

$$\frac{\partial}{\partial x} H_y(x,y) - \frac{\partial}{\partial y} H_x(x,y) = -i\,\omega\,\varepsilon_r\,E_z(x,y). \tag{4}$$

We introduce a curvilinear coordinate system O-uvz, which is related to the original coordinate system O-xyz by continuous transformation equations:

$$u = \int_0^x \frac{d}{d + \varDelta(\eta)}\,d\eta \tag{5}$$

$$v = y - g(x) \tag{6}$$

where $\varDelta(\eta)$ is supposed to provide onto and one-to-one correspondence between x and u. Using the chain rule, the differential operators in terms of x and y are transformed as

$$\frac{\partial}{\partial x} = (1 + f(u))\left(\frac{\partial}{\partial u} - \dot{g}_p(u)\,\frac{\partial}{\partial v}\right) \tag{7}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial v} \tag{8}$$

with

$$f(u) = \frac{\partial u}{\partial x} - 1 \tag{9}$$

$$\dot{g}_p(u) = \frac{d\,g_p(u)}{du}. \tag{10}$$

Then, Eqs. (2)–(4) are transformed as follows:

$$\frac{\partial}{\partial v} E_z(u,v) = i\,\omega\,\mu_r\,H_x(u,v) \tag{11}$$

$$(1 + f(u))\left(\frac{\partial}{\partial u} - \dot{g}_p(u)\,\frac{\partial}{\partial v}\right) E_z(u,v) = -i\,\omega\,\mu_r\,H_y(u,v) \tag{12}$$

$$(1 + f(u))\left(\frac{\partial}{\partial u} - \dot{g}_p(u)\,\frac{\partial}{\partial v}\right) H_y(u,v) - \frac{\partial}{\partial v} H_x(u,v) = -i\,\omega\,\varepsilon_r\,E_z(u,v). \tag{13}$$

Here, we introduce the PPFT. Let $\varphi(u)$ be a function of u and d be a positive real constant. Then the PPFT and its inverse transform are formally defined by

$$\overline{\varphi}(u;\xi) = \sum_{m=-\infty}^{\infty} \varphi(u - m\,d)\,e^{i\,m\,d\,\xi} \tag{14}$$

$$\varphi(u) = \frac{1}{k_d}\int_{-k_d/2}^{k_d/2} \overline{\varphi}(u;\xi)\,d\xi \tag{15}$$

where ξ is the transform parameter and $k_d = 2\,\pi/d$. The transformed functions have pseudo-periodic property in terms of u: $f(u - md;\xi) = f(u;\xi)\,\exp(-i\,m\,d\,\xi)$ for any integer m, and also have periodic property in terms of ξ: $f(u;\xi - m\,k_d) = f(u;\xi)$. We use the period of $g_p(x)$ for the positive real constant d for the PPFT. Then, k_d becomes the inverse lattice constant and the periodicity cell of the transformed function gives the Brillouin zone. Applying the PPFT to Eqs. (11)–(13) and using the formula shown in [2], we obtain the following relations:

$$\frac{\partial}{\partial v}\,\overline{E}_z(u;\xi,v) = i\,\omega\,\mu_r\,\overline{H}_x(u;\xi,v) \tag{16}$$

$$\left(\frac{\partial}{\partial u} - \dot{g}_p(u)\,\frac{\partial}{\partial v}\right)\overline{E}_z(u;\xi,v) + \frac{1}{k_d}\int_{-k_d/2}^{k_d/2} \overline{f}(u;\xi-\xi')\left(\frac{\partial}{\partial u} - \dot{g}_p(u)\,\frac{\partial}{\partial v}\right)\overline{E}_z(u;\xi',v)\,d\xi' = -i\,\omega\,\mu_r\,\overline{H}_y(u;\xi,v) \tag{17}$$

$$\left(\frac{\partial}{\partial u} - \dot{g}_p(u)\,\frac{\partial}{\partial v}\right)\overline{H}_y(u;\xi,v) + \frac{1}{k_d}\int_{-k_d/2}^{k_d/2} \overline{f}(u;\xi-\xi')\left(\frac{\partial}{\partial u} - \dot{g}_p(u)\,\frac{\partial}{\partial v}\right)\overline{H}_y(u;\xi',v)\,d\xi' - \frac{\partial}{\partial v}\,\overline{H}_x(u;\xi,v) = -i\,\omega\,\varepsilon_r\,\overline{E}_z(u;\xi,v). \tag{18}$$

Since the transformed functions have pseudo-periodic property in terms of u, the transformed fields are expressed in the generalized Fourier series [3]. For example, the z-component of electric field is approximately written as

$$\overline{E}_z(u;\xi,v) = \sum_{n=-N}^{N} \overline{E}_{z,n}(\xi,v)\,e^{i\,\alpha_n(\xi)\,u} \tag{19}$$

$$\alpha_n(\xi) = \xi + n\,k_d \tag{20}$$

where N denotes the truncation order and $\overline{E}_{z,n}(\xi,v)$ are the nth-order coefficients. To treat the coefficients systematically, we introduce $(2\,N+1)\times 1$ column matrices; for example, the coefficients of $\overline{E}_{z,n}(\xi,v)$ are expressed by a column matrix $\overline{\boldsymbol{e}}_z(\xi,v)$ in such a way that its nth-components are given by $\overline{E}_{z,n}(\xi,v)$. Then Eqs. (16)–(18) yield the following relations:

$$\frac{\partial}{\partial v}\,\overline{\boldsymbol{e}}_z(\xi,v) = i\,\omega\,\mu_r\,\overline{\boldsymbol{h}}_x(\xi,v) \tag{21}$$

$$\left(i\,\overline{\boldsymbol{X}}(\xi) - [\![\dot{g}_p]\!]\,\frac{\partial}{\partial v}\right)\overline{\boldsymbol{e}}_z(\xi,v) + \frac{1}{k_d}\int_{-k_d/2}^{k_d/2} [\![\overline{f}]\!](\xi-\xi')\left(i\,\overline{\boldsymbol{X}}(\xi') - [\![\dot{g}_p]\!]\,\frac{\partial}{\partial v}\right)\overline{\boldsymbol{e}}_z(\xi',v)\,d\xi' = -i\,\omega\,\mu_r\,\overline{\boldsymbol{h}}_y(\xi,v) \tag{22}$$

$$\left(i\,\overline{\boldsymbol{X}}(\xi) - [\![\dot{g}_p]\!]\,\frac{\partial}{\partial v}\right)\overline{\boldsymbol{h}}_y(\xi,v) + \frac{1}{k_d}\int_{-k_d/2}^{k_d/2} [\![\overline{f}]\!](\xi-\xi')\left(i\,\overline{\boldsymbol{X}}(\xi') - [\![\dot{g}_p]\!]\,\frac{\partial}{\partial v}\right)\overline{\boldsymbol{h}}_y(\xi',v)\,d\xi' - \frac{\partial}{\partial v}\,\overline{\boldsymbol{h}}_x(\xi,v) = -i\,\omega\,\varepsilon_r\,\overline{\boldsymbol{e}}_z(\xi,v) \tag{23}$$

with

$$\left([\![\overline{f}]\!](\xi)\right)_{n,m} = \frac{1}{d}\int_{-\infty}^{\infty} f(u)\, e^{-i\,\alpha_{n-m}(\xi)\,u}\, du \qquad (24)$$

$$\left([\![\dot{g}_p]\!]\right)_{n,m} = \frac{1}{d}\int_{0}^{d} \dot{g}_p(u)\, e^{-i(n-m)k_d\,u}\, du \qquad (25)$$

$$\left(\overline{\boldsymbol{X}}(\xi)\right)_{n,m} = \delta_{n,m}\,\alpha_n(\xi) \qquad (26)$$

where the symbol $\delta_{n,m}$ stands for Kronecker's delta.

Considering the periodicity in terms of ξ, Eqs. (21)–(23) have to be satisfied for arbitrary $\xi \in (-k_d/2, k_d/2]$. However, we take L sample points inside the Brillouin zone and assume that Eqs. (21)–(23) are satisfied at these sample points. Also the integrations in Eqs. (22) and (23) are approximated by an appropriate numerical integration scheme using the same sample points. Let $\{\xi_l\}_{l=1}^{L}$ and $\{w_l\}_{l=1}^{L}$ be the sample points and the weights chosen by a numerical integration scheme. Then we may obtain the following relations:

$$\frac{d}{dv}\,\widetilde{\boldsymbol{e}}_z(v) = i\,\omega\,\mu_r\,\widetilde{\boldsymbol{h}}_x(v) \qquad (27)$$

$$\left(\widetilde{\boldsymbol{C}}_1 + \widetilde{\boldsymbol{C}}_2\,\frac{d}{dv}\right)\widetilde{\boldsymbol{e}}_z(v) = -\omega\,\mu_r\,\widetilde{\boldsymbol{h}}_y(v) \qquad (28)$$

$$\left(\widetilde{\boldsymbol{C}}_1 + \widetilde{\boldsymbol{C}}_2\,\frac{d}{dv}\right)\widetilde{\boldsymbol{h}}_y(v) + i\,\frac{d}{dv}\,\widetilde{\boldsymbol{h}}_x(v) = -\omega\,\varepsilon_r\,\widetilde{\boldsymbol{e}}_z(v) \qquad (29)$$

with

$$\widetilde{\boldsymbol{e}}_z(v) = \begin{pmatrix} \overline{\boldsymbol{e}}_z(\xi_1, v) \\ \vdots \\ \overline{\boldsymbol{e}}_z(\xi_L, v) \end{pmatrix} \qquad (30)$$

$$\widetilde{\boldsymbol{C}}_1 = \widetilde{\boldsymbol{F}}\,\widetilde{\boldsymbol{X}} \qquad (31)$$

$$\widetilde{\boldsymbol{C}}_2 = i\,\widetilde{\boldsymbol{F}}\,[\![\dot{g}_p]\!] \qquad (32)$$

$$\widetilde{\boldsymbol{F}} = \boldsymbol{I} + \begin{pmatrix} \frac{w_1}{k_d}\,[\![\overline{f}]\!](\xi_1-\xi_1) & \cdots & \frac{w_L}{k_d}\,[\![\overline{f}]\!](\xi_1-\xi_L) \\ \vdots & \ddots & \vdots \\ \frac{w_1}{k_d}\,[\![\overline{f}]\!](\xi_L-\xi_1) & \cdots & \frac{w_L}{k_d}\,[\![\overline{f}]\!](\xi_L-\xi_L) \end{pmatrix} \qquad (33)$$

$$\widetilde{\boldsymbol{X}} = \begin{pmatrix} \overline{\boldsymbol{X}}(\xi_1) & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \overline{\boldsymbol{X}}(\xi_L) \end{pmatrix} \qquad (34)$$

$$[\![\dot{g}_p]\!] = \begin{pmatrix} [\![\dot{g}_p]\!] & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & [\![\dot{g}_p]\!] \end{pmatrix} \qquad (35)$$

where the definition of the column matrices $\widetilde{\boldsymbol{h}}_x(v)$ and $\widetilde{\boldsymbol{h}}_x(v)$ are similar to $\widetilde{\boldsymbol{e}}_z(v)$. After a simple calculation, Eqs. (27)–(29) yield the following coupled differential-equation set:

$$\begin{pmatrix} \widetilde{\boldsymbol{e}}_z(v) \\ -i\,\frac{d}{dv}\,\widetilde{\boldsymbol{e}}_z(v) \end{pmatrix} = -i\,\frac{d}{dv}\,\boldsymbol{M}_r \begin{pmatrix} \widetilde{\boldsymbol{e}}_z(v) \\ -i\,\frac{d}{dv}\,\widetilde{\boldsymbol{e}}_z(v) \end{pmatrix} \qquad (36)$$

with

$$\boldsymbol{M}_r = \begin{pmatrix} i\,\widetilde{\boldsymbol{D}}_r\left(\widetilde{\boldsymbol{C}}_1\,\widetilde{\boldsymbol{C}}_2 + \widetilde{\boldsymbol{C}}_2\,\widetilde{\boldsymbol{C}}_1\right) & -\widetilde{\boldsymbol{D}}_r\left(\widetilde{\boldsymbol{C}}_2^2 - \boldsymbol{I}\right) \\ \boldsymbol{I} & \boldsymbol{0} \end{pmatrix} \qquad (37)$$

$$\widetilde{\boldsymbol{D}}_r = \left({k_r}^2\,\boldsymbol{I} - \widetilde{\boldsymbol{C}}_1^2\right)^{-1} \qquad (38)$$

where $\boldsymbol{I}$ denotes the identity matrix and the superscript "-1" stands for the matrix inverse. The general solution to the coupled differential-equation set (36) can be obtained by solving the eigenvalue-eigenvector problems because the matrix of coefficients $\boldsymbol{M}_r$ is constant. The $2\,L\,(2\,N+1)$ eigenvalues can be divided into two sets, each containing $L\,(2\,N+1)$ eigenvalues. The first set contains the negative real eigenvalues and the complex eigenvalues with positive imaginary parts, and the second set contains those with the opposite signs. We denote the reciprocals of the eigenvalues of $\boldsymbol{M}_r$ by $\{\eta_{r,n}\}_{n=1}^{2L(2N+1)}$, in which $\{\eta_{r,n}\}_{n=1}^{L(2N+1)}$ correspond to the first set and $\{\eta_{r,n}\}_{n=L(2N+1)+1}^{2L(2N+1)}$ correspond to the second set. Let $\boldsymbol{p}_{r,n}$ denote the eigenvector of $\boldsymbol{M}_r$ associating with the eigenvalue $1/\eta_{r,n}$. Then the matrix for diagonalization is constructed as

$$\begin{pmatrix} \boldsymbol{P}_{r,11} & \boldsymbol{P}_{r,12} \\ \boldsymbol{P}_{r,21} & \boldsymbol{P}_{r,22} \end{pmatrix} = \begin{pmatrix} \boldsymbol{p}_{r,1} & \cdots & \boldsymbol{p}_{r,2L(2N+1)} \end{pmatrix}, \qquad (39)$$

and the general solution to the coupled differential-equation set (36) is written in the following form:

$$\widetilde{\boldsymbol{e}}_z(v) = \boldsymbol{P}_{r,11}\,\boldsymbol{a}_{e,r}^{(-)}(v) + \boldsymbol{P}_{r,12}\,\boldsymbol{a}_{e,r}^{(+)}(v) \qquad (40)$$

$$\frac{d}{dv}\,\widetilde{\boldsymbol{e}}_z(v) = i\,\boldsymbol{P}_{r,21}\,\boldsymbol{a}_{e,r}^{(-)}(v) + i\,\boldsymbol{P}_{r,22}\,\boldsymbol{a}_{e,r}^{(+)}(v) \qquad (41)$$

where the column matrices $\boldsymbol{a}_{f,r}^{(-)}(v)$ and $\boldsymbol{a}_{f,r}^{(+)}(v)$ are the amplitudes of the eigenmodes propagating in the negative and the positive v-directions, respectively. The relation between the modal amplitudes at $v = v'$ and $v = v''$ is given by

$$\begin{pmatrix} \boldsymbol{a}_{e,r}^{(-)}(v') \\ \boldsymbol{a}_{e,r}^{(+)}(v') \end{pmatrix} = \boldsymbol{U}_r(v - v'') \begin{pmatrix} \boldsymbol{a}_{e,r}^{(-)}(v'') \\ \boldsymbol{a}_{e,r}^{(+)}(v'') \end{pmatrix} \qquad (42)$$

$$\left(\boldsymbol{U}_r(v)\right)_{n,m} = \delta_{n,m}\,e^{i\,\eta_{r,n}\,v} \qquad (43)$$

For the TM-polarization, following the same process, we may understand that $\widetilde{\boldsymbol{h}}_z(v)$ satisfies the same coupled differential-equation set with Eq. (36). Therefore, the general solution is written as follows

$$\widetilde{\boldsymbol{h}}_z(v) = \boldsymbol{P}_{r,11}\,\boldsymbol{a}_{h,r}^{(-)}(v) + \boldsymbol{P}_{r,12}\,\boldsymbol{a}_{h,r}^{(+)}(v) \qquad (44)$$

$$\frac{d}{dv}\,\widetilde{\boldsymbol{h}}_z(v) = i\,\boldsymbol{P}_{r,21}\,\boldsymbol{a}_{h,r}^{(-)}(v) + i\,\boldsymbol{P}_{r,22}\,\boldsymbol{a}_{h,r}^{(+)}(v) \qquad (45)$$

with

$$\begin{pmatrix} \boldsymbol{a}_{h,r}^{(-)}(v') \\ \boldsymbol{a}_{h,r}^{(+)}(v') \end{pmatrix} = \boldsymbol{U}_r(v - v'') \begin{pmatrix} \boldsymbol{a}_{h,r}^{(-)}(v'') \\ \boldsymbol{a}_{h,r}^{(+)}(v'') \end{pmatrix}. \qquad (46)$$

IV. Scattering-Matrix

The general solutions separately obtained in the regions s and c can be matched at the grating surface $v = 0$ by using the boundary conditions, which are given by the continuities of the tangential components of the fields. For the TE-polarization, the covariant component of the magnetic field in terms of u is given by $H_t(u,v) = H_x(u,v) + \dot{g}(u)\,H_y(u,v)$, which gives the tangential component of the magnetic field on the grating surface $v = 0$. From Eqs. (27), (28), (40), and

(41), the generalized Fourier coefficients of $\overline{E}_z(u;\xi_l,v)$ and $\overline{H}_t(u;\xi_l,v)$ are expressed in the following form:

$$\begin{pmatrix}\widetilde{\boldsymbol{e}}_z(v)\\ \widetilde{\boldsymbol{h}}_t(v)\end{pmatrix}=\begin{pmatrix}\boldsymbol{P}_{r,11} & \boldsymbol{P}_{r,12}\\ \boldsymbol{Q}_{e,r,21} & \boldsymbol{Q}_{e,r,22}\end{pmatrix}\begin{pmatrix}\boldsymbol{a}_{e,r}^{(-)}(v)\\ \boldsymbol{a}_{e,r}^{(+)}(v)\end{pmatrix} \tag{47}$$

with

$$\boldsymbol{Q}_{e,r,2q}=\frac{i}{\omega\,\mu_r}\left[\widetilde{\boldsymbol{C}}_2\,\widetilde{\boldsymbol{C}}_1\,\boldsymbol{P}_{r,1q}+i\left(\widetilde{\boldsymbol{C}}_2^2-\boldsymbol{I}\right)\boldsymbol{P}_{r,2q}\right] \tag{48}$$

for $q=1,2$. On the other hand, the tangential components of the fields on the grating surface $v=0$ are $H_z(x,y)$ and $E_t(x,y)=E_x(x,y)+\dot{g}(x)\,E_y(x,y)$ for the TE-polarization. The generalized Fourier coefficients of $\overline{H}_z(u;\xi_l,v)$ and $\overline{E}_t(u;\xi_l,v)$ are expressed in the following form:

$$\begin{pmatrix}\widetilde{\boldsymbol{h}}_z(v)\\ \widetilde{\boldsymbol{e}}_t(v)\end{pmatrix}=\begin{pmatrix}\boldsymbol{P}_{r,11} & \boldsymbol{P}_{r,12}\\ \boldsymbol{Q}_{h,r,21} & \boldsymbol{Q}_{h,r,22}\end{pmatrix}\begin{pmatrix}\boldsymbol{a}_{h,r}^{(-)}(v)\\ \boldsymbol{a}_{h,r}^{(+)}(v)\end{pmatrix} \tag{49}$$

$$\boldsymbol{Q}_{h,r,2q}=-\frac{i}{\omega\,\varepsilon_r}\left[\widetilde{\boldsymbol{C}}_2\,\widetilde{\boldsymbol{C}}_1\,\boldsymbol{P}_{r,1q}+i\left(\widetilde{\boldsymbol{C}}_2^2-\boldsymbol{I}\right)\boldsymbol{P}_{r,2q}\right]. \tag{50}$$

Since the incident field illuminates the corrugated surface from the region s, the boundary conditions at $v=0$ provide the following relation:

$$\begin{pmatrix}\boldsymbol{P}_{s,11} & \boldsymbol{P}_{s,12}\\ \boldsymbol{Q}_{f,s,21} & \boldsymbol{Q}_{f,s,22}\end{pmatrix}\begin{pmatrix}\boldsymbol{a}_{f,s}^{(-)}(+0)\\ \boldsymbol{a}_{f,s}^{(+)}(+0)\end{pmatrix}=\begin{pmatrix}\boldsymbol{P}_{c,11}\\ \boldsymbol{Q}_{f,c,21}\end{pmatrix}\boldsymbol{a}_{f,c}^{(-)}(-0) \tag{51}$$

for $f=e,h$. Then, the relation between the amplitudes of the incoming and outgoing fields is derived as

$$\begin{pmatrix}\boldsymbol{a}_{f,s}^{(+)}(+0)\\ \boldsymbol{a}_{f,c}^{(-)}(-0)\end{pmatrix}=\begin{pmatrix}\boldsymbol{S}_{f,11}\\ \boldsymbol{S}_{f,21}\end{pmatrix}\boldsymbol{a}_{f,s}^{(-)}(+0), \tag{52}$$

where the scattering matrices are given by

$$\boldsymbol{S}_{f,11}=-\left(\boldsymbol{P}_{c,11}\,\boldsymbol{Q}_{f,c,21}^{-1}\,\boldsymbol{Q}_{f,s,22}-\boldsymbol{P}_{s,12}\right)^{-1}\left(\boldsymbol{P}_{c,11}\,\boldsymbol{Q}_{f,c,21}^{-1}\,\boldsymbol{Q}_{f,s,21}-\boldsymbol{P}_{s,11}\right) \tag{53}$$

$$\boldsymbol{S}_{f,21}=\boldsymbol{Q}_{f,c,21}^{-1}\left(\boldsymbol{Q}_{f,s,21}+\boldsymbol{Q}_{f,s,22}\,\boldsymbol{S}_{f,11}\right). \tag{54}$$

Here, we denote the $E_z(x,y)$ for the TE-polarization and the $H_z(x,y)$ for the TM-polarization by $\psi(x,y)$ to express both polarizations simultaneously, and the incident field is written as $\psi^{(i)}(x,y)$. The PPFT is applied to the incident field in the O-uvz coordinate system, and the transformed field is expressed in the generalized Fourier series. Then, the nth-order coefficient is obtained by

$$\overline{\psi}_n^{(i)}(\xi_l,v)=\frac{1}{d}\int_{-\infty}^{\infty}\psi^{(i)}(u,v)\,e^{-i\,\alpha_n(\xi_l)\,u}\,du. \tag{55}$$

If $L\,(2\,N+1)\times 2\,L\,(2\,N+1)$ matrices $\boldsymbol{R}_{s,1}$ is defined by the inversion of the diagonalization matrix given in Eq. (39) as

$$\begin{pmatrix}\boldsymbol{R}_{s,1}\\ \boldsymbol{R}_{s,2}\end{pmatrix}=\begin{pmatrix}\boldsymbol{P}_{s,11} & \boldsymbol{P}_{s,12}\\ \boldsymbol{P}_{s,21} & \boldsymbol{P}_{s,22}\end{pmatrix}^{-1}, \tag{56}$$

where the column matrix $\boldsymbol{a}_{f,c}^{(-)}(+0)$, which contains the modal amplitudes of the incident field, are given by

$$\boldsymbol{a}_{f,s}^{(-)}(v)=\boldsymbol{R}_{s,1}\begin{pmatrix}\widetilde{\boldsymbol{\psi}}^{(i)}(v)\\ -i\,\frac{d}{dv}\,\widetilde{\boldsymbol{\psi}}^{(i)}(v)\end{pmatrix}. \tag{57}$$

This means that we may obtain $\boldsymbol{a}_{f,s}^{(-)}(+0)$ from Eq. (57) for a known incident field, and $\boldsymbol{a}_{f,c}^{(+)}(+0)$ and $\boldsymbol{a}_{f,s}^{(-)}(-0)$, which contains the modal amplitudes of the scattered field, are computed by Eq. (52).

V. Conclusion

This paper presents a formulation of the electromagnetic scattering from the surface-relief grating, in which the structural period is locally changed. The present formulation is based on the C-method with the help of the PPFT, and the discretization for the numerical computation is introduced on the transform parameter ξ, which is related to the wavenumber. The main problem on the spectral-domain analysis is generally summarized to the discretization scheme on the wavenumber space. In the present formulation, the discretization scheme can be considered only inside the Brillouin zone owing to the PPFT.

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