Numerical Analysis of Conical Horn with Dielectric Lens in Free-Space Method

Junichi Tsutsumi #1, Koichi Hirayama #2, Takashi Yasui #3, Yasuhide Tsuji *4

Electrical and Electronic Eng., Kitami Institute of Technology
165 Koen-cho, Kitami-shi, 090-8507 Japan
2 hirakc@mail.kitami-it.ac.jp
3 yasui@mail.kitami-it.ac.jp

* Information and Electronic Eng., Muroran Institute of Technology
27-1 Mizumoto-cho, Muroran-shi, 050-8585 Japan
4 y-tsuji@mmm.muroran-it.ac.jp

***Abstract*—A free-space method where a pair of conical horns with dielectric lenses are used has an advantage that one can estimate material constants without electromagnetic absorbers. Since the analysis region for the free-space method in a 3-dimensional (3-D) electromagnetic analysis is very large, we investigate the usefulness of 2-D FEM analysis and the plane wave spectrum method in 3- and 2-D FEM analysis for a conical horn with a dielectric lens in the setup of the free-space method, and illustrate the difference of the focusing property against operating frequencies.**

I. Introduction

Free-space method is one of the measurement methods of permittivity and permeability for materials in microwave and millimeter-wave regime [1], [2]. In a free-space method where a pair of conical horns with dielectric lenses are used, as shown in Fig. 1, sidelobes are low and it has an advantage that one can estimate material constants without electromagnetic absorbers.

A horn antenna with a dielectric lens has been analyzed using FDTD [3] and FEM [4]. However, since both the diameter and the focal length of a dielectric lens in the free-space method are greater than ten times the wavelength of the incident wave, it is difficult to directly analyze an analysis region which includes the focal point of the dielectric lens by a 3-D analysis method. In Ref. [3], the electromagnetic field around the focal point of the lens is estimated using the plane wave spectrum (PWS) method.

In this paper, employing COMSOL MULTIPHYSICS [5], an FEM software package, we investigate the usefulness of 2-D FEM analysis and PWS in 3- and 2-D FEM analysis for a conical horn with a dielectric lens in the setup of the free-space method, and illustrate the change of focusing property against operating frequencies.

II. 3- and 2-Dimensional Finite Element Analysis for Conical Horn with Dielectric Lens

A. Dielectric Lens Design Based on Geometrical Optics

A dielectric lens designed by the geometrical optics, as shown in Fig. 2, is specified as

$$r = \frac{(F-T)(n-1)}{n\cos\theta - 1} \tag{1}$$

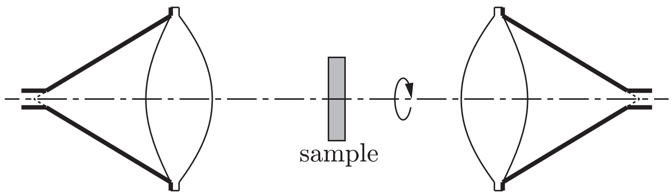


Fig. 1. Free-space method.

where r represents the distance between the lens surface and the origin of the coordinate system at angle θ from the z axis, and F, T, and n are the focal length, thickness, and refractive index of the lens, respectively. Thickness T of the lens is given as

$$T = \frac{\sqrt{F^2 + (D/2)^2} - F}{n-1} \tag{2}$$

where D is the diameter of the lens.

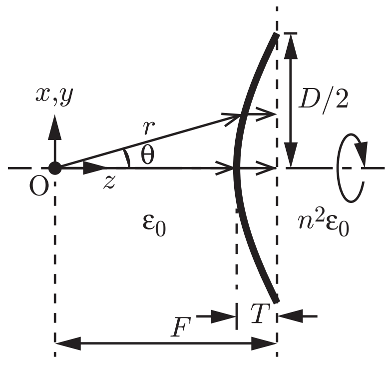


Fig. 2. Geometry of dielectric lens.

B. 3-Dimensional Analysis

A conical horn with a dielectric lens shown in Fig. 3 is analyzed using COMSOL, where the operating frequency is 20 GHz, and the diameter, the focal length, and the relative permittivity of the lens are D=150 mm, F=280 mm, and

ε_r=2.1, respectively. The incident wave is TE_{11} mode of the cylindrical hollow waveguide with diameter $2a$=9.53 mm. Here we analyze a quarter region because of the symmetry of the geometry and the incident wave, where the yz and xz planes are specified as the perfect magnetic and electric walls, respectively. Fig. 4 shows the magnetic and electric field distributions on the yz and xz planes, respectively, where E_{inc} and H_{inc} represent the amplitude of the incident TE_{11} mode on the z axis.

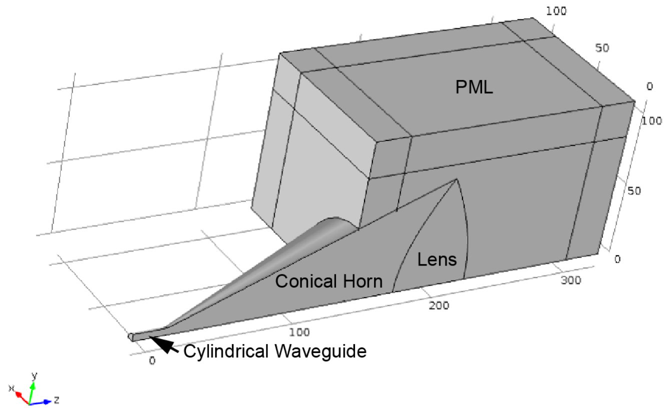
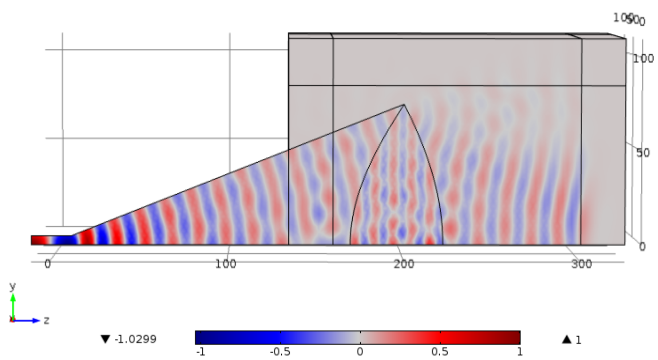
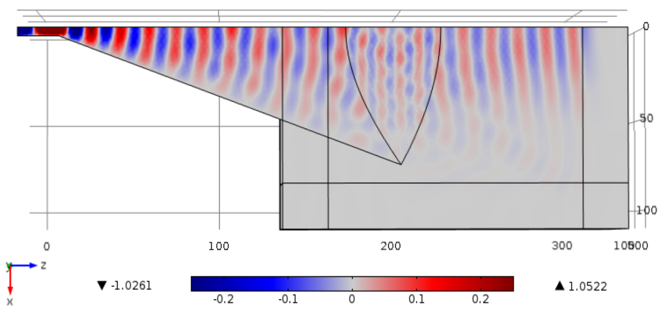


Fig. 3. Geometry of conical horn with dielectric lens.



(a) Magnetic field distribution, H_x/H_{inc}, on the yz plane.



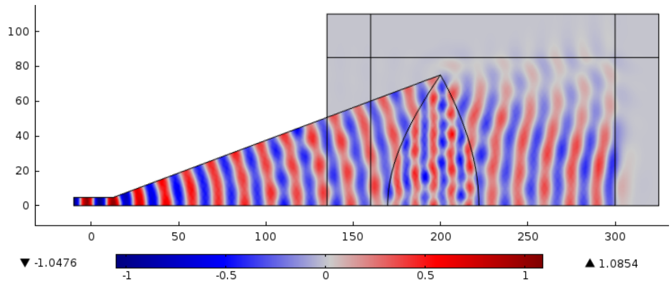
(b) Electric field distribution, E_y/E_{inc}, on the xz plane.

Fig. 4. Field distribution for conical horn with dielectric lens by 3-D FEM.

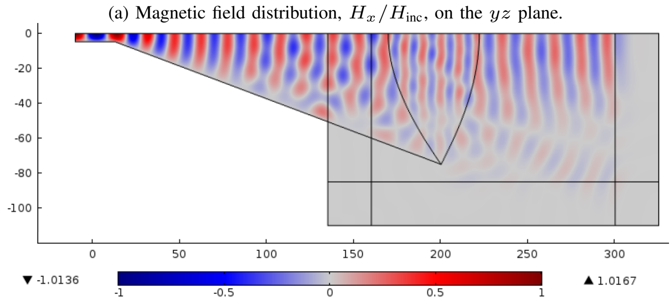
C. 2-Dimensional Analysis

Since the diameter and the focal length of the dielectric lens are greater than ten times the wavelength of the incident wave, it is difficult to extend the analysis region in 3-D FEM to include the focal point of the dielectric lens in our computational resource. Now, we analyze the conical horn with the dielectric lens by 2-D FEM, and compare the results with those in 3-D FEM.

We analyze the yz and xz plane of Fig. 4 by 2-D FEM for the incident TEM and TE_1 mode of the parallel plate waveguide, respectively. Fig. 5 shows the magnetic and electric field distributions by 2-D FEM for the yz and xz plane. The field distributions are in good agreement with those of 3-D FEM shown in Fig. 4, but the amplitude of the field is greater, because the electromagnetic wave extends on the plane in the 2-D region while 3-dimensionally in the 3-D region.



(a) Magnetic field distribution, H_x/H_{inc}, on the yz plane.



(b) Electric field distribution, E_y/E_{inc}, on the xz plane.

Fig. 5. Field distribution for conical horn with dielectric lens by 2-D FEM.

III. Plane Wave Spectrum Method

A. Formulation

Since it is difficult to extend the analysis region in 3-D FEM to include the focal point of the dielectric lens in our computational resource, we compute the electromagnetic fields around the focal point of the dielectric lens by employing the PWS method.

A component of electromagnetic fields at $z = z_1$ in free space is expressed using PWS as follows:

$$F(x, y, z_1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(k_x, k_y)\mathrm{e}^{-j(k_x x + k_y y)}\,\mathrm{d}k_x \mathrm{d}k_y \quad (3)$$

When we obtain the electromagnetic fields at $z = z_1$ by 3-D FEM, we can calculate the spectrum A from the above equation as follows:

$$A(k_x, k_y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(x, y, z_1) \mathrm{e}^{j(k_x x + k_y y)} \, \mathrm{d}x\mathrm{d}y \tag{4}$$

Since the electromagnetic fields are obtained as discrete values in 3-D FEM, the integral of the above equation is calculated by the trapezoidal rule.

After calculating A, we can compute the electromagnetic field at $z = z_2 (> z_1)$ in free space as follows:

$$F(x, y, z_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(k_x, k_y) \mathrm{e}^{-j\beta(z_2 - z_1)} \times \mathrm{e}^{-j(k_x x + k_y y)} \, \mathrm{d}k_x \mathrm{d}k_y \tag{5}$$

where

$$\beta = \begin{cases} \sqrt{k_0^2 - k_x^2 - k_y^2} & \text{for } k_x^2 + k_y^2 \le k_0^2 \\ -j\sqrt{k_x^2 + k_y^2 - k_0^2} & \text{for } k_x^2 + k_y^2 > k_0^2 \end{cases} \tag{6}$$

and k_0 is the free-space wavenumber. When we ignore the attenuation wave to the z direction, the integral of Eq. (5) is calculated using $k_x = k_0 \sin\theta \cos\phi$, $k_y = k_0 \sin\theta \sin\phi$, and $\beta = k_0 \cos\theta$ as follows:

$$F(x, y, z_2) \simeq k_0^2 \int_0^{\frac{\pi}{2}} \mathrm{e}^{-jk_0(z_2 - z_1)\cos\theta} \sin\theta \cos\theta \left\{ \int_0^{2\pi} A(k_x(\theta, \phi), k_y(\theta, \phi)) \times \mathrm{e}^{-jk_0(x \sin\theta \cos\phi + y \sin\theta \sin\phi)} \mathrm{d}\phi \right\} \mathrm{d}\theta \tag{7}$$

The integral of the above equation is also calculated by the trapezoidal rule.

Similarly, we can obtain the expression for a component of electromagnetic fields in 2-D analysis using PWS as follows:

$$F(x, z_2) \simeq k_0 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathrm{e}^{-jk_0(z_2 - z_1)\cos\theta} \cos\theta \times A(k_0 \sin\theta) \mathrm{e}^{-jk_0 x \sin\theta} \, \mathrm{d}\theta \tag{8}$$

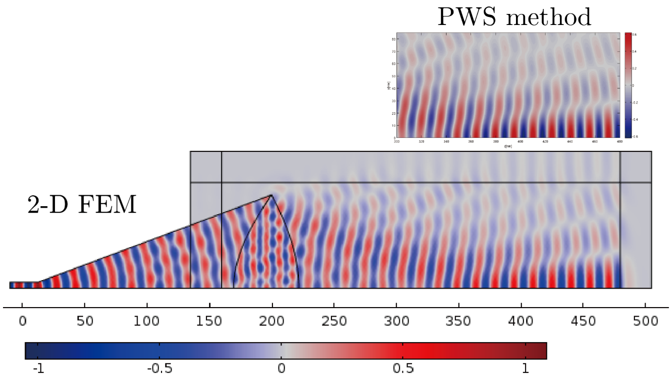
with

$$A(k_x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(x, z_1) \mathrm{e}^{jk_x x} \, \mathrm{d}x \tag{9}$$

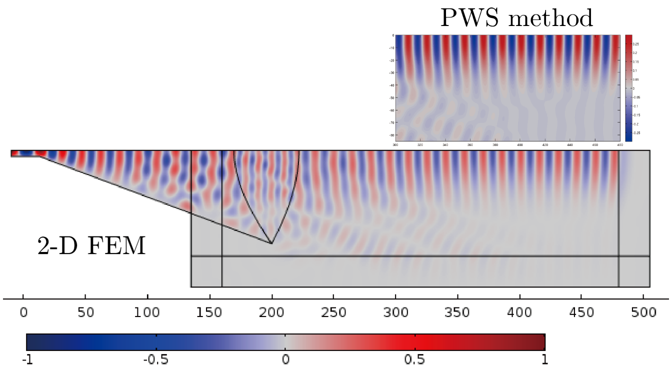
B. Numerical Results

First, we compare in 2-D analysis the fields of the PWS method with those of 2-D FEM, where in the PWS method we use the fields at z_1=300 mm obtained from 2-D FEM. Fig. 6 shows the magnetic and electric field on the yz and xz plane, respectively. We notice that the fields of the PWS method are in good agreement with those of 2-D FEM over 300 mm$<$ z $<$480 mm, where z=480 mm is the designed focal point.

Next, we calculate in 3-D analysis the fields of the PWS method. Since we cannot compute the fields for z $>$300 mm in 3-D FEM, we arrange the fields of 3-D FEM and the PWS method and confirm the validity of the results of the PWS method. Fig. 7 shows the magnetic and electric field on the yz and xz plane, respectively. We notice that the fields of the PWS method are smoothly connected with those of 3-D FEM.



(a) Magnetic field distribution, H_x/H_{inc}, on the yz plane.



(b) Electric field distribution, E_y/E_{inc}, on the xz plane.

Fig. 6. Field distribution for conical horn with dielectric lens by 2-D FEM.

IV. Analysis for Setup of Free-Space Method by 2-D FEM

Finally we compute the electromagnetic field distribution for a pair of conical horns with dielectric lenses by 2-D FEM, as in the setup of the free-space method. Fig. 8 shows the electric field distributions for 20, 50, and 80 GHz, and Fig. 9 shows those on the z axis. We notice from these figures that the electric field has the maximum value around z=400, 430, and 460 mm for 20, 50, and 80 GHz and the location tends to the designed focal point of z=480 mm as the operating frequency increases. Also, from Fig. 9, since a pair of dielectric lenses face each other, the amplitude of the electric field greatly varies due to multiple reflections.

V. Conclusion

We presented several numerical results of 3- and 2-D FEM for a conical horn with a dielectric lens, and confirmed the validity of the PWS method for this case. Also, we computed the electric field distribution for the setup of free-space method against some operating frequencies, and illustrated that the focal point tends to the designed one as an operating frequency increases.

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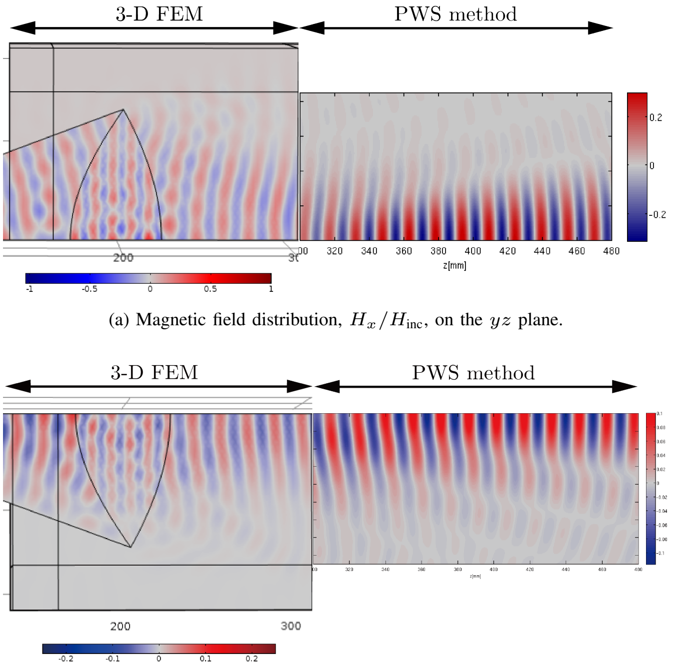
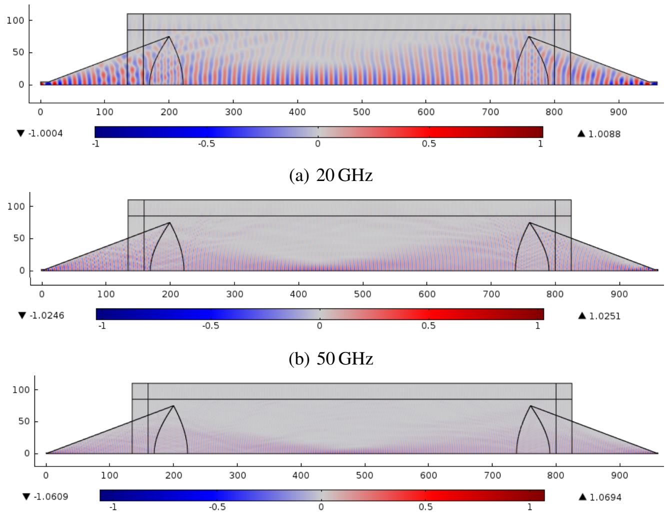


(a) Magnetic field distribution, $H_x/H_{\rm inc}$, on the yz plane.

(b) Electric field distribution, $E_y/E_{\rm inc}$, on the xz plane.

Fig. 7. Field distribution for conical horn with dielectric lens by 3-D FEM.



(a) 20 GHz

(b) 50 GHz

(c) 80 GHz

Fig. 8. Electric field distribution, $E_y/E_{\rm inc}$, on the xz plane for a pair of conical horns with dielectric lenses by 2-D FEM.

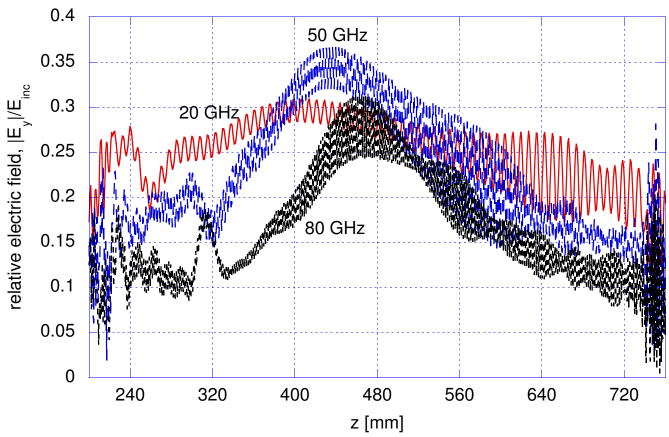


Fig. 9. Electric field distribution, $E_y/E_{\rm inc}$, on the z axis for a pair of conical horns with dielectric lenses by 2-D FEM.

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