Multivariate Regression Analysis for Estimating the Channel Capacity of Small MIMO Antennas Regarding Correlation, Power Imbalance, and SNR

Takeshi Kitamura, Kazuhiro Honda, and Koichi Ogawa

Graduate School of Engineering, Toyama University
3190 Gofuku, Toyama-shi, Toyama, 930-8555 Japan
m1271006@ems.u-toyama.ac.jp

***Abstract*— A simple and efficient method of the evaluation of MIMO channel capacity has been proposed. First, the channel capacity of a half wavelength dipole array antenna was calculated using a Monte Carlo method by changing incident wave SNR, power difference between two elements, and correlation coefficient. Using the calculated results, a polynomial function of estimating the channel capacity was given by the multivariate regression analysis. Effectiveness of the developed function was verified by comparing the channel capacity estimated by the developed function and that calculated by the Monte Carlo method using a MIMO array antenna mounted on a mobile terminal close to the human body. Finally, GUI software has been developed, so that simple and efficient calculation of the MIMO channel capacity can be performed without a large amount of computer and human resources.**

I. Introduction

A simple and appropriate evaluation for the channel capacity of a multiple input multiple output (MIMO) system for mobile terminals is indispensable to the success of upcoming 3G-LTE and IMT-Advanced systems. In recent years, there have been a number of studies on over the air (OTA) testing for the purpose of evaluating the channel capacity of MIMO antennas, which includes fading simulators [1] and reverberation chambers [2]. However, to construct a test environment using an OTA apparatus, a great deal of investment and preparation of appropriate implementation space is needed. Furthermore, it takes a long time to measure the channel capacity using OTA testing because a number of snap shots has to be performed during the measurement. In order to overcome these difficulties, a simple and efficient method is desired to evaluate the channel capacity.

With regard to analytical approaches, a Monte Carlo simulation is commonly performed to analyse a MIMO antenna for cellular handsets [3]. A simulation-based approach, however, can only be applied to a limited range of problems in which the mobile terminal has a simple configuration because a commercially available MIMO handset includes complicated circuit board layout, batteries, and packaging. It is thus important to develop an evaluation tool that allows the channel capacity of a commercial MIMO handset to be properly predicted in a simplified procedure and a shorter period of time.

This paper presents a simple method of the evaluation of MIMO channel capacity. The final objective of our study is to provide a useful and efficient tool for the evaluation of the MIMO channel capacity, particularly for engineers responsible for a developing and designing work in a factory.

In the first step, the channel capacity of a half wavelength dipole array antenna was calculated using a Monte Carlo method by changing incident wave SNR, power difference between two elements, and correlation coefficient. Using the calculated results, a polynomial function for estimating the channel capacity was given by the multivariate regression analysis.

Once mean effective gain (MEG) of MIMO array elements and correlation coefficient between the elements are measured in an anechoic chamber using ordinary measurement instruments and procedure for radiation patterns, the channel capacity is readily obtained using the developed polynomial function for an arbitrary SNR or plural of SNRs. Because MEG and correlation coefficient are obtained in an empirical or experimental way using a usual anechoic chamber, it is possible to evaluate the channel capacity of a commercial MIMO handset in a very simple way.

Effectiveness of the developed polynomial function was verified by comparing the channel capacity estimated by the developed function and that calculated by the Monte Carlo method using a MIMO array antenna mounted on a mobile terminal close to the human body. Finally, GUI software has been developed, so that simple and efficient calculation of the MIMO channel capacity can be performed without a large amount of computer and human resources.

II. Analytical Model

The MIMO array antenna used for the analysis is comprised of quasi-half wavelength dipole array antennas, constructed with a single dipole antenna being placed at two different locations. The frequency for the analysis was set at 900 MHz. The radiation characteristics were calculated using the method of moments.

The Monte Carlo simulation is conducted using a channel model of the two-dimensional angular power spectrum with a uniform distribution in azimuth simulating a Rayleigh fading environment [3]. In the simulation, the channel response is created in consideration for both vertical and horizontal components of radiation patterns of the antenna elements, and the XPR in a fading environment is taken into account. In the analysis of the quasi dipole array, the XPR is set at 50 dB, and thus only the vertical polarization is considered.

Because there is no electromagnetic coupling between the array elements, the correlation coefficient calculated by the Monte Carlo method coincides exactly with the Jake's theoretical curve. Hence, we can obtain an arbitrary correlation coefficient between 0 and 1 by setting an appropriate separation of the quasi dipole array. For example, the array spacing is set to 0.26 wavelengths for the correlation coefficient of 0.4.

III. A METHOD OF THE MULTIVARIATE REGRESSION ANALYSIS

Fig. 1 shows a method of the multivariate regression analysis conducted in this study. As seen in Fig. 1, there are four steps (Fig. 1(a), (b), (c), and (d)) in the multivariate regression analysis, as follows:

(1) First, 150 combinations of the channel capacity defined as Eq. (1) are calculated by the Monte Carlo method. The calculated results are indicated by the black circles in Fig. 1(a), where ΔG is the ratio of MEGs between two antenna elements.

$$\begin{cases} \mathrm{SNR}_i = 10, 20, 30, 40, 50\,[\mathrm{dB}]\ (i=1,2,3,4,5) \\ \Delta G_j = 0, 5, 10, 15, 20\,[\mathrm{dB}]\ (j=1,2,3,4,5) \\ \rho_k = 0, 0.2, 0.4, 0.6, 0.8, 0.9\ (k=1,2,3,4,5,6) \end{cases} \quad (1)$$

(2) The least mean square (LMS) method is applied to the 150 combinations of the channel capacity to yield lines linking points with the same power imbalance ΔG with the correlation coefficient ρ as parameters, as in shown in Fig. 1(b). This procedure creates the function C(SNR) with a variable of SNR.

(3) The LMS method is applied to the lines produced by the procedure (2) to form planes linking all the lines with different ΔG values with a fixed correlation coefficient ρ, as shown in Fig. 1(c). This procedure creates the function C(SNR,ΔG) with variables of SNR and ΔG.

(4) The LMS method is applied to the planes constructed by the procedure (3) to give solids linking all the planes with different ρ values, as shown in Fig. 1(d). This procedure creates the function C(SNR, ΔG, ρ) with variables of SNR, ΔG, and ρ.

Now, we have the polynomial function C(SNR, ΔG, ρ) that provides the channel capacity for arbitrary values of SNR, ΔG, and ρ, in the following form:

$$C(SNR',\Delta G,\rho') = \alpha(\Delta G,\rho') + \beta(\Delta G,\rho')SNR' + \gamma(\Delta G,\rho')SNR'^2$$
$$\because \begin{cases} SNR' = SNR - 10 \\ \rho' = 10\log_{10}(1-\rho) \end{cases} \quad (2)$$

$$\begin{bmatrix} \alpha(\Delta G,\rho') \\ \beta(\Delta G,\rho') \\ \gamma(\Delta G,\rho') \end{bmatrix} = \begin{bmatrix} \alpha_0(\rho') & \alpha_1(\rho') & \alpha_2(\rho') & \alpha_3(\rho') \\ \beta_0(\rho') & \beta_1(\rho') & \beta_2(\rho') & \beta_3(\rho') \\ \gamma_0(\rho') & \gamma_1(\rho') & \gamma_2(\rho') & \gamma_3(\rho') \end{bmatrix} \begin{bmatrix} 1 \\ \Delta G \\ \Delta G^2 \\ \Delta G^3 \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} \alpha_m(\rho') \\ \beta_m(\rho') \\ \gamma_m(\rho') \end{bmatrix} = \begin{bmatrix} \alpha_{m0} & \alpha_{m1} & \alpha_{m2} \\ \beta_{m0} & \beta_{m1} & \beta_{m2} \\ \gamma_{m0} & \gamma_{m1} & \gamma_{m2} \end{bmatrix} \begin{bmatrix} 1 \\ \rho' \\ \rho'^2 \end{bmatrix} \quad (4)$$
$$\because m = 0, 1, 2, 3$$

where the variables SNR and ρ are transformed to the variables SNR' and ρ'. α(ΔG, ρ'), β(ΔG, ρ'), and γ(ΔG, ρ') in Eq. (3) represent the coefficients used in the Eq. (2). $\alpha_m(\rho')$, $\beta_m(\rho')$, and $\gamma_m(\rho')$ shown in Eq. (4) represent the coefficients used in Eq. (3), which is the quadratic equation with respect to the variable ρ' with α_m, β_m, and γ_m (m=0,1,2,3) as coefficients. These coefficients are summarized in Table 1, which are rounded off to four significant figures using the coefficients obtained by double-precision calculations with

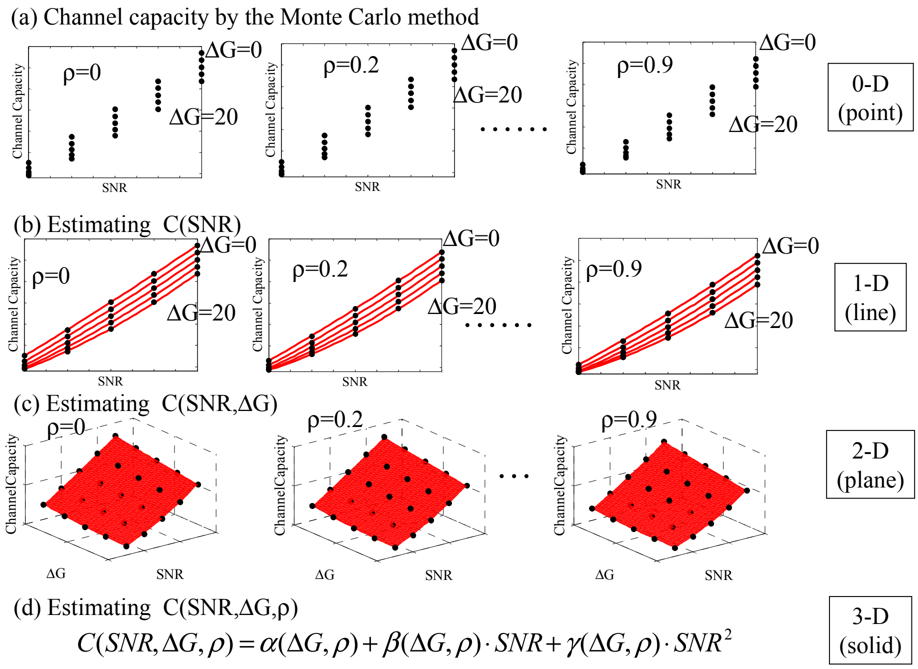

Figure 1 Method of the multivariate regression analysis

Table 1(a) Coefficients of $\alpha_m(\rho')$

$\alpha_m(\rho')$	coefficient		
	α_{m0}	α_{m1}	α_{m2}
$\alpha_0(\rho')$	8.933E-04	-4.841E-06	1.127E-05
$\alpha_1(\rho')$	3.803E-05	-2.894E-06	3.293E-07
$\alpha_2(\rho')$	1.071E-05	-6.735E-07	-4.719E-08
$\alpha_3(\rho')$	-2.109E-07	2.650E-08	-1.625E-10

m=0,1,2,3

Table 1(b) Coefficients of $\beta_m(\rho')$

$\beta_m(\rho')$	coefficient		
	β_{m0}	β_{m1}	β_{m2}
$\beta_0(\rho')$	6.130E-01	3.281E-04	-6.666E-04
$\beta_1(\rho')$	-2.431E-03	1.556E-04	-2.095E-05
$\beta_2(\rho')$	-5.831E-04	3.475E-05	1.382E-06
$\beta_3(\rho')$	7.806E-06	-1.208E-06	3.656E-08

m=0,1,2,3

Table 1(c) Coefficients of $\gamma_m(\rho')$

$\gamma_m(\rho')$	coefficient		
	γ_{m0}	γ_{m1}	γ_{m2}
$\gamma_0(\rho')$	7.554E+00	1.251E-01	-2.583E-03
$\gamma_1(\rho')$	-2.938E-01	-1.437E-03	3.615E-04
$\gamma_2(\rho')$	6.604E-03	-4.109E-04	5.414E-06
$\gamma_3(\rho')$	9.300E-06	1.008E-05	-7.282E-07

m=0,1,2,3

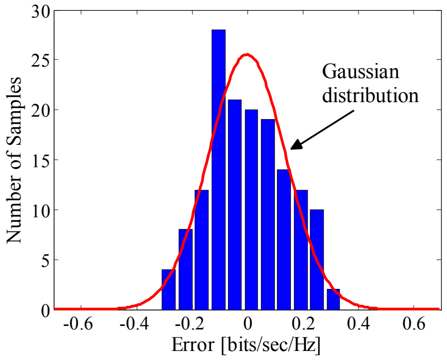

Figure 2 Error distribution of REC

Table 2 Error of REC

Maximum	0.31
Minimum	-0.28
Average	0
Standard Deviation	0.14

bits/sec/Hz

MATLAB. It is confirmed that the coefficients shown in Table 1 has a sufficient accuracy in estimating the channel capacity using Eq. (2), as will be mentioned in the next section.

IV. Error of REC

In this section, the estimated error of the developed polynomial function with the coefficients shown in Table 1 is evaluated by comparing the channel capacity estimated by the developed function and that calculated by the Monte Carlo method. Now, we call the channel capacity calculated by the developed polynomial function, Eq. (2), as the REC (Regression Based Channel Capacity) in the discussion that follows.

The estimated error is evaluated by subtracting the channel capacity calculated by the Monte Carlo method from that calculated by the REC using the 150 combinations of the channel capacity defined as Eq. (1). Fig. 2 and Table 2 show the error of the REC. It can be seen from Fig. 2 that the distribution of the error agrees well with the Gaussian distribution, with the average value of 0 and the standard deviation of 0.14 bits/sec/Hz. The result indicates that the REC has a high accuracy in evaluating the MIMO channel capacity.

V. Application of REC to a Handset MIMO Antenna

In this section, effectiveness of the developed function is verified by analyzing a MIMO antenna mounted on a small mobile terminal close to the human body at 900 MHz. Fig. 3 shows the analytical model that represents realistic use scenarios for a small notebook-sized handset, commonly referred to as a tablet type handset. Two inverted-L antennas are mounted on the left and right sides of the terminal.

As seen in Fig. 3, the hand is located in the vicinity of the element #1. We have examined two scenarios; one is the case when the element #1 is completely covered with the hand, referred to as "Full cover." The other is the case when approximately half of the element #1 is covered with the hand, with the hand moving 6 cm to the –z direction from the location depicted in Fig. 3, called "Half cover." These two scenarios produce different imbalanced power conditions between the elements #1 and #2, as described in Table 3. This allows us to evaluate the accuracy of the REC under very realistic use conditions for a tablet type handset.

When Eq. (2) is applied to a mobile terminal, SNR should be changed by the following equation:

$$SNR_{i2} = SNR_{i1} - \Delta MEG \quad \because \Delta MEG = MEG_d - MEG_t \tag{5}$$

where MEG_d and MEG_t are the mean effective gain of the half wavelength dipole antenna used in Sec. III and that of a DUT (mobile terminal), respectively. SNR_{i1} indicates the signal-to-noise ratio of incident waves at the input port of the DUT, and SNR_{i2} is the signal-to-noise ratio to be substituted in Eq. (2). This transformation regarding the SNR is needed because the channel capacity is determined by the SNR at the output port of the array antenna, whereas the SNR appearing in Eq. (2) is defined as an SNR at the input port of the array antenna, which is equivalent to the incident wave SNR.

The use of Eq. (5) is exampled as follows; supposing that MEG_t = –3 dBi and the incident wave SNR of the DUT is SNR_{i1} = 20 dB. Since MEG of the half wavelength dipole antenna used in this study is MEG_d = 2.15 dBi, ΔMEG is calculated to be ΔMEG = 2.15 – (–3) = 5.15 dB. Hence, SNR_{i2} is calculated to be SNR_{i2} = SNR_{i1} – ΔMEG = 20 – 5.15 = 14.85 dB, which must be substituted in Eq. (2).

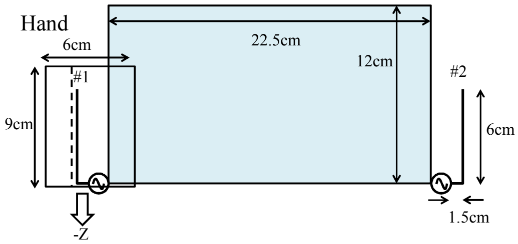

Figure 3 Configuration of the handset MIMO antenna

Table 3 Characteristics of the handset MIMO antenna

	state	element	MEG [dBi]	ΔG [dB]	ρ
(a)	Full cover	#1	-6.05	5.02	0.72
		#2	-1.03		
(b)	Half cover	#1	-1.71	1.17	0.18
		#2	-0.54		

@900MHz

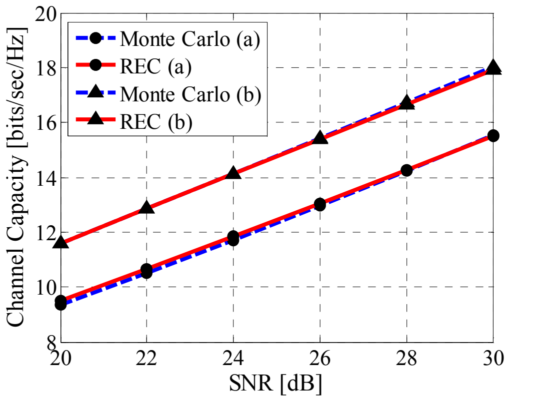

Figure 4 Channel capacity of the handset MIMO antenna

Fig. 4 shows the channel capacity as a function of SNR for the two hand locations. The solid and broken lines indicate the results calculated by the REC and the Monte Carlo method, respectively. It can be seen from Fig. 4 that there is a good agreement between the two lines, confirming a high accuracy of the developed polynomial function in an actual use scenario of a small MIMO antenna.

VI. Development of Graphical User Interface

The final objective of our study is to provide a useful and efficient tool for the evaluation of the MIMO channel capacity, particularly for engineers responsible for a developing work in a factory. Thus we have developed a GUI software environment using MATLAB that facilitates the use of the developed formulation.

Fig. 5 shows the GUI console panel where a user enters various input parameters and receives the calculated results. Fig. 6 shows the calculated channel capacity in a three-dimensional fashion with the power imbalance and correlation coefficient as variables for SNR = 23 dB. The GUI software has a variety of functions that can yield various versatile formatted figures, which helps engineers to develop and design MIMO antennas in an efficient way.

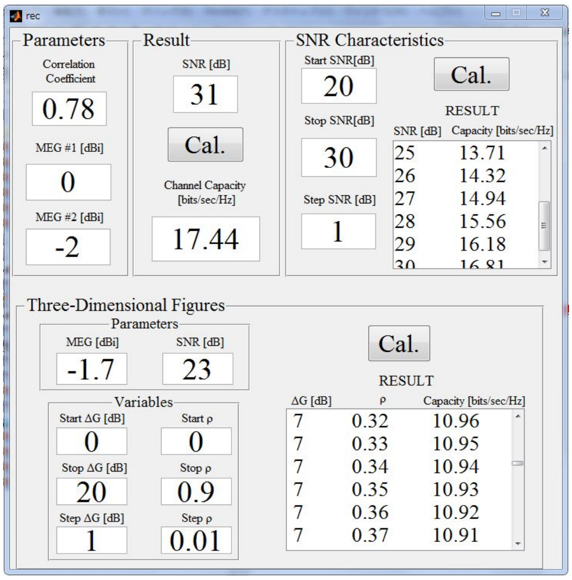

Figure 5 GUI console panel

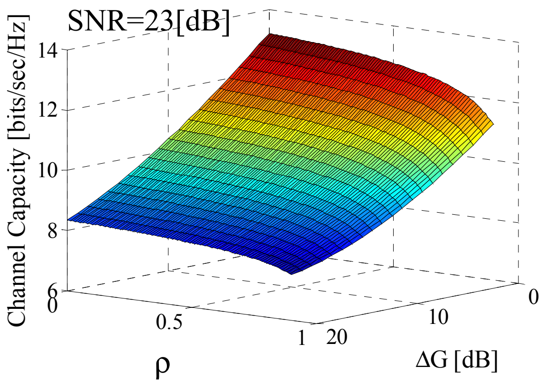

Figure 6 Three-dimensional representation of the channel capacity

VII. Conclusion

A simple and efficient tool for the evaluation of MIMO channel capacity was provided. The estimated error was found to be the average value of 0 and the standard deviation of 0.14 bits/sec/Hz, confirming that a high accuracy in the evaluation of the MIMO channel capacity has been achieved. The developed GUI environment is particularly useful for engineers responsible for a developing and designing work on the front line in a factory.

References

[1] T. Sakata, A. Yamamoto, K. Ogawa, and J. Takada, "MIMO Channel Capacity Measurement in the Presence of Spatial Clusters Using a Fading Emulator," PIMRC2009 Intl. Symp. Digest (Tokyo, Japan), Session D1, no. 2, Sep. 2009.

[2] P.-S. Kildal and K. Rosengren, "Correlation and capacity of MIMO systems and mutual coupling, radiation efficiency, and diversity gain of their antennas: simulations and measurements in a reverberation chamber," IEEE Communications Magazine, vol.42, 12, pp. 104-112, Dec. 2004.

[3] K. Ogawa, H. Iwai, A. Yamamoto, and J. Takada, "Channel Capacity of a Handset MIMO Antenna Influenced by the Effects of 3D Angular Spectrum, Polarization, and Operator," IEEE AP-S Intl. Symp. Digest, pp. 153-156, July 2006.