Proceedings of the "2013 International Symposium on Electromagnetic Theory"



# Recent Advances in the Analytical Modeling of Wire Media Based Metamaterials with Microwave and Terahertz Applications

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***Abstract*—Here, we review our recent progress on the analytical modeling of wire media based metamaterial structures and demonstrate a variety of microwave and terahertz applications. The analysis is based on the homogenization theory of spatially dispersive wire media with additional boundary conditions derived for different interface scenarios. The applications include subwavelength imaging, super lenses, wideband absorbers, artificial impedance surfaces, carbon-based nanomaterials, and many others.**

## I. Introduction

Artificial media in the form of an array of parallel metallic rods has been known for many decades, for example, the "rodded medium" [1], which is understood as an artificial plasma with an effective index of refraction less than unity. It has been revisited in the last years because of its profound relevance in *metamaterials* technology, and has received special attention as *wire media* (WM). The WM are important constituents in various metamaterial structures, including artificial mushroom-type impedance surfaces [2]-[4], wire lenses to provide a canalization of the near field to distances of several wavelengths at microwave [5], [6] and terahertz [7] frequencies, wire lenses based on the evanescent waves amplification [8] negative refraction phenomena [9]-[12] and broadband microwave absorbers with stable angle characteristics [13]-[15], among many others.

The most characteristic feature of WM is its ability to conduct electric current along sets of wires such that the wave propagation in WM is accompanied with the propagation of wave of a conduction current (leading to the so-called "extraordinary wave"). This results in the local accumulation of charge along the wires, leading to strong *spatial dispersion* (SD) in the WM at microwave and low-terahertz frequencies, and even in the very long-wavelength limit, i.e., in the quasi-static regime. These SD effects in WM, associated with the dependence of the effective permittivity on the wave vector along the wires, cannot be neglected [16]. Various analytical and numerical approaches have been proposed for the analysis of WM based metamaterial structures, which in general lack physical insight or are limited to some canonical applications. Very recently, a homogenization theory of WM, based on the quasi-static approximation as a local model for a nonlocal (spatially-dispersive) material, has been developed [17], and the additional boundary conditions (ABCs) at various interface scenarios (including truncation, termination, lumped impedance insertions, resistive loads or even one atom-thick graphene) have been formulated in a general framework [18], [15]. In another "local" model the spatial drift-diffusion equation leads to a current density that includes both the local electric field via a local conductivity and the gradient of charge [19]. This method is equivalent in wavenumber space to the usual nonlocal permittivity model, and provides a tractable method to consider scattering from three-dimensional WM structures.

The intention of this review paper is to demonstrate that the interaction of electromagnetic waves with complex WM based metamaterial structures can be analyzed in a most elegant, physically insightful manner using the effective medium approximation, resulting in accurate analytical solutions for a wide range of frequencies. In the presentation, the basic concepts of the homogenization theory of WM will be discussed, with a variety of examples and applications demonstrating different physical phenomena and practical realization of metamaterial structures.

## II. Homogenization Theory of Wire Media

The homogenization theory of WM is based on the quasi-static approximation, such that the relation between the full electromagnetic description of WM and its transmission line (TL) analogy (the WM can be understood as equivalent to a multiwire TL) can be established. By considering a WM with the wires oriented along the $z-$ direction, the following system of macroscopic (averaged) Maxwell's equations coupled to TL-type equations associated with the TL modes propagating along the wires can be obtained [17],

$$\nabla \times \mathbf{E} = -j\omega\mu_h \mathbf{H} \quad (1)$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon_h \mathbf{E} + \mathbf{J} \quad (2)$$

$$\frac{\partial\langle\varphi\rangle}{\partial z} = -\left(j\omega L + Z_w\right)\langle I_z\rangle + E_z \quad (3)$$

$$\frac{\partial\langle I_z\rangle}{\partial z} \equiv -j\omega\langle q\rangle = -j\omega C\langle\varphi\rangle \quad (4)$$

where $\mathbf{J} = \hat{\mathbf{z}}\langle I_z\rangle / A_{\text{cell}}$ is the macroscopic polarization current density in the wire medium, $A_{\text{cell}} = a^2$ is the area of the unit cell of the square lattice of wires with $a << \lambda$ being the lattice period, $\varepsilon_h$ and $\mu_h$ are the permittivity and permeability of the host material, $\mathbf{E}$ and $\mathbf{H}$ are the averaged (macroscopic) electric and magnetic fields in the WM, and the angular brackets $\langle ...\rangle$ represent any suitable interpolating (averaging) operator that smooths the microscopic quantities defined at discrete wires and makes them continuous through the entire volume. In (3) and (4), $L$, $C$, and $Z_w$ are the effective inductance, effective capacitance, and self-impedance per unit length of a wire, respectively.

It should be noted that Eqs. (1)-(4) constitute a *local* framework in which the effects of spatial dispersion are taken into account by introducing additional parameters via derivatives: wire current $I_z$ and the additional potential $\varphi$ at the wire surface, associated with the accumulation of the electric charge $q$ on the wire [17]. The usual *nonlocal* framework of WM described with a nonlocal dyadic permittivity can be obtained by considering a macroscopic electric displacement along the wires

$$D_z = \varepsilon_h E_z + \frac{J_z}{j\omega} \quad (5)$$

and substituting Eqs. (3) and (4) in the above equation. Here, we assume that the current along the wires is of the form $I_z(z) = I_0 e^{-jk_z z}$, where $k_z$ is the $z-$component of the wave vector $\mathbf{k}$. Following the notations defined in [17] we obtain a nonlocal dielectric function for the permittivity of the WM along the wires:

$$\frac{\varepsilon_{zz}}{\varepsilon_h} = 1 - \frac{k_p^2}{k_h^2 - j\xi k_h - k_z^2 / n^2} \quad (6)$$

where $k_p^2 = \mu_h / \left(A_{\text{cell}} L\right)$ is the plasma wave number, $\xi = Z_w \sqrt{\varepsilon_h \mu_h} / L$, and $n^2 = LC / \left(\varepsilon_h \mu_h\right)$ is the square of the slow-wave factor, which determines the degree of nonlocality of the material response. The formalism of the nonlocal dielectric function (6) can be applied only to unbounded uniform WM, and in the case of truncation, termination, or loading wires with impedance insertions, the appropriate ABCs are necessary at the interfaces. The generalized ABCs for the most common cases of WM terminations are presented in [18], with an extension to the cases of WM loaded with impedance insertions in [11]. The ABCs for interface scenarios when wires are connected to a thin resistive sheet or in general a monolayer such as graphene have been derived in [15].

With a general homogenization theory framework developed for WM with a variety of ABCs at the most common cases of terminations and loading, below we present several examples of practical metamaterial structures analyzed at microwave and terahertz frequencies.

## III. Microwave and Terahertz Metamaterial Applications

Here we present a few examples of WM based metamaterial structures, and many others will be discussed in the presentation.

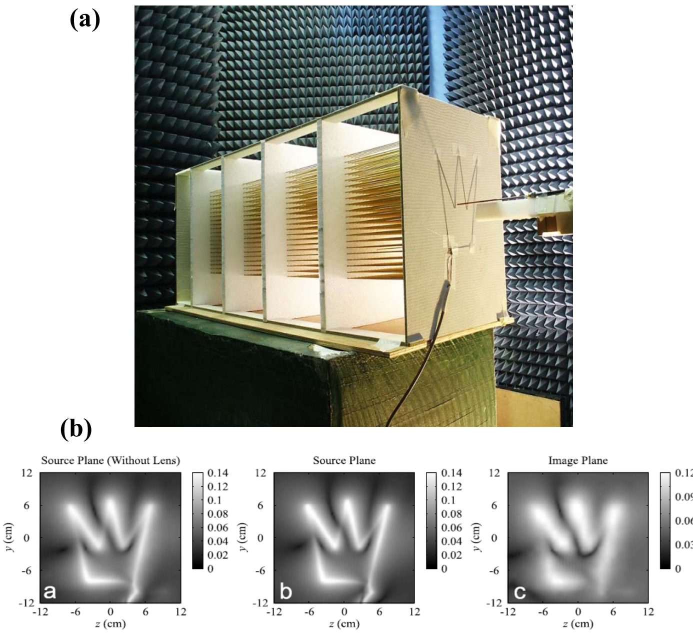


Fig. 1. (a) Practical realization of WM for subwavelength imaging. (b) Near-field at the source and image planes [5].

In Figure 1(a), a practical realization of WM is shown demonstrating the concept of near-field transport (canalization) to distances of several wavelengths at microwave frequencies [5]. Long densely-packed wires transform the evanescent waves from source into propagating waves in the WM, wherein the near-field from the source plane is canalized to the image plane (Fig. 1(b)).

In the second example, a crossed wire mesh structure is shown in Fig. 2(a), which has been analyzed based on the homogenization theory of WM [9]. The two perpendicular sets of metallic wires provide two coupled propagation channels, and the physical channel mainly excited is the one that provides a negative refraction of the incoming wave (Fig. 2(b)).

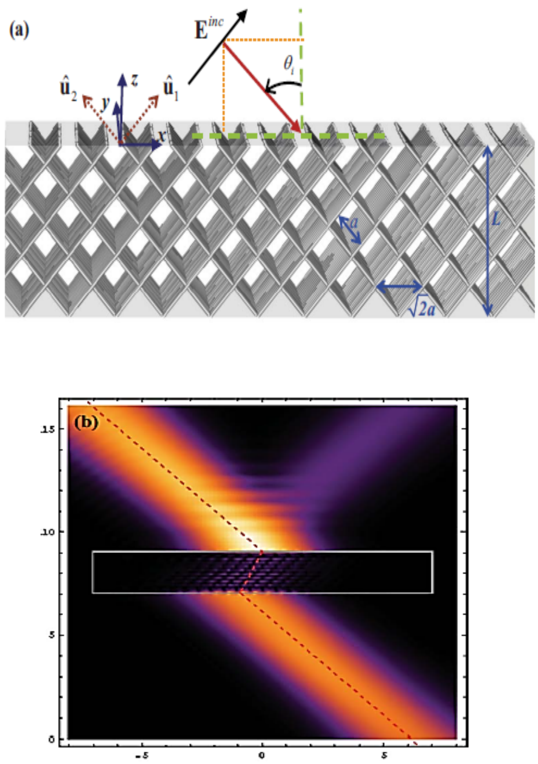


Fig. 2. (a) Crossed wire mesh. (b) Demonstration of negative refraction with the Gaussian beam excitation [9].

In the next example, a bi-layer mushroom structure with the vias loaded with lumped impedance insertions (with the geometry shown in Fig. 3(a)) has been proposed as an ultra-thin WM structure which enables to achieve all-angle negative refraction (with the results demonstrated in Fig. 3(b) for the Gaussian beam excitation) [11].

Next example relates to the wire media consisting of metallic carbon nanotubes (CNTs). Let us apply Eq. (6) to array of single-wall CNTs. In this case in addition to the electrostatic capacitance and the electromagnetic inductance we should take into account the quantum capacitance and the kinetic inductance, connected serially with their classic counterparts. The most important is the role of the kinetic inductance which exceeds by three orders the electromagnetic one. It was shown [20, 21] that the term $k_z^2/n^2 << k_h$ in denominator of (6) can be neglected at terahertz frequencies. As a result, the spatial dispersion is completely suppressed and arrays of metallic CNTs behave as ε-negative uniaxial crystals. We do not need any reactive loadings [22] introducing parasitic inductances and parasitic capacitances. This leads to a dramatically increased bandwidth of backward wave propagation and gives the ground to consider CNT arrays as *perfect backward-wave metamaterials.* Fig. 4 illustrates the hyperbolic (conic) dispersion of eigenwaves propagating in the CNT array.

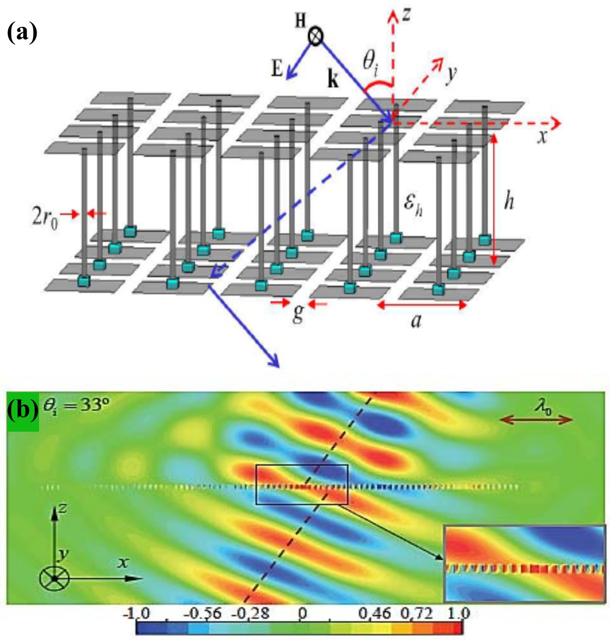


Fig. 3. (a) Ultra-thin design based on a bi-layer mushroom structure with loaded vias. (b) Demonstration of negative refraction with the Gaussian beam excitation [11].

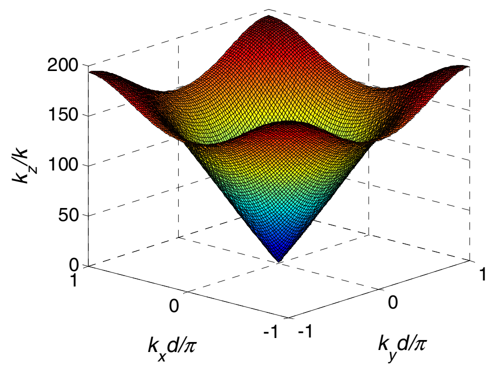


Fig. 4. The real part of the slow-wave factor $k_z/k$ in plane of normalized wave vector components $k_x$ and $k_y$, calculated for the period of square CNT lattice $d$ = 2.5 nm at the frequency 1 THz.

Fig. 5 illustrates the dispersion properties of three modes propagating in the CNT slab placed between PEC and PMC planes. There are three embedded hyperbolic surfaces. The top and bottom parts of these surfaces are cut. One can see that the backward waves propagate in the slab in a very wide frequency range. Their properties are quite isotropic in the *xy* plane due to a very small period of the CNT lattice.

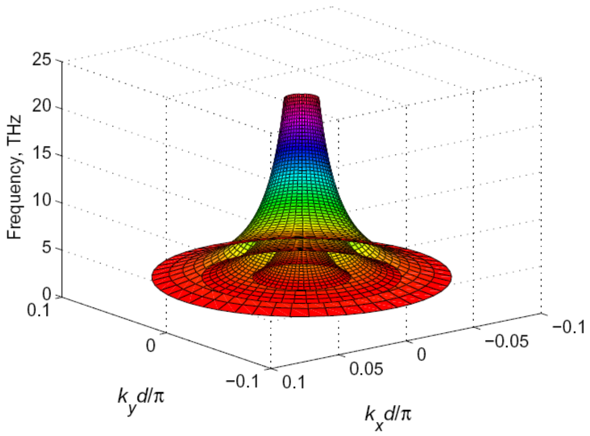


Fig. 5. Frequency as a function of the normalized wavenumbers in the transverse plane, calculated at $d$ = 15 nm, and the slab thickness $h$ = 1.5 µm.

Still another application of the effect of strong spatial dispersion in wire media is in the control and optimization of radiative heat transport [24]. More information about terahertz and especially optical applications can be found in recent review [25].

## IV. Conclusion

In this contribution we review our recent progress on the homogenization theory of WM, and demonstrate the application to a variety of practical metamaterial structures operating at microwave and terahertz frequencies. The analysis is based on the quasi-static approximation resulting in a local framework of nonlocal (spatially-dispersive) WM. The ABCs are derived for various interface terminations and loadings (including the cases of lumped impedance insertions and thin resistive sheets) which enable to analytically model complex WM based matematerial structures. We also discuss carbon nanotubes WM, where the spatial dispersion is suppressed, and the effects caused by the spatial dispersion suppression.

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