Antiresonance in the Retrieved Material Parameters of Periodic and Aperiodic Composite Materials

Pekka Alitalo #1, Ali E. Culhaoglu *2, Constantin R. Simovski #3, Sergei A. Tretyakov #4

Department of Radio Science and Engineering/SMARAD Center of Excellence, Aalto University
P.O. Box 13000, FI-00076 AALTO Finland
1 pekka.alitalo@aalto.fi
3 konstantin.simovski@aalto.fi
4 sergei.tretyakov@aalto.fi

* *Microwaves and Radar Institute, German Aerospace Center (DLR)*
82234 Wessling, Germany
2 ali.culhaoglu@dlr.de

***Abstract*—We study the occurrence of the so-called antiresonance in the material parameters (permittivity, permeability) characterizing the electromagnetic response of a composite material slab. The material parameters are retrieved using the well-known reflection/transmission method. As many previous works have either claimed or assumed that the antiresonance is an effect caused by the periodicity of the composite material, we study in this work both periodic and aperiodic material samples.**

I. Introduction

The Nicolson-Ross-Weir method, commonly applied in the experimental characterization of properties of electromagnetic materials, such as permittivity and permeability, is widely used in the characterization of metamaterials [1]. This method uses the complex reflection and transmission coefficients of a slab of the material under test to determine the permittivity and permeability of the material composing the slab.

In the case of composite materials, especially metamaterials that exhibit a resonant material response, the appearance of the so-called antiresonance [2] in (usually) one of the retrieved material parameters is a common event. Often this unphysical behaviour of the material parameter(s) is explained by the periodicity of the composite material (e.g., [2], [3], [4]).

In this work we study the emergence of the antiresonant material parameters by designing a composite material with a resonant permittivity, i.e., a material comprising small resonant electric dipoles. Two cases are studied: a material with a periodic arrangement of the dipoles and a material with aperiodic arrangement of the dipoles. It is shown by numerical results as well as experimental results that both cases exhibit a resonant permittivity (with a Lorentzian resonance) and an antiresonant (unphysical) permeability.

To calculate the effective material parameters from the simulated and measured reflection and transmission data, we use a recently proposed material parameter extraction algorithm that is based on the Nicolson-Ross-Weir method [1]. Using this algorithm, we can mitigate the problems related to the branch-seeking algorithms commonly used in the retrieval of resonant material parameters.

II. Studied Composite Material

A. Periodic Material Sample

A representative example material was chosen to be comprised of resonant inclusions, such as the one illustrated in Fig. 1. This particle is essentially a very small electric dipole, when the exciting electric field is parallel to the y-axis. Note that materials comprising particles with a similar geometry have been studied previously [5]. Here this specific geometry is chosen simply due to its very small electrical size (at the resonance frequency, the particle dimensions are very small electrically).

The inclusions are printed with 0.035 mm thick copper on a 0.8 mm thick FR4 substrate ($\epsilon_\mathrm{r} = 4.4 - j0.088$). The structure is made periodical so that the period in both x and y directions is 4.2 mm (i.e., there is a 1.2 mm gap

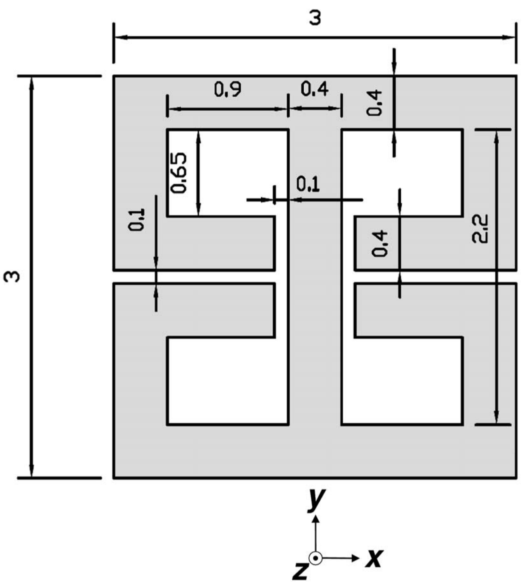


Fig. 1. Illustration of the metallic inclusion that is essentially a small electric dipole when the exciting electric field is parallel to the y-axis. The dimensions are in [mm].

between adjacent inclusions). To make the unit cell of our artificial material symmetric with respect to the xy-plane, we add another 0.8 mm thick FR4 board (no metallization in this board) on the other side of the inclusion, therefore sandwiching the inclusion between two 0.8 mm thick FR4 boards.

The material sample is made volumetric by stacking four such pairs of FR4 substrate boards on top of each other. To allow for the variation of the distances between adjacent FR4 pairs, the volume between these is filled by stacks of thin sheets of teflon ($\epsilon_{\rm r} = 2.1 - j0.002$). With the teflon sheets, the periodicity of the material (along the z direction) can be controlled with a 0.1 mm step, which is the thickness of a single teflon sheet. See Fig. 2 for an illustration of the material sample and the unit cell. The dimensions of the periodic sample are: $d = 4.2$ mm, $p = 0.8$ mm, $s = 1.2$ mm, and $t = 0.035$ mm.

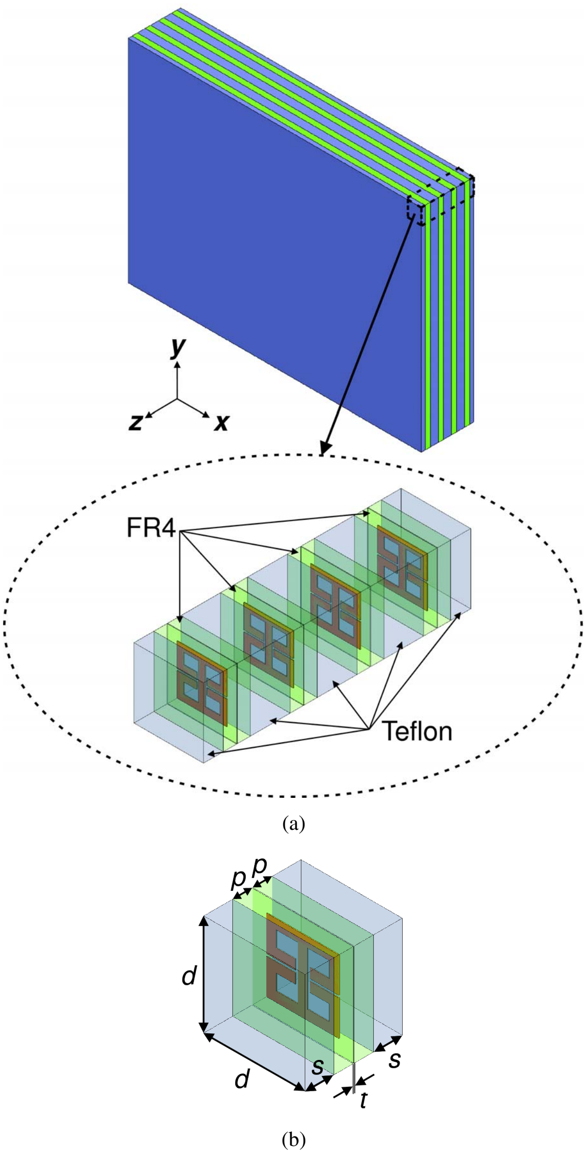


Fig. 2. (a) Illustration of the material sample (slab). (b) Unit cell of the periodic material.

The periodic material sample is simulated to be infinite along the transverse directions (x and y). This is accomplished by applying proper boundary conditions to the simulation model comprising only a small section of the whole slab (the inset of Fig. 2a). All the numerical results presented in this paper are obtained with the Ansys HFSS software.

The experimental periodic material sample has the same dimensions as presented above, the only difference is that the sample is finite in the transverse plane (xy-plane). Namely, the substrates that the resonant inclusions are printed on have the sizes 360 mm and 260 mm along the x and y directions, respectively. There are $85 \times 61 = 5185$ inclusions printed on four 0.8 mm thick FR4 substrates, and another set of four FR4 substrates with the same thickness (but without metallization) is used to obtain the symmetrical slab and unit cell shown in Fig. 2.

B. Aperiodic Material Samples

The aperiodic material samples comprise the same FR4 boards as the periodic sample and the aperiodicity is inflicted only in the z-direction (wave propagation direction). This is accomplished in practice by varying the thickness of the teflon insulation between adjacent FR4 layers (see Fig. 2a). Let us consider that each of the unit cell positions along the z-axis is independently moved by a random number $\Delta z_{\rm n}$ (where $n = 1, 2, 3, 4$). Then, obviously, the thicknesses of the teflon between adjacent cells can be smaller or larger than in the periodic case ($2s$). See Fig. 3 for an illustration of the aperiodicity in the z direction. Note that as the center of each unit cell is moved by $\Delta z_{\rm n}$, the teflon thicknesses at the edges of the sample remain always as $s = 1.2$ mm.

We have computed 4 sets of random values for $\Delta z_{\rm n}$, with normal distribution and standard deviation of 0.5 mm. The resulting values are shown in Table I. Note that the case illustrated in Fig. 3 is sample #1 of Table I. Similar to the

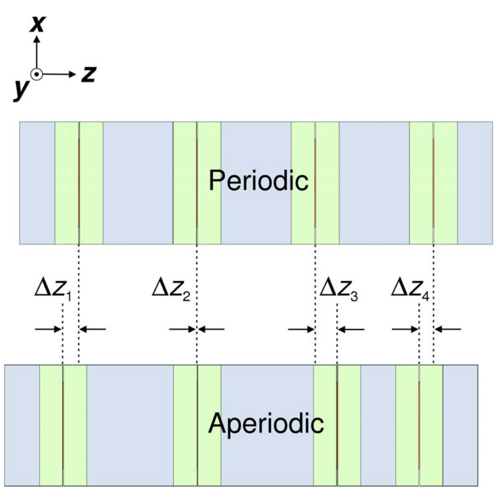


Fig. 3. Periodic and aperiodic samples (aperiodicity in the z direction).

TABLE I
VARIATIONS IN UNIT CELL POSITIONS ALONG THE z DIRECTION.

	Δz_1	Δz_2	Δz_3	Δz_4
Sample #1	-0.6 mm	0 mm	0.8 mm	-0.4 mm
Sample #2	-0.1 mm	-1 mm	-0.4 mm	0.7 mm
Sample #3	0.4 mm	-0.1 mm	-0.1 mm	0.7 mm
Sample #4	-0.4 mm	0.7 mm	-0.9 mm	-0.1 mm

periodic sample, the aperiodic samples are simulated with proper boundary conditions, as explained above, in order to simulate a transversally infinite slab. Also the experimental aperiodic material samples are assembled according to Table I. This is accomplished by varying the number of the 0.1 mm thick teflon sheets that are placed between the FR4 boards. The FR4 boards are the same as used in the periodical sample.

III. MEASUREMENT SETUP

We have used a bistatic measurement setup [6] at the Microwaves and Radar Institute of the German Aerospace Center (DLR) to conduct reflection and transmission measurements for our periodic and aperiodic samples. The reflection and transmission measurements are conducted for the frequency range from 5.8 GHz to 8.8 GHz. From the measured scattering parameter data, we obtain the refractive index, as well as the relative complex permittivity and permeability, using the material parameter extraction method presented in [1]. It should be noted that both the numerical and experimental scattering parameters exhibit a stopband (minimum of $|S_{21}|$) near the frequencies where the material parameters are resonant. The numerical results are accurate enough for the extraction method to keep on the correct branch of the phase [1], but, as is expected, the measured scattering parameters are not accurate enough to achieve this. Therefore we omit a small frequency band of the measured data to avoid erroneous results in the refractive index and the material parameters. The correctness of the branch [1] of the extracted refractive index and material parameters is evident from the comparison of the numerical and experimental results of the refractive index.

IV. NUMERICAL AND EXPERIMENTAL RESULTS

The numerical as well as experimental results for the periodic sample are presented in Fig. 4. The experimental values of the magnitudes of the scattering parameters as well as the retrieved complex refractive index, relative permittivity, and relative permeability are compared to the numerical results. It can be concluded that there is a small shift in the resonance frequencies when comparing the numerical and experimental results, but overall the experimental results are in good agreement with the numerical ones. Most importantly, it is evident that the relative permittivity is resonant whereas the relative permeability experiences the "antiresonance". Especially it should be noted that the sign of the imaginary part of the relative permeability is positive at the antiresonance. This is of course a clear indication of an unphysical material parameter, as the material itself is known to be passive.

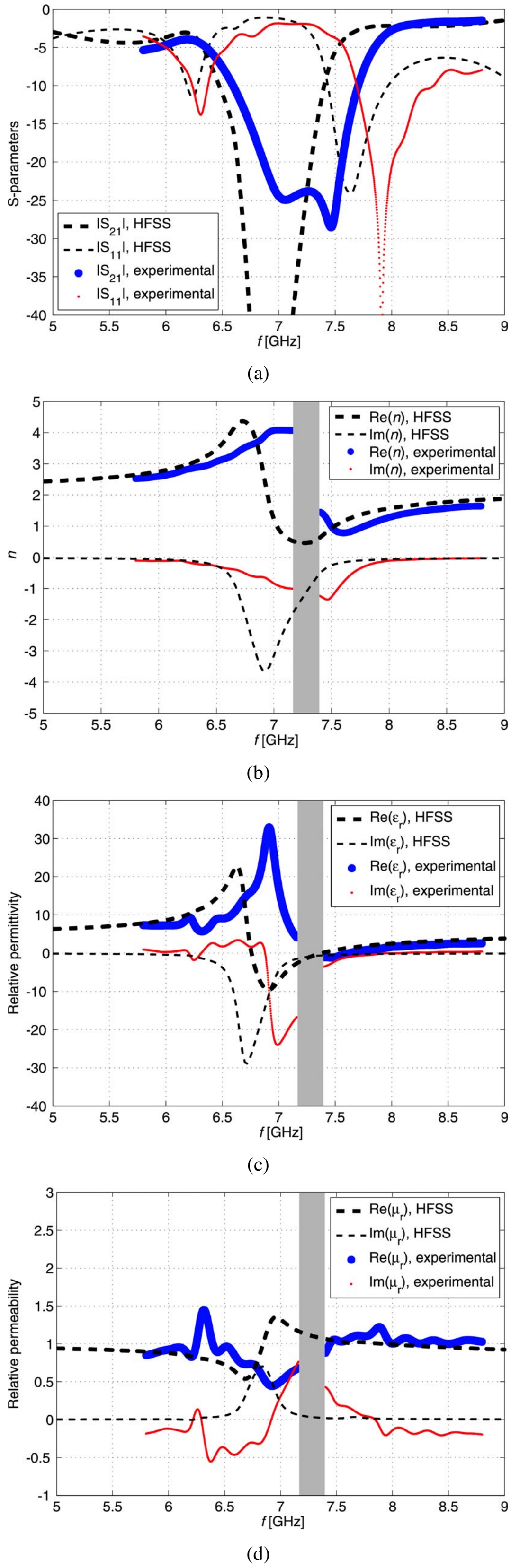


Fig. 4. Results for the periodic sample. (a) S-parameters. (b) Refractive index. (c) Relative permittivity. (d) Relative permeability. The grey areas show the frequency band where the experimental scattering parameter data is omitted due to a stopband.

The numerical as well as experimental results for the aperiodic sample #1 are presented in Fig. 5. It should be noted that the results for all the four aperiodic samples are similar, only slight changes in the frequencies and magnitudes of the resonances are observed. Again the experimental results are shown together with the numerical ones for comparison purposes and it can be concluded that the experimental results are in good agreement with the numerical ones. The same shift in the frequency, as for the periodic sample, is observed. It is also clear that even for the aperiodic case, the relative permittivity is resonant and the relative permeability still experiences the "antiresonance".

V. Conclusion

Material slabs made of periodic as well as aperiodic arrangements of resonant inclusions are studied numerically and experimentally. It is shown that one of the extracted material parameters, the permeability in these cases, is antiresonant in both the periodic and aperiodic material. The fundamental reason for the emergence of the antiresonance in extracted material parameters is still undergoing much discussion in the literature. The present results indicate that at least one of the most commonly used explanations of the antiresonance, namely, the periodicity of the material, is not likely to be the main factor behind this phenomenon.

Acknowledgment

This work has been partially funded by the Academy of Finland and Nokia through the center-of-excellence program. The work of P. Alitalo has been supported by the Academy of Finland via post-doctoral project funding.

References

[1] O. Luukkonen, S. I. Maslovski, and S. A. Tretyakov, "A stepwise nicolsonrossweir-based material parameter extraction method," *IEEE Antennas Wireless Propag. Lett.*, vol. 10, pp. 1295–1298, 2011.

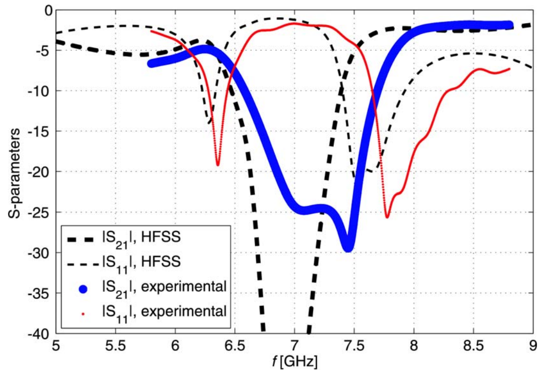
[2] T. Koschny, P. Markos, D. R. Smith, and C. M. Soukoulis, "Resonant and antiresonant frequency dependence of the effective parameters of metamaterials," *Phys. Rev. E*, vol. 68, p. 065602(R), 2003.

[3] T. Koschny, P. Markos, E. N. Economou, D. R. Smith, D. C. Vier, and C. M. Soukoulis, "Impact of inherent periodic structure on effective medium description of left-handed and related metamaterials," *Phys. Rev. B*, vol. 71, p. 245105, 2005.

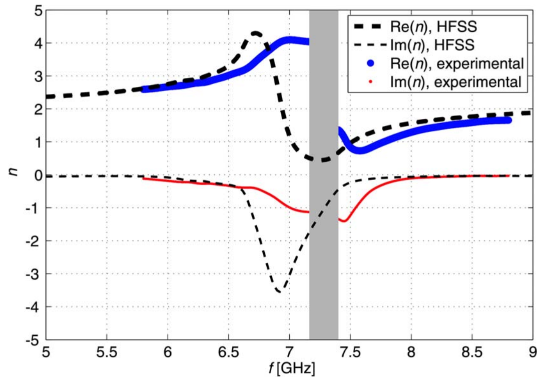
[4] J. Zhou, T. Koschny, M. Kafesaki, E. N. Economou, J. B. Pendry, and C. M. Soukoulis, "Saturation of the magnetic response of split-ring resonators at optical frequencies," *Phys. Rev. Lett.*, vol. 95, p. 223902, 2005.

[5] W. J. Padilla, M. T. Aronsson, M. Lee, C. Highstrete, A. J. Taylor, and R. D. Averitt, "Electrically resonant terahertz metamaterials: Theoretical and experimental investigations," *Phys. Rev. B*, vol. 75, p. 041102(R), 2007.

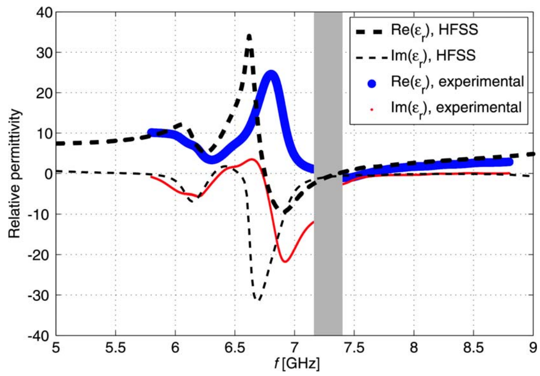
[6] E. Kemptner and S. Thurner, "Free space material characterization for microwave frequencies," in *Proc. 6th European Conference on Antennas and Propagation (EuCAP2012)*, Prague, Czech Republic, Mar. 2012, pp. 3513–3515.



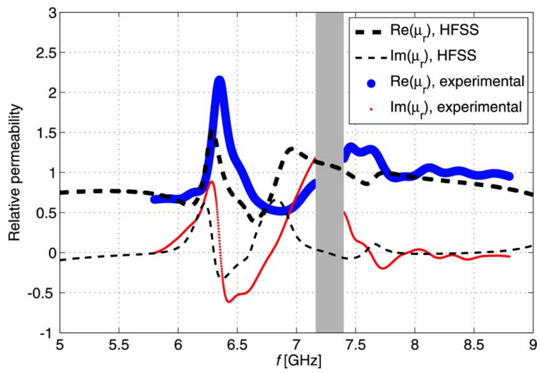
(a)



(b)



(c)



(d)

Fig. 5. Results for the aperiodic sample #1. (a) S-parameters. (b) Refractive index. (c) Relative permittivity. (d) Relative permeability. The grey areas show the frequency band where the experimental scattering parameter data is omitted due to a stopband.