Design of an Optimized Biomagnetic Measurement Surface by a Spherical-Multipole Approach

Fatih Argin [1], Henning Ahrens [2], Ludger Klinkenbusch [3]

Institute for Electrical and Information Engineering, Kiel University
Kaiserstr. 2, D-24143 Kiel, Germany
[1] far@tf.uni-kiel.de
[2] ha@tf.uni-kiel.de
[3] lbk@tf.uni-kiel.de

***Abstract*—The spherical-multipole technique is applied to find the best distribution of the magnetometers in an array for biomagnetic measurements, e.g., for Magnetoencephalography (MEG) and Magnetocardiography (MCG). By investigating the condition number of a matrix depending only on the measurement points and not on the source points, we find that the optimum choice consists of equally located magnetometers distributed on a connected double-layer surface. The results are of interest particularly in the context of new passive and uncooled magneto-electric (ME) sensors which might be able to replace conventional SQUID magnetometers in the future.**

I. Introduction

The measurement of the magnetic field produced by physiological currents in the brain or heart can be used to identify the origin of these currents and the related functional activity by means of a Magnetoencephalography (MEG) or Magnetocardiography (MCG), respectively [1]. Compared to the electrical counterparts, that is, to the Electroencephalography (EEG) and Electrocardiography (ECG) the main advantages of MEG and MCG are that the magnetic field is directly related to the primary currents and that there is no need to have electrodes, that is, a direct contact between sensors and skin is not necessary. However, as such extracorporeal biomagnetic fields are extremely weak, i.e., in the pT (MCG) or fT (MEG) range, commonly SQUIDs (Super Conducting Quantum Interference Devices) have been used as magnetometers and gradiometers in existing systems. The recent development of passive, uncooled, and highly sensitive magnetoelectric (ME) sensors [2] offers not only more practicable and cheaper biomagnetic measurement systems but also the opportunity to introduce more flexible measurement surfaces for the corresponding arrays. In this paper we investigate the performance of a special surface which is particularly suited for biomagnetic measurements, that is, a double layer surface. By using a spherical-multipole representation of the biomagnetic field we establish a system of linear equations where the corresponding matrix only depends on the number, locations, and polarizations of the sensors - but it does not depend on the location of the sources. We show that the condition number of that matrix is significantly lower (better) for magnetometers on a double layer surface than for the conventionally used magnetometers distributed on a single layer surface.

II. Spherical-Multipole Expansion of the Magnetic Field

In spherical coordinates r, ϑ, φ, the quasi-static magnetic flux density $\mathbf{B}$ in a homogeneous domain outside of a sphere centered at $r = 0$ and containing all current sources $\mathbf{J}$ (with $\nabla \cdot \mathbf{J} = 0$) can be written as a spherical-multipole expansion [3]

$$\mathbf{B}(\mathbf{r}) = -\nabla \left\{ \sum_{l=1}^{\infty} \sum_{m=-l}^{+l} a_{l,m} r^{-(l+1)} Y_{l,m}(\vartheta, \varphi) \right\} \quad (1)$$

where $a_{l,m}$ denote the expansion coefficients (multipole amplitudes), and the normalized surface spherical harmonics are defined as

$$Y_{l,m}(\vartheta, \varphi) = \sqrt{\frac{(2l+1}{4\pi} \frac{(l+m)!}{(l-m)!}} P_l^m(\cos\vartheta) \exp\{jm\varphi\} \quad (2)$$

The P_l^m are associated Legendre functions of the first kind of degree l and order m. Performing the gradient operations in (1) yields

$$\mathbf{B}(\mathbf{r}) = \sum_{l=1}^{\infty} \sum_{m=-l}^{+l} a_{l,m} r^{-(l+2)} \left\{ l+1) Y_{l,m}(\vartheta, \varphi) \hat{r} - \mathbf{n}(\vartheta, \varphi) \right\} \quad (3)$$

Here, $\hat{r}$ represents the radial unit vector, and the transverse vector function is defined as

$$\mathbf{n} = \frac{\partial Y_{l,m}}{\partial \vartheta} \hat{\vartheta} + \frac{1}{\sin\vartheta} \frac{\partial Y_{l,m}}{\partial \varphi} \hat{\varphi} \quad (4)$$

III. Multipole Formulation of the Inverse Biomagnetic Problem

Supposed that in the biological domain there is no accumulation of charge it holds that

$$\nabla \cdot \mathbf{J}(\mathbf{r}) = 0 \quad (5)$$

Consequently, any source (primary current) and any related secondary current can be approximated by a finite number of properly distributed magnetic dipoles [4]. For each of these dipoles the corresponding multipole amplitudes can be found, and it holds that the maximum degree of multipole amplitudes necessary to represent the magnetic field within a given error bound monotonously increases with the distance of that dipole from the origin. Finally, by superposition the magnetic field of all dipoles can be represented by means of a multipole expansion (3) where the maximum degree l_{max} corresponds to the maximum distance of all dipoles to the origin.

Now we suppose that we have an arbitrary distribution of dipoles and a corresponding multipole expansion of maximum degree l_{max}. If a suitable component of the magnetic field is measured at K different measurement locations $\mathbf{r}_m^{(k)}$ with $k = 1, 2, 3, ..., K$, then obviously we can reconstruct the magnetic field within the given error bound if we have $K = l_{max}(l_{max+2})$ independent measurements.

For the evaluation of all multipole amplitudes we set up a quadratic system of K linear equations according to

$$\mathbf{A} \cdot \mathbf{a} = \mathbf{B} \qquad (6)$$

The right-hand side of (6) is the vector of all measurements of the magnetic flux density at the K measurement point while $\mathbf{a}$ is the vector of the unknown multipole amplitudes $a_{l,m}$. Since the matrix $\mathbf{A}$ does depend only on the measurement locations and not on the location of the sources, it is characteristic for the topology of the entire measurement system and well suited for the determination of the optimum sensor locations and polarizations.

IV. Numerical Results

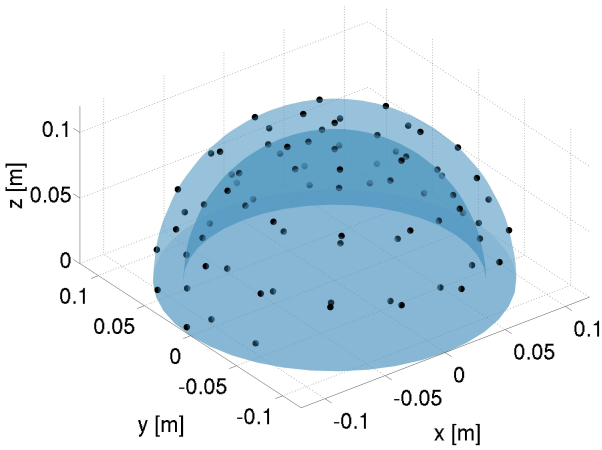

Fig. 1. Double-layer surface with measurement points distributed on both of the two spherical layers.

For the numerical evaluation we have chosen the geometry in Fig. 1, consisting of two concentric spherical caps with a distance of 2 cm between the caps. We have studies the condition number of the matrix in (6) for different distributions of the measurement point on these two layers. The condition number is a quantitative measure how well-posed or ill-posed a matrix is: For a given number of unknowns, condition numbers for different distributions of measurement points are compared, but the absolute value of the condition number is not relevant.

V. Numerical Results

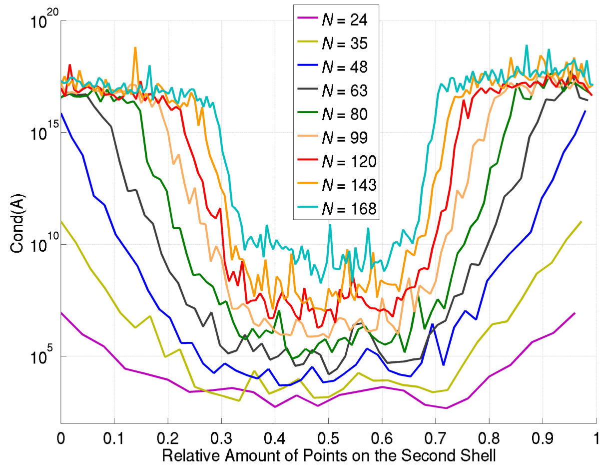

Fig. 2. Condition numbers for different total numbers N of measurement points on two concentrical spherical layers (see Fig. 1 as a function of the relative amount of measurement points on the outer spherical cap .

Figure 2 shows the behaviour of the condition number for different total amounts of measurement points each as a function of the relative amount of measurement points on the outer spherical cap. On each cap the measurement points are equally distributed. If all measurement points are either on the inner or the outer cap, the condition numbers increase, i.e., the reconstruction of the magnetic field from these measurements is difficult or even impossible. The best results are obtained if the measurement points are distributed on both spherical caps. Note that the maximum values of the condition number at about 10^{17} are due to numerical limitations of the used MATLAB code *cond*.

VI. Conclusions

A systematic multipole-based strategy has been presented to find the optimal sensor locations and polarizations for biomagnetic measurements. For a double-layer surface consisting of two spherical caps we have shown that the optimal distribution appears if the sensors are equally distributed on both surfaces. Further studies will include the effect of polarization and the case of non-spherically shaped surfaces which are as close as possible to the head (for MEG) or chest (MCG).

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