A Novel Theoretical Approach of Fishnet-type Material Composed of Multilayer Metallic Patterns by the Equivalent Circuit

Tsunayuki Yamamoto [#1], Hiroshi Kubo [#2], Atsushi Sanada [#3]

[#] *Graduate School of Science and Engineering, Yamaguchi University*
2-16-1 Tokiwadai, Ube-shi, Yamaguchi Japan

[1] t_y@yamaguchi-u.ac.jp
[2] kubo@yamaguchi-u.ac.jp
[3] as@yamaguchi-u.ac.jp

***Abstract*—In this paper, an equivalent circuit of a fishnet-type material composed of multilayer metallic patterns is proposed. The fishnet-type material can be considered as a rectangular waveguide with stubs, and the operation principles can be simply and clearly explained by a proposed equivalent circuit. Proposed equivalent circuit can be useful for analyzing and designing of fishnet-type material.**

I. INTRODUCTION

Negative refractive index artificial materials have been actively studied [1], [2], for potential applications exploiting the unique property. The materials have to be composed of two kinds of elements: an element with negative dielectric property and an element with negative magnetic property or series capacitance and shunt inductance in the equivalent circuit expression. It is troublesome to make two kinds of elements and to assemble them. Recently, a simple structure made by stacking sub-wavelength hole arrays, the so-called fishnet structure, has shown to work as negative refractive index materials [3]. The experimental confirmation of negative refractive index characteristics in near-infrared region has attracted many interests. Studies of the same kind of structures and the numerical analyses have followed the experiment with strong impact [4], [5]. In the field of physics the transmission phenomena through the fishnet-type structure is understood as a natural evolution of well-known phenomena of extraordinary optical transmission. However, another approach for understanding the operation mechanism is necessary for microwave and millimetre wave engineering conscious of designing devices as well as grasping physical image.

For these reasons, in this paper, an equivalent circuit of a fishnet-type material composed of multilayer metallic patterns [6] is proposed. In the equivalent circuit, the fishnet-type material is expressed as a conventional rectangular waveguide with shunt stubs that operate as parallel plate waveguides. The operation principles of the fishnet-type material can be simply and clearly explained by a proposed equivalent circuit. Moreover, the equivalent circuit shows that a composite right/left-handed (CRLH) fishnet-type material can be designed by adjusting the parameter.

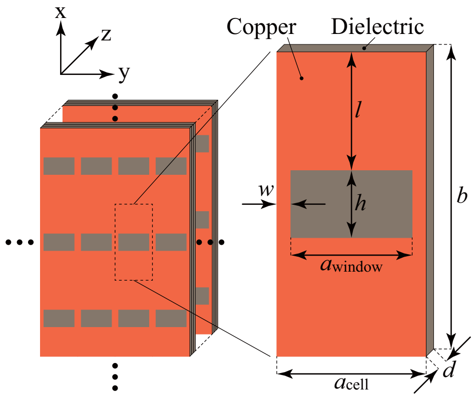

Fig. 1. The fishnet-type material composed of multilayer metallic patterns and the unit cell

II. UNIT CELL STRUCTURE AND EQUIVALENT CIRCUIT

Fig. 1 shows a fishnet-type material composed of multilayer metallic patterns. The metallic patterns are printed on the Teflon substrate where the conductor is copper, the relative permittivity ε_r is 2.17, and the thickness d is 1.525 mm. The width of a unit cell is shown by a_{cell} and the height is shown by b. The metallic pattern has rectangular windows with the height h and the width a_{window} put periodically in two-dimensional arrangement. If the fishnet-type material operates at a dominant propagating mode, then the electric distribution in the window is the same as TE_{10} mode of a conventional rectangular waveguide. For this reason, it is considered that the window works like a rectangular waveguide filled up by dielectric with ε_r for x-polarized wave propagating in the z-direction. The space between metallic patterns works as stubs for the propagating wave. Two stubs with the length l are connected to the window at the top and the bottom, respectively.

Fig. 2 shows a proposed equivalent circuit of the fishnet-type material. The equivalent circuit of unit length d is composed of a series impedance $Z(\omega)$, a shunt admittance $Y(\omega)$. $Z(\omega)$ is composed of an inductor L_{se} and a transformer

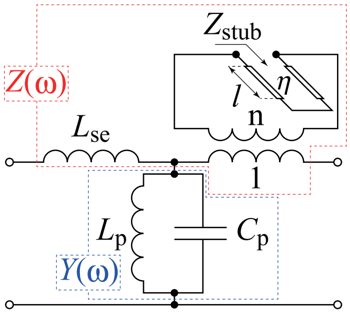

Fig. 2. The equivalent circuit of the unit cell

with two stubs that the impedance is Z_{stub}. Two stubs are connected through the transformer with turn ratio n. $Y(\omega)$ is composed of an inductor L_p and a capacitor C_p. For TE_{10} mode of a rectangular waveguide, L_{se}, C_p and L_p are expressed as follows:

$$L_{se} = \mu_0 d, C_p = \varepsilon_r \varepsilon_0 d, L_p = \frac{\mu_0 \left(\frac{a_{\mathrm{window}}}{\pi}\right)^2}{d}. \quad (1)$$

The stub can be treated as a parallel plate waveguide where the width is a_{cell}, the height is d and length l. When the electric fields are excited in the same phase at all windows, then the top of the stub becomes short equivalently, so the parallel plate waveguide is terminated with short. The impedance looking into two stubs from the rectangular waveguide is shown by Eq. (2).

$$Z_{\mathrm{stub}} = j\frac{\eta \cdot \tan(\beta \cdot l)}{2n^2}, \quad (2)$$

Where η and β are the characteristic impedance and the phase constant in the parallel plate waveguide, respectively, and these are expressed as follows:

$$\eta = \frac{d}{a_{\mathrm{cell}}}\sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}}, \beta = \frac{\omega}{\sqrt{\frac{1}{\varepsilon_r \varepsilon_0 \mu_0}}}. \quad (3)$$

For these equations, the series impedance $Z(\omega)$ and the shunt admittance $Y(\omega)$ of the equivalent circuit in Fig. 2 are expressed by next equations.

$$Z(\omega) = j\left\{\omega \mu_0 d + \frac{\eta \cdot \tan(\beta \cdot l)}{2n^2}\right\}, \quad (4)$$

$$Y(\omega) = j\left\{\omega \varepsilon_r \varepsilon_0 d - \frac{d}{\omega \mu_0 \left(\frac{a_{\mathrm{window}}}{\pi}\right)^2}\right\}. \quad (5)$$

When the same four-port circuits are connected infinitely, the phase constant is given by Eq. (6) [7].

$$\beta d = \cos^{-1}\left\{1 + \frac{1}{2} Z(\omega) \cdot Y(\omega)\right\}$$
$$= \cos^{-1}\left\{1 - \frac{1}{2}\left\{\omega \mu_0 d + \frac{\eta \cdot \tan(\beta \cdot l)}{2n^2}\right\}\left\{\omega \varepsilon_r \varepsilon_0 d - \frac{d}{\omega \mu_0 \left(\frac{a_{\mathrm{window}}}{\pi}\right)^2}\right\}\right\}. \quad (6)$$

III. Calculations of Dispersion Characteristics

TABLE 1
STRUCTURAL PARAMETERS FOR THE CALCULATIONS BY THE PROPOSED EQUIVALENT CIRCUIT

Parameters	Param. (1)	Param. (2)
a_{cell}	$a_{\mathrm{window}} + 2w$	20.00 mm
b	40.85 mm	$h + 2l$
d	1.524 mm	1.524 mm
a_{window}	----	19.50 mm
w	0.25 mm	0.25 mm
h	9.00 mm	9.00 mm
l	15.93 mm	----
n	0.318	0.318

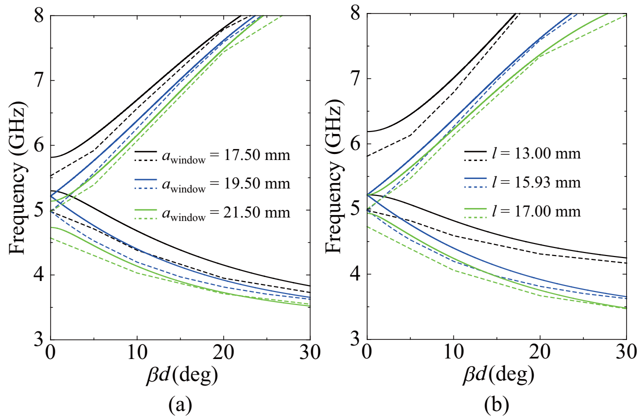

Fig. 3. The calculated results by using the proposed equivalent circuit. (a) The results with Param. (1) shown in Table 1. (b) The results with

For verification for validity of the proposed equivalent circuit shown in Fig. 2, some dispersion characteristics of the fishnet-type material are calculated by Eq. (6). In this time, the dependences on a_{window} and l are investigated. These calculated results are shown in Fig. 3(a) and Fig. 3(b), respectively. Table 1 shows the structural parameters for the calculations. The calculated results are compared with the simulated results by full-wave FEM simulations.

These results lead the facts that the dependence of the calculated dispersion curves on a_{window} and l is qualitatively the same as that of the simulated dispersion curves though the calculated curves are slightly different from the simulated curves. Therefore, the operation principles of the fishnet-type material can be discussed based on the equivalent circuit. By the way, the value of n is chosen as 0.318. When n = 0.318, the balanced condition is satisfied for a_{window}=19.50 mm and l = 15.93 mm. However, the value of n does not affect the qualitative discussion in the next paragraph.

When $Z(\omega)$ is inductive and $1 / Y(\omega)$ is capacitive, a right-handed (RH) dispersion curve is obtained. On the other hand, when $Z(\omega)$ is capacitive and $1 / Y(\omega)$ is inductive, then a left-handed (LH) dispersion curve is obtained. For obtaining of capacitive $Z(\omega)$, it is necessary that l is above quarter-wavelength below half-wavelength. ω_{0Z} and ω_{0Y} are the values of ω that satisfy $Z(\omega) = 0$ and $Y(\omega) = 0$, respectively. Eq. (4)

shows that $Z(\omega)$ is capacitive for $\omega < \omega_{0Z}$ and is inductive for $\omega > \omega_{0Z}$. Eq. (5) shows that $1/Y(\omega)$ is inductive for $\omega < \omega_{0Y}$ and is capacitive for $\omega > \omega_{0Y}$. When $\omega_{0Z} < \omega_{0Y}$, the dispersion curve of a LH mode is obtained below ω_{0Z} that is the Γ-point frequency, and the dispersion curve of a RH mode is obtained above ω_{0Y} that is the Γ-point frequency. A bandgap exists between the two frequencies. Eq. (5) shows that ω_{0Y} decreases by making a_{window} longer. When ω_{0Y} decreases to ω_{0Z}, the bandgap disappears and balanced dispersion curves are obtained. When $\omega_{0Z} > \omega_{0Y}$, the dispersion curve of a LH mode is obtained below ω_{0Y}, and the dispersion curve of a RH mode is obtained above ω_{0Z}. ω_{0Y} increases by making a_{window} shorter. When ω_{0Y} increases to ω_{0Z}, balanced dispersion curves are obtained. After all, a CRLH fishnet-type metamaterial can be designed by adjusting a_{window} as shown in Fig. 3 (a). Eq. (4) shows that ω_{0Z} is changed by the stub length l. Thus, a CRLH fishnet-type metamaterial can be similarly designed by adjusting l as shown in Fig. 3 (b).

The smulated curves in Fig. 3 (a) and Fig. 3 (b) show that the bandgap between the LH and the RH modes can disappear by adjusting of a_{window} and l, and the CRLH fishnet-type metamaterial with the balanced dispersion curves is obtained when $a_{\text{window}} = 19.50$ mm and $l = 15.93$ mm.

IV. Conclusion

In this paper, the equivalent circuit of the fishnet-type material composed of multilayer metallic patterns has been proposed for designing of the CRLH fishnet-type metamaterial. The fishnet-type material can be expressed as the circuit of a rectangular waveguide with stubs. The calculated dispersion characteristics of the fishnet-type material by the proposed equivalent circuit qualitatively agree with the simulated results by full-wave FEM simulators. The operation principles of the fishnet-type material can be simply and clearly explained by the proposed equivalent circuit. The proposed equivalent circuit has shown that a CRLH metamaterial is designed by adjusting the window width and the stub length.

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