IEICE Proceeding Series

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Vol. 1 pp. 239-242
Publication Date: 2014/03/17
Online ISSN: 2188-5079

2012 International Symposium on Nonlinear Theory and its Applications
NOLTA2012, Palma, Majorca, Spain, October 22-26, 2012

Characterizing global dynamics on time-evolving networks of networks

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Abstract—A network of networks, which consists of interconnected sub-networks, is ubiquitous in the real world. Methods for analyzing such networks have been proposed. Although these analyses focused on static structures of networks, most of real networks often change with time. Moreover, we often observe the dynamics on each node rather than the topology of the network. However, time-varying interactions among nodes have not been well characterized yet. Here, we propose a method to delineate time-evolving global topological structures on networks of networks. The proposed method roughly estimates the time-evolving structures, and characterizes the dynamics of their time-evolution using a distance.

1. Introduction

The network is ubiquitous structure in the real world. Many real networks share the structure called "networks of networks", where subnetworks representing subsystems are connected with each other. Some methods to characterize the structure of the network of networks have been proposed [1]. These methods have focused on static structures of networks. However, most of real networks often change their structures with time. These existing methods ignore the dynamics of time evolution of networks. Moreover, we often observe the dynamics on each node rather than the exact topology of the network of networks. Hence, it is required to estimate the interaction between measured time series. Although many attempts have been made at such a problem [2–7], it is difficult to reconstruct the exact topology of the network from measured time series. We proposed a new method to characterize the time evolution of networks even if the topology of the network could not be estimated exactly [8].

Most of networks of networks have high modularity, i.e., nodes are densely connected with nodes in the same subnetwork and sparsely connected with nodes in other subnetworks. There are many indirect interactions between nodes in such networks. Many indirect interactions result in strong interdependencies between two time series, even if two nodes corresponding to these time series are not directly connected. In addition, the existing methods to estimate network topologies assume the sparseness of the underlying network explicitly or implicitly. Hence, the estimation of such networks with high modularity is more difficult than that of the ordinaly networks.

Here, we show results of numerical simulations of application of the previously proposed method [8] to the dynamics on networks of networks with high modularity. These results revealed that the global dynamics of time evolution of networks of networks with high modularity can also be captured. However, higher modularity of networks of networks results in lower accuracy of characterization. This result indicates that high modularity makes it difficult to understand the global dynamics.

2. Method

In this section, we briefly describe the method which we used [8]. We consider the system on the network of networks, where each of nodes obeys the certain dynamics respectively and interacts through links each other. We assume that we can simultaneously measure the dynamics at each node. The purpose is to understand the dynamics of the time evolution of the network of networks from such multivariate time series.

The method to characterize the dynamics of time-evolving networks of networks is composed of two steps. The first step is the rough estimation of the time-evolving structure of the network of networks, which does not completely correspond to the underlying network. In the second step, we characterize the dynamics of the network of networks using the roughly estimated time-evolving network.

We let $\boldsymbol{x}^k(m)$ denote the time series measured from the kth node. To estimate the time-evolving structure, we divide measured time-series into some short segments. For each segment, we estimate the structure of network in the following manner. First, we obtain the recurrence plot [9, 10] of time series measured at each node. The recurrence plot of kth time series is the matrix defined as

$$R^k_{i,j} = \begin{cases} 1, & \text{if} \quad d(\boldsymbol{x}^k(i), \boldsymbol{x}^k(j)) < \epsilon, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

Here, $d(\boldsymbol{x}^k(i), \boldsymbol{x}^k(j))$ is the distance between the states at times i and j. When the distance between values at two times is smaller than a certain threshold ϵ, we plot a dot at (i, j) and (j, i). Second, we evaluate the interdependency between each pair of time series using joint recurrence plots [11]. The joint recurrence plot of the kth and lth time series is defined as

$$JR^{k,l}_{i,j} = R^k_{i,j} R^l_{i,j}. \quad (2)$$

Hence, in the joint recurrence plot, a dot is plotted at (i, j) if and only if there are dots there in both recurrence plots. The stronger the correlation between two time series is, the more black dots are plotted in their joint recurrence plot. The number of dots plotted in the joint recurrence plot of two independent time series follows a binomial distribution of size $N(N-1)$ and probability $q_1 q_2$, where q_1 and q_2 denote the probabilities that a dot is plotted in the recurrence plots, respectively. If the joint recurrence plot contains a sufficiently large number of dots (0.05 significance level), two time series are defined to be significantly correlated. Then, two nodes, where these time series are observed, are connected with each other. Otherwise, these nodes are not connected.

Let $\boldsymbol{A}(m)$ be the adjacency matrix of the mth segment. We visualize the time evolution of the network topology by the recurrence plot of the time-series of the estimated network $\boldsymbol{A}(m)$. A definition of distances is required to obtain recurrence plots. We use the hamming distance [12]:

$$d(\boldsymbol{A}(i), \boldsymbol{A}(j)) = \sum_{k>l} \|A_{k,l}(i) - A_{k,l}(j)\|. \quad (3)$$

This distance corresponds to the total number of links which exist in only one of two networks $\boldsymbol{A}(i)$ and $\boldsymbol{A}(j)$.

3. Simulation

We simulated the coupled logistic map on time-evolving networks of networks. The dynamics of the kth node in the time-evolving networks of networks is described as

$$\begin{aligned} x^k(t+1) &= f(x^k(t)) + K \sum_l W_{lk}(t) f(x^l(t)), \quad (4) \\ f(x) &= ax(1-x), \end{aligned}$$

where $x^k(t)$ is the state of the kth node at time t, a is the parameter of the logistic map, $\boldsymbol{W}(t)$ is the adjacency matrix at time t and K determines the strength of the interactions. We set $K = 0.1$. The parameter a of each node is randomly selected from $[3.7, 4.0]$.

We prepared four instances of random networks of networks ($\boldsymbol{W}^1, \ldots, \boldsymbol{W}^4$) that consist of five subnetworks. Each subnetwork contains ten nodes. Hence, the total number of nodes in networks is fifty. First, we constructed the network, where all of pairs of nodes within the same subnetwork are connected and there are no links between nodes belong to different subnetworks. Second, we rewired the link to the randomly selected node with a certain probability p. Smaller p results in more links within subnetworks and less links interconnecting subnetworks. On the other hand, if the rewiring probability p is equal to 1, the network is the ordinary random network. Hence, the rewiring probability p controls the modularity of the network. We simulated with $p = 0.2, 0.4, \ldots, 1$. We set the initial network as $\boldsymbol{W}(1) = \boldsymbol{W}^1$, and abruptly switched the network to another instance with probability 0.0002 at each time step.

We divided each time series $x^k(t)$ into segments whose length is 500 and constructed recurrence plots of each segment whose threshold is determined to obtain a probability of 0.05 for plotting a dot. Then, we obtain the time series of networks $\boldsymbol{A}(m)$ using joint recurrence plots. The threshold of the recurrence plot of the estimated time-evolving network $\boldsymbol{A}(m)$ is determined to obtain a probability of 0.2 for plotting a dot.

To validate the effectiveness of the method, we compared recurrence plots of estimated time-evolving networks to 'raw recurrence plots', which are recurrence plots of resampled raw time series,

$$\boldsymbol{x}(m) = (x^1(500m), x^2(500m), \ldots, x^{50}(500m)), \quad (5)$$

instead of estimated time-evolving networks. Thresholds of raw recurrence plots are also determined to obtain a probability of 0.2 for plotting a dot. Precisions of recurrence plots are calculated in the following way. We consider the network $\widehat{\boldsymbol{W}}(m)$, which appeared most frequently in the mth segment, as the network at the mth segment. We define the true recurrence plot of network patterns as

$$R^t_{i,j} = \begin{cases} 1, & \text{if} \quad \widehat{\boldsymbol{W}}(i) = \widehat{\boldsymbol{W}}(j), \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

Then, the precision of the recurrence plot $\boldsymbol{R}$ is the ratio of the number of dots contained in both the $\boldsymbol{R}$ and the true recurrence plot $\boldsymbol{R}^t$ to the total number of dots of $\boldsymbol{R}$, that is,

$$\frac{\sum_{i=1}^N \sum_{j=i+1}^N R_{i,j} R^t_{i,j}}{\sum_{i=1}^N \sum_{j=i+1}^N R_{i,j}}, \quad (7)$$

where N is the size of recurrence plots.

We performed the simulation 50 times. The recurrence plot of $\boldsymbol{A}(m)$, the raw recurrence plot, and the true recurrence plot are illustrated in Fig. 1. The recurrence plot of the estimated time-evolving network (Fig. 1(a)) reconstructs fine structures in the true recurrence plot (Fig. 1(c)) better than the raw recurrence plot (Fig. 1(b)).

Precisions with various rewiring probabilities are shown in Fig. 2(a)-2(e). In all of cases, precisions of recurrence plots of time-evolving networks are significantly higher than those of raw recurrence plots ($p < 5 \times 10^{-9}$ for all of cases, Wilcoxon signed-rank test).

Medians of precisions of recurrence plots of time-evolving networks with each rewiring probability are plotted in Fig. 2. Precisions in the case where the rewiring probability is equal to 0.2 are significantly smaller than those in other cases ($p < 5 \times 10^{-8}$, Wilcoxon signed-rank test). This result indicates that the high modularity of networks of networks make it difficult to characterize the dynamics of time-evolving networks.

4. Conclusion

There are many networks of networks in this world. In many cases, structures of these networks are time-evolving and unknown. We tried to capture the dynamics of time evolution of networks of networks from time series measured on individual nodes. To characterize the dynamics of time evolution of networks, we estimate the network topology from measured time series. However, many networks of networks have high modularity which makes it more difficult to estimate network topologies.

We performed numerical simulations, where individual dynamics interact with each other on the network of networks whose structure abruptly changes. We used recurrence plots of time series of networks estimated from observed time series. The network of networks was obtained by rewiring links of isolated complete subnetworks with a certain rewiring probability p. We investigated the relation between precisions of recurrence plots of estimated time-evolving networks and modularity of underlying networks by changing the rewiring probability. Although higher modularity of networks of networks result in lower precisions of recurrence plots of time-evolving networks, these recurrence plots extracted significantly more information than recurrence plots of raw time series. These results indicate the possibility to characterize the dynamics of time evolution of a variety of networks of networks, even if their topologies are unknown.

Acknowledgments

This research is supported by the Aihara Innovative Mathematical Modelling Project, the Japan Society for the Promotion of Science (JSPS) through the "Funding Program for World-Leading Innovative R&D on Science and Technology (FIRST Program)," initiated by the Council for Science and Technology Policy (CSTP).

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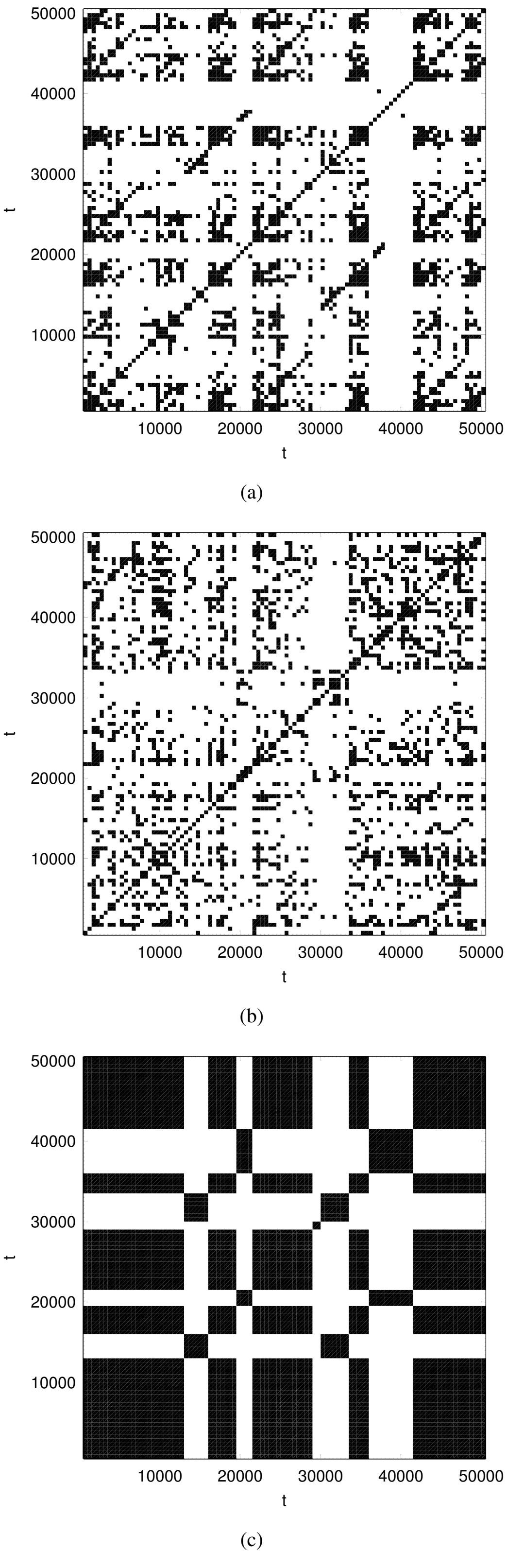

Figure 1: Recurrence plots of one of numerical simulations with the rewiring probability $p = 0.2$. (a) The recurrence plot of the time series of estimated networks $\boldsymbol{A}(m)$. (b) The raw recurrence plot. (c) The true recurrence plot $\boldsymbol{R}^t$.

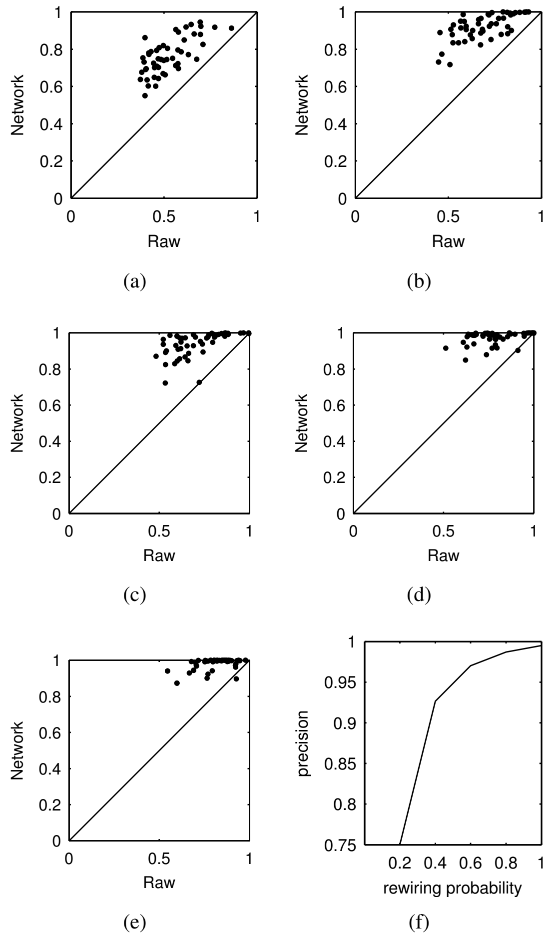

Figure 2: (a-e) Plots of precisions of recurrence plots with various rewiring probabilities. In these plots, vertical axes mean precisions of recurrence plots of time-evolving networks $\boldsymbol{A}(m)$ and horizontal axes mean those of raw recurrence plots. (a) $p = 0.2$. (b) $p = 0.4$. (c) $p = 0.6$. (d) $p = 0.8$. (e) $p = 1.0$. (f) Medians of precisions of recurrence plots of time-evolving networks.