Spectral-Domain Formulation of Electromagnetic Scattering from Circular Cylinder Located near Periodically Corrugated Surface

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1. Introduction

Periodic structures are widely used in microwave, millimeter-wave, and optical wave regions, and many analytical and numerical approaches have been developed to analyze the scattering from periodic structures. When a plane-wave illuminates a perfectly periodic structure, the Floquet theorem asserts that the scattered fields are pseudo-periodic (namely, each field component is a product of a periodic function and an exponential phase factor). This implies that the scattered fields have discrete spectra in the wavenumber space. The field components can be therefore expressed in the generalized Fourier series expansions, and the analysis region can be reduced to only one periodicity cell. Then most of the approaches for periodic structures are based on the Floquet theorem. However, when the structural periodicity is broken even if locally, the Floquet theorem is no longer applicable and the analysis region has to generally cover all the scattering structure under consideration.

This paper presents an approach in spectral-domain for the electromagnetic scattering problem of an imperfectly periodic structure, in which a circular cylinder is located near a periodically corrugated surface. The fields in imperfectly periodic structures have continuous spectra, and an artificial discretization in the wavenumber space is necessary for numerical computation. When perfectly periodic structures are illuminated by incident fields with continuous spectra, the spectra of scattered fields are known to have infinite number of non-smooth points in the wavenumber space, which are called the Wood anomalies. They do not vanish if the structural periodicity is locally collapsed, and should be taken into account on the discretization in the wavenumber space. The present approach uses the pseudo-periodic Fourier transform (PPFT) [1] to consider the discretization scheme in the wavenumber space. PPFT converts an arbitrary function into a pseudo-periodic one, and the transformed function can be expressed in the generalized Fourier series expansion. Then the conventional formulations for perfectly periodic structures based on the Floquet theorem can be applied to analyze the scattering problem of imperfectly periodic structures. The transformed function has also a periodic property in terms of the transform parameter, which is related to the wavenumber, and the analysis region in the spectral domain is reduced to the Brillouin zone. Therefore, the discretization scheme in terms of the transform parameter can be considered inside the Brillouin zone.

2. Settings of the Problem

We consider time-harmonic fields assuming a time-dependence in $e^{-i\omega t}$ and the electromagnetic scattering problem from a circular cylinder located near a periodically corrugated surface schematically shown in Fig. 1. The structure is uniform in the z-direction and the y-axis is perpendicular to the surface plane. The corrugated surface is given by $y = g(x)$, where $g(x)$ is a known periodic function with a period d. The minimum and the maximum values of $g(x)$ are respectively denoted by y_a and y_b, and $g(x)$ is supposed to be a continuous function with continuous derivative for simplification. The substrate region $y < g(x)$ is filled with a homogeneous and isotropic medium described by a permittivity ε_b and

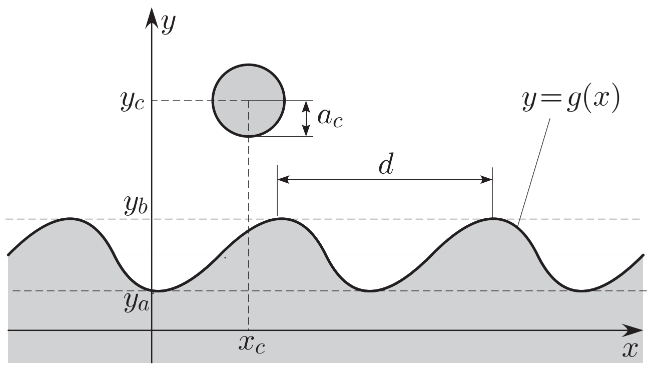

Figure 1: Geometry under consideration.

a permeability μ_b. The surrounding region $y > g(x)$ is also filled with a homogeneous and isotropic medium with the permittivity ε_s and a permeability μ_s, and a circular cylinder with the permittivity ε_c, the permeability μ_c, and the radius a_c is located in this region at $(x, y) = (x_c, y_c)$ for $y_c > y_b + a_c$. The wavenumber in each region is denoted by $k_r = \omega \sqrt{\varepsilon_r \mu_r}$ for $r = b, s, c$. The electromagnetic fields are uniform in the z-direction and two-dimensional scattering problem is here considered. Two fundamental polarizations are expressed by TE and TM, in which the electric and the magnetic fields are respectively parallel to the z-axis. Here, we denote the z-component of electric field for the TE-polarization and the z-component of magnetic field for the TM-polarization by $\psi(x, y)$, and show the formulation. The incident field is supposed to illuminate the scatterers from the upper or lower regions and there exists no source inside the scatterer region $y_a \leq y \leq y_c + a_c$.

3. Outline of Formulation

PPFT of $\psi(x, y)$ and its inverse transform are formally defined by

$$\overline{\psi}(x; \xi, y) = \sum_{m=-\infty}^{\infty} \psi(x - m\,d, y)\, e^{i\,m\,d\,\xi}, \qquad \psi(x, y) = \frac{1}{k_d} \int_{-k_d/2}^{k_d/2} \overline{\psi}(x; \xi, y)\, d\xi \tag{1}$$

where ξ denotes the transform parameter. The transformed field $\overline{\psi}(x; \xi, y)$ has a pseudo-periodic property in terms of x: $\overline{\psi}(x - d; \xi, y) = \overline{\psi}(x; \xi, y)\, e^{-i\,d\,\xi}$ and also have periodic property in terms of ξ: $\overline{\psi}(x; \xi - k_d, y) = \overline{\psi}(x; \xi, y)$. On the benefit of the pseudo-periodicity, the transformed field is approximately expressed in the truncated generalized Fourier series expansion:

$$\overline{\psi}(x; \xi, y) = \sum_{n=-N}^{N} \overline{\psi}_n(\xi, y)\, e^{i\,\alpha_n(\xi)\,x} \tag{2}$$

with $\alpha_n(\xi) = \xi + n\,k_d$ where N denotes the truncation order.

The fields in the homogeneous media satisfy the Helmholtz equation, and the transformed field can be expressed in the plane-wave expansion [1]:

$$\overline{\psi}(x; \xi, y) = \boldsymbol{f}_r^{(+)}(x; \xi, y - y')^t\, \overline{\boldsymbol{a}}_r^{(+)}(\xi, y') + \boldsymbol{f}_r^{(-)}(x; \xi, y - y')^t\, \overline{\boldsymbol{a}}_r^{(-)}(\xi, y') \tag{3}$$

with

$$\left(\boldsymbol{f}_r^{(\pm)}(x; \xi, y) \right)_n = e^{i(\alpha_n(\xi)\,x \pm \beta_{r,n}(\xi)\,y)}, \qquad \beta_{r,n}(\xi) = \sqrt{{k_r}^2 - \alpha_n(\xi)^2} \tag{4}$$

where $\overline{\boldsymbol{a}}_r^{(\pm)}(\xi, y')$ are column matrices that are generated by the amplitudes of the plane-waves at $y = y'$. The subscript r ($r = b, s$) indicates the region with the wavenumber k_r, and the superscripts $(+)$ and

$(-)$ indicate the propagation in the $+y$- and $-y$-directions, respectively. The y-dependence of $\overline{\boldsymbol{a}}_r^{(\pm)}(\xi, y)$ is given by

$$\overline{\boldsymbol{a}}_r^{(\pm)}(\xi, y) = \overline{\boldsymbol{V}}_r(\xi, \pm\left(y - y'\right))\,\overline{\boldsymbol{a}}_r^{(\pm)}(\xi, y'), \qquad \left(\overline{\boldsymbol{V}}_r(\xi, y)\right)_{n,m} = \delta_{n,m}\, e^{i\,\beta_{r,n}(\xi)\,y} \tag{5}$$

where $\delta_{n,m}$ denotes Kronecker's delta.

Since the transformed fields are expressed in the generalized Fourier series expansions as shown in Eq. (2), the scattering problem of the periodically corrugated surface can be solved by the conventional grating theories. Here, we use the differential method with following Li's Fourier factorization rules [2], and derive the scattering relation in the following form:

$$\begin{pmatrix} \overline{\boldsymbol{a}}_s^{(+)}(\xi, y_b + 0) \\ \overline{\boldsymbol{a}}_b^{(-)}(\xi, y_a - 0) \end{pmatrix} = \begin{pmatrix} \overline{\boldsymbol{S}}_{g,11}(\xi) & \overline{\boldsymbol{S}}_{g,12}(\xi) \\ \overline{\boldsymbol{S}}_{g,21}(\xi) & \overline{\boldsymbol{S}}_{g,22}(\xi) \end{pmatrix} \begin{pmatrix} \overline{\boldsymbol{a}}_s^{(-)}(\xi, y_b + 0) \\ \overline{\boldsymbol{a}}_b^{(+)}(\xi, y_a - 0) \end{pmatrix}. \tag{6}$$

This relation does not include any convolution like expression because we have used the structural period d to define PPFT. On the other hand, the scattering by the additional cylinder located at $(x, y) = (x_c, y_c)$ is described by the transition-matrix and the plane-wave amplitudes at $y = y_c \pm 0$ are related in the following form:

$$\begin{aligned} \begin{pmatrix} \overline{\boldsymbol{a}}_s^{(+)}(\xi, y_c + 0) \\ \overline{\boldsymbol{a}}_s^{(-)}(\xi, y_c - 0) \end{pmatrix} &= \begin{pmatrix} \overline{\boldsymbol{a}}_s^{(+)}(\xi, y_c - 0) \\ \overline{\boldsymbol{a}}_s^{(-)}(\xi, y_c + 0) \end{pmatrix} \\ &\quad + \frac{1}{k_d} \int_{-k_d/2}^{k_d/2} \begin{pmatrix} \overline{\boldsymbol{S}}_{c,11}(\xi, \xi') & \overline{\boldsymbol{S}}_{c,12}(\xi, \xi') \\ \overline{\boldsymbol{S}}_{c,21}(\xi, \xi') & \overline{\boldsymbol{S}}_{c,22}(\xi, \xi') \end{pmatrix} \begin{pmatrix} \overline{\boldsymbol{a}}_s^{(-)}(\xi', y_c + 0) \\ \overline{\boldsymbol{a}}_s^{(+)}(\xi', y_c - 0) \end{pmatrix} d\xi'. \end{aligned} \tag{7}$$

The structure near $y = y_c$ is not periodic in the x-direction and the convolution like expression remains. The expressions of the scattering matrices $\boldsymbol{S}_{p,nm}(\xi)$ and $\boldsymbol{S}_{c,nm}(\xi, \xi')$ for $n, m = 1, 2$ are omitted in this paper because of the page limitation.

Equations (6) and (7) that relate the plane-wave amplitudes have to be satisfied for arbitrary ξ. Here, considering the periodicity in terms of the transform parameter ξ, we take L sample points $\{\xi_l\}_{l=1}^{L}$ in the first Brillouin zone $-k_d/2 < \xi \le k_d/2$, and Eqs. (6) and (7) are satisfied only at the sample points. Also, the integration in Eq. (7) is approximated by an appropriate numerical integration scheme. To treat the discretized Rayleigh coefficients systematically, we introduce the following column matrices:

$$\widetilde{\boldsymbol{a}}_r^{(\pm)}(y) = \begin{pmatrix} \overline{\boldsymbol{a}}_r^{(\pm)}(\xi_1, y) \\ \vdots \\ \overline{\boldsymbol{a}}_r^{(\pm)}(\xi_L, y) \end{pmatrix} \tag{8}$$

for $r = b, s$, and then Eqs. (6) and (7) are rewritten as follows:

$$\begin{pmatrix} \widetilde{\boldsymbol{a}}_s^{(+)}(y_b + 0) \\ \widetilde{\boldsymbol{a}}_b^{(-)}(y_a - 0) \end{pmatrix} = \begin{pmatrix} \widetilde{\boldsymbol{S}}_{g,11} & \widetilde{\boldsymbol{S}}_{g,12} \\ \widetilde{\boldsymbol{S}}_{g,21} & \widetilde{\boldsymbol{S}}_{g,22} \end{pmatrix} \begin{pmatrix} \widetilde{\boldsymbol{a}}_s^{(-)}(y_b + 0) \\ \widetilde{\boldsymbol{a}}_b^{(+)}(y_a - 0) \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} \widetilde{\boldsymbol{a}}_s^{(+)}(y_c + 0) \\ \widetilde{\boldsymbol{a}}_s^{(-)}(y_c - 0) \end{pmatrix} = \begin{pmatrix} \widetilde{\boldsymbol{S}}_{c,11} & \widetilde{\boldsymbol{S}}_{c,12} \\ \widetilde{\boldsymbol{S}}_{c,21} & \widetilde{\boldsymbol{S}}_{c,22} \end{pmatrix} \begin{pmatrix} \widetilde{\boldsymbol{a}}_s^{(-)}(y_c + 0) \\ \widetilde{\boldsymbol{a}}_s^{(+)}(y_c - 0) \end{pmatrix} \tag{10}$$

with

$$\widetilde{\boldsymbol{S}}_{g,nm} = \begin{pmatrix} \overline{\boldsymbol{S}}_{g,nm}(\xi_1) & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \overline{\boldsymbol{S}}_{g,nm}(\xi_L) \end{pmatrix} \tag{11}$$

$$\widetilde{\boldsymbol{S}}_{g,nm} = \frac{1}{k_d} \begin{pmatrix} w_1\,\overline{\boldsymbol{S}}_{c,nm}(\xi_1, \xi_1) & \cdots & w_L\,\overline{\boldsymbol{S}}_{c,nm}(\xi_1, \xi_L) \\ \vdots & \ddots & \vdots \\ w_1\,\overline{\boldsymbol{S}}_{c,nm}(\xi_L, \xi_1) & \cdots & w_L\,\overline{\boldsymbol{S}}_{c,nm}(\xi_L, \xi_L) \end{pmatrix} + \begin{cases} \boldsymbol{0} & \text{for } n = m \\ \boldsymbol{I} & \text{for } n \neq m \end{cases} \tag{12}$$

where $\mathbf{0}$ and $\boldsymbol{I}$ denote the null and the identity matrices, respectively, and $\{w_l\}_{l=1}^{L}$ denotes the weight factor. From Eqs. (5), (9), and (10), we finally obtain the scattering relation for the entire structure as

$$\begin{pmatrix} \widetilde{\boldsymbol{a}}_s^{(+)}(y_c+0) \\ \widetilde{\boldsymbol{a}}_b^{(-)}(y_a-0) \end{pmatrix} = \begin{pmatrix} \widetilde{\boldsymbol{S}}_{11} & \widetilde{\boldsymbol{S}}_{12} \\ \widetilde{\boldsymbol{S}}_{21} & \widetilde{\boldsymbol{S}}_{22} \end{pmatrix} \begin{pmatrix} \widetilde{\boldsymbol{a}}_s^{(-)}(y_c+0) \\ \widetilde{\boldsymbol{a}}_b^{(+)}(y_a-0) \end{pmatrix} \tag{13}$$

with

$$\widetilde{\boldsymbol{S}}_{11} = \widetilde{\boldsymbol{S}}_{c,11} + \widetilde{\boldsymbol{S}}_{c,12}\,\widetilde{\boldsymbol{V}}_s\,\left(\boldsymbol{I} - \widetilde{\boldsymbol{S}}_{g,11}\,\widetilde{\boldsymbol{V}}_s\,\widetilde{\boldsymbol{S}}_{c,22}\,\widetilde{\boldsymbol{V}}_s\right)^{-1}\widetilde{\boldsymbol{S}}_{g,11}\,\widetilde{\boldsymbol{V}}_s\,\widetilde{\boldsymbol{S}}_{c,21} \tag{14}$$

$$\widetilde{\boldsymbol{S}}_{12} = \widetilde{\boldsymbol{S}}_{c,12}\,\widetilde{\boldsymbol{V}}_s\,\left(\boldsymbol{I} - \widetilde{\boldsymbol{S}}_{g,11}\,\widetilde{\boldsymbol{V}}_s\,\widetilde{\boldsymbol{S}}_{c,22}\,\widetilde{\boldsymbol{V}}_s\right)^{-1}\widetilde{\boldsymbol{S}}_{g,12} \tag{15}$$

$$\widetilde{\boldsymbol{S}}_{21} = \widetilde{\boldsymbol{S}}_{g,21}\,\widetilde{\boldsymbol{V}}_s\left[\boldsymbol{I} + \widetilde{\boldsymbol{S}}_{c,22}\,\widetilde{\boldsymbol{V}}_s\,\left(\boldsymbol{I} - \widetilde{\boldsymbol{S}}_{g,11}\,\widetilde{\boldsymbol{V}}_s\,\widetilde{\boldsymbol{S}}_{c,22}\,\widetilde{\boldsymbol{V}}_s\right)^{-1}\widetilde{\boldsymbol{S}}_{g,11}\,\widetilde{\boldsymbol{V}}_s\right]\widetilde{\boldsymbol{S}}_{c,21} \tag{16}$$

$$\widetilde{\boldsymbol{S}}_{22} = \widetilde{\boldsymbol{S}}_{g,21}\,\widetilde{\boldsymbol{V}}_s\,\widetilde{\boldsymbol{S}}_{c,22}\,\widetilde{\boldsymbol{V}}_s\,\left(\boldsymbol{I} - \widetilde{\boldsymbol{S}}_{g,11}\,\widetilde{\boldsymbol{V}}_s\,\widetilde{\boldsymbol{S}}_{c,22}\,\widetilde{\boldsymbol{V}}_s\right)^{-1}\widetilde{\boldsymbol{S}}_{g,12} + \widetilde{\boldsymbol{S}}_{g,22} \tag{17}$$

$$\widetilde{\boldsymbol{V}}_s = \begin{pmatrix} \overline{\boldsymbol{V}}_s(\xi_1, y_c - y_b) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \overline{\boldsymbol{V}}_s(\xi_L, y_c - y_b) \end{pmatrix}. \tag{18}$$

It is worth noting that, if the sample points of the transform parameter $\{\xi_l\}_{l=1}^{L}$ are taken with the constant interval and the weights $\{w_l\}_{l=1}^{L}$ are identical constants, the convergence in terms of the sample number L becomes very slow and the practical computation is impossible. This means that, from the sampling theorem, the analysis region with finite width in the spatial region cannot supply the practical computation. The trapezoidal scheme is known to usually provide accurate results to integrate smooth periodic functions over one period. However, the integrands are not smooth at the Wood-Rayleigh anomalies that are known to occur when diffraction field of a spectral order propagates along the grating surface and cause abrupt changes in the power diffracted into the other orders. Then the x-directional propagation constant ξ has to satisfy $\alpha_n(\xi) = \pm k_s$ or $\alpha_n(\xi) = \pm k_b$ at the anomalies. In the present formulation, the anomalies are degenerated to four points in the Brillouin zone. It is well known, if there are discontinuities or singularities of the integrand or of its derivative and we know where they are, the integration range should be split at these points and analyze each subinterval. We split the integration interval at the Wood-Rayleigh anomalies, and the sample points and weights are decided by applying the Gauss-Legendre scheme or the double exponential scheme for each subinterval.

4. Concluding Remarks

This paper has presents a formulation of the two-dimensional electromagnetic scattering problem from a circular cylinder located near a periodically corrugated surface. The formulation is based on PPFT and the fields in homogeneous media are expressed in the plane-wave expansions. The scattering matrices of the periodic surface and the additional cylinder are separately calculated by the conventional methods, and the plane-wave amplitudes are matched by the technique for multilayer structure. PPFT introduces a transform parameter ξ and we need to discretize it for practical computation. The transformed fields are periodic in terms of ξ and the discretization scheme can be considered inside the Brillouin zone.

References

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