Parallel Higher-Order DG-FETD Simulation of Antennas

(Invited Paper)

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***Abstract*—A discontinuous Galerkin finite-element time-domain (DG-FETD) code is developed to simulate some interesting and challenging antennas. It incorporates several advanced techniques, such as higher-order tetrahedral elements, conformal perfectly matched layer (PML), and local time-stepping scheme. To further speed up the calculation, the DG-FETD method is parallelized by using Message Passing Interface (MPI).**

I. Introduction

The discontinuous Galerkin finite-element time-domain (DG-FETD) method [1]-[5] is one of the most important time-domain methods for solving complex electromagnetic (EM) problems. The DG-FETD method is not only globally explicit, but also capable of dealing with arbitrarily-shaped and inhomogeneously-filled objects. In this paper, the well-developed DG-FETD code is applied to simulate some interesting and challenging antennas. It involves the implementation of the higher-order tetrahedral element technique [6]-[8], conformal perfectly matched layer (PML) technique [1], local time-stepping (LTS) scheme [4], and parallelization scheme using Message Passing Interface (MPI) [3]. Good numerical results demonstrate the validity and capability of the parallel higher-order DG-FETD method.

II. Basic formulation

The Maxwell's curl equations in the non-PML region are written as

$$\nabla \times \mathbf{E} = -\frac{\mu_r}{c}\frac{\partial \bar{\mathbf{H}}}{\partial t}, \quad \nabla \times \bar{\mathbf{H}} = \frac{\epsilon_r}{c}\frac{\partial \mathbf{E}}{\partial t} \tag{1}$$

where $\bar{\mathbf{H}} = \eta_0 \mathbf{H}$. The boundary conditions is imposed on the interface of elements [1]-[3]

$$\bar{\mathbf{J}}_s = \hat{n} \times (\bar{\mathbf{H}}^+ - \bar{\mathbf{H}}), \qquad \mathbf{M}_s = -\hat{n} \times (\mathbf{E}^+ - \mathbf{E}) \tag{2}$$

Applying the Galerkin's approach in each element V_i [1]-[2] and taking advantage of the leap-frog (LF) scheme [2], one can obtain the matrix equations [2]

$$A_{hhv}(h^{n+\frac{1}{2}} - h^{n-\frac{1}{2}}) = -(A_{hev}e^n + A^+_{hes}e^{+n}) - b_{hs} \tag{3a}$$

$$A_{eev}(e^{n+1} - e^n) = A_{ehv}h^{n+\frac{1}{2}} + A^+_{ehs}h^{+(n+\frac{1}{2})} - b_{es} \tag{3b}$$

In the simulation, the conformal PML is applied to terminate waveguide. The corresponding differential equations of auxiliary variables $\tilde{\mathbf{E}}$, $\tilde{\mathbf{H}}$, $\mathbf{P}$, and $\mathbf{Q}$ are given by [1]-[2]

$$\nabla \times \widetilde{\mathbf{E}} = -\frac{\mu_r}{c}\frac{\partial \bar{\bar{A}}_1 \cdot \widetilde{\mathbf{H}}}{\partial t} - \mu_r \bar{\bar{A}}_2 \cdot \widetilde{\mathbf{H}} - \mu_r \bar{\bar{A}}_3 \cdot \mathbf{P} \tag{4a}$$

$$\nabla \times \widetilde{\mathbf{H}} = \frac{\epsilon_r}{c}\frac{\partial \bar{\bar{A}}_1 \cdot \widetilde{\mathbf{E}}}{\partial t} + \epsilon_r \bar{\bar{A}}_2 \cdot \widetilde{\mathbf{E}} + \epsilon_r \bar{\bar{A}}_3 \cdot \mathbf{Q} \tag{4b}$$

$$\bar{\bar{A}}_5^{-1} \cdot \widetilde{\mathbf{H}} - \bar{\bar{A}}_4 \cdot \mathbf{P} = \frac{1}{c}\frac{\partial \mathbf{P}}{\partial t} \tag{4c}$$

$$\bar{\bar{A}}_5^{-1} \cdot \widetilde{\mathbf{E}} - \bar{\bar{A}}_4 \cdot \mathbf{Q} = \frac{1}{c}\frac{\partial \mathbf{Q}}{\partial t} \tag{4d}$$

where $\bar{\bar{A}}_j = \bar{\bar{J}}^T \Lambda_j \bar{\bar{J}}, \quad j = 1, \cdots, 5$. Λ_j are 3×3 diagonal matrices. $\bar{\bar{J}}$ is a tensor related to the local coordinate system on the interface between the PML and non-PML regions. Discretizing the above differential equations yields [2]

$$\begin{aligned}(A_{hha} + A_{hhb})h^{n+\frac{1}{2}} =& (A_{hha} - A_{hhb})h^{n-\frac{1}{2}} - A_{hp}p^n \\ & - (A_{hev}e^n + A^+_{hes}e^{+n}) - b_{hs}\end{aligned} \tag{5a}$$

$$(A_{qq} + A_{qqd})q^{n+\frac{1}{2}} = (A_{qq} - A_{qqd})q^{n-\frac{1}{2}} + A_{qe}e^n \tag{5b}$$

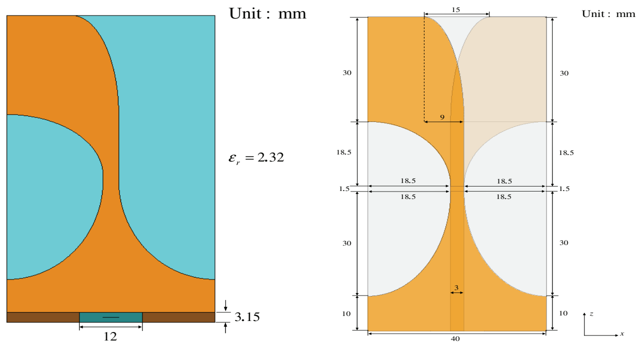
$$\begin{aligned}&(A_{eea} + A_{eeb})e^{n+1} = (A_{eea} - A_{eeb})e^n - A_{eq}q^{n+\frac{1}{2}} \\ &+ (A_{ehv}h^{n+\frac{1}{2}} + A^+_{ehs}h^{+(n+\frac{1}{2})}) - b_{es}\end{aligned} \tag{5c}$$

$$(A_{pp} + A_{ppd})p^{n+1} = (A_{pp} - A_{ppd})p^n + A_{ph}h^{n+\frac{1}{2}} \tag{5d}$$

The higher-order interpolatory vector basis functions on tetrahedral elements are applied in the above coefficient matrices. The surface magnetic current $\mathbf{M}_s$ is imposed on the excitation port. The incident electric fields can be found in terms of $\mathbf{M}_s$. Hence, pre-simulation of uniform waveguide can be avoided [2].

III. Numerical results

As an example, the Vivaldi antenna shown in Fig. 1 is simulated. This antenna is fed by a shielded microstrip line that is homogeneously filled with the dielectric $\epsilon_r = 2.32$. Figs. 2 and 3 show the incident and reflection coefficients in the time domain, respectively. The S-parameters are shown in Fig. 4. Fig. 5 shows the directivity patterns at 10 GHz. The DG-FETD results agree well with the HFSS results.



(a) 3-D view (b) Top view

Fig. 1. Configuration of the antipodal Vivaldi antenna.

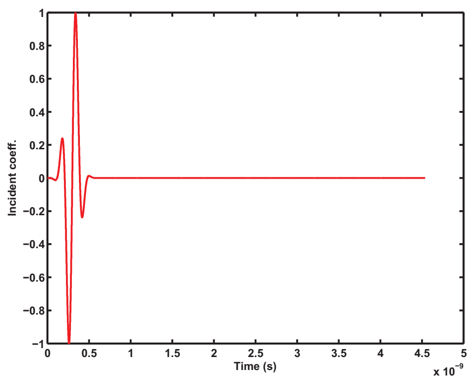


Fig. 2. Time-domain incident coefficients.

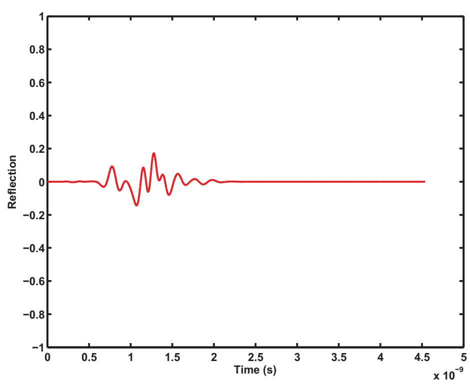


Fig. 3. Time-domain scattered coefficients.

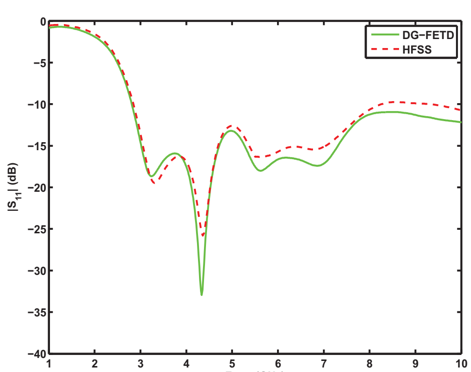
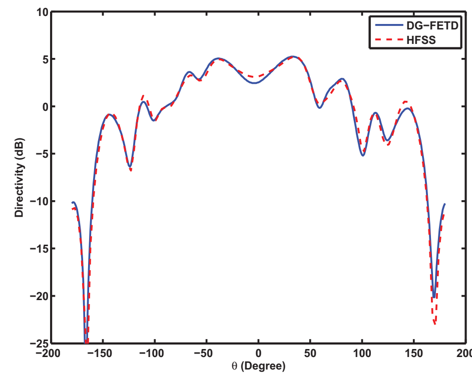
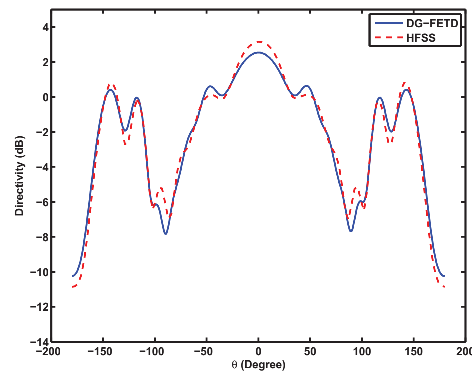


Fig. 4. S-parameters of the Vivaldi antenna.



(a) E-plane



(b) H-plane

Fig. 5. Directivity of the Vivaldi antenna at 10 GHz.

IV. Conclusions

This paper presents the simulation of antennas using the parallel higher-order DG-FETD method. The successful simulation of the challenging antennas demonstrates the capability of the DG-FETD method as an important time-domain technique in the computational electromagnetics.

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