Reflection and Transmission Analysis on Single Conductor Line

Takashi Hisakado, Naoki Takayama, and Osami Wada

Departement of Electrical Engineering, Kyoto University Kyotodaigakukatsura Nishikyo, Kyoto, 615-8510, Japan hisakado@kuee.kyoto-u.ac.jp

Abstract—This paper describes a model of current propagation on single conductor line which does not have explicit return path. As a fundamental element, we use the current on a finite length straight line and we derive the electric field caused by the current. Using models consists of the fundamental element, we obtain an integral equation in the complex frequency domain based on the boundary conditions on the conductor surface. The time domain current and charge waveforms are obtained by the numerical Laplace transform. The method figures out the source of electric field on the conductor surface and makes clear the mechanism of the current propagation based on the recursive induction of the current. First, we apply the proposed method to a uniform cylindrical single conductor line and clarify the mechanism of current propagation without explicit return path. Next, we analyze the case that the radius of single conductor line changes at a point and confirm the reflection and transmission phenomena. The mechanism of these phenomena is revealed by the charge distribution.

I. INTRODUCTION

It is well known that radiated emissions due to commonmode is very important for the noise problem [1], [2]. The common-mode which has no explicit return path produces strong emissions. In order to clarify the mechanism of the common mode propagation, it is essential to model the current propagation without explicit return path.

As a fundamental model of the current propagation without explicit return path, we consider a single conductor line. Several modes on infinite single conductor line were originally studied by Sommerfeld [3] and G.Goubau analyzed the surface waves in detail [4]. Recently, the current propagation on a semi-infinite single conductor line with a current source was studied in order to reveal the transient phenomena in time domain [5]. However, the model in [5] analyzes only uniform single conductor line since the model is based on the thin wire approximation. In order to clarify the mechanism of reflection and transmission on nonuniform single conductor line, we propose a novel model which consists of finite length line currents and accumulated charges.

II. ELECTRIC FIELD CAUSED BY CURRENT PROPAGATION ON FINITE LENGTH STRAIGHT LINE ELEMENT

As a fundamental element of current, we consider the current on a finite length straight conductor line as shown in Fig.1. The length of the line is ΔL . We assume that at t = 0 a step current of magnitude I_0 with a velocity of light

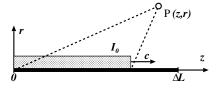


Fig. 1. Current on finite length straight conductor line E_{2} E_{2} t_{1} t_{2} t_{2} t_{1}

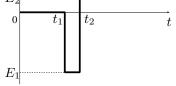


Fig. 2. Electric field E_z at a point P(z, r).

c is applied to the origin and that the current is removed at $z = \Delta L$. The current on the line is represented by

$$I(z,t) = I_0 H(t - z/c) \{ H(z) - H(z - \Delta L) \},$$
(1)

where the function $H(\cdot)$ is a Heaviside step function. From the relation between charge density and current, charge density $\rho_0 = I_0/c$ is also applied to the origin and removed at $z = \Delta L$. The charge density ρ due to the current I is

$$\rho(z,t) = \rho_0 H(t - z/c) \{ H(z) - H(z - \Delta L) \}.$$
 (2)

The conservation of charge is satisfied except at the origin and at $z = \Delta L$.

Let us consider the electric field caused by the current. The z-component of the electric field E_z at a point P(z, r) shown in Fig.1 is given by Fig.2 [6].

1) $0 \le t < t_1 \equiv \sqrt{z^2 + r^2}/c$ Because the sphere of influence does not arrive at P,

$$E_z = 0. (3)$$

 t₁ ≤ t < t₂ ≡ (ΔL + √(z − ΔL)² + r²)/c Because only the sphere of influence from the origin arrives at P, the electric field E_z is equal to

$$E_1 = -\frac{I_0}{4\pi\varepsilon_0 c} \frac{1}{\sqrt{z^2 + r^2}}.$$
(4)

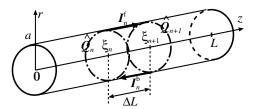


Fig. 3. Model of single conductor line

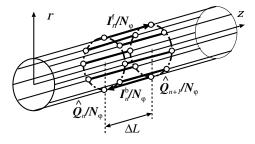


Fig. 4. Approximated surface current and accumulated charge.

3) $t_2 \le t$ Because the sphere of influence from $z = \Delta L$ arrives at

P, the electric field E_z is equal to

$$E_2 = \frac{I_0}{4\pi\varepsilon_0 c} \left\{ \frac{1}{\sqrt{z^2 + r^2}} - \frac{1}{\sqrt{(z - \Delta L)^2 + r^2}} \right\}.$$
 (5)

Although the expressions (4) and (5) do not contain ρ_0 explicitly, the effect of ρ is included by the relation $\rho_0 = I_0/c$.

If we apply a current I(t) which satisfies I(t) = 0, t < 0 instead of the step current I_0 , Eq.(3), Eq.(4) and Eq.(5) are replaced by

$$E_{z}(t) = \frac{1}{4\pi\varepsilon_{0}c} \left\{ \frac{I(t-t_{1})}{\sqrt{z^{2}+r^{2}}} - \frac{I(t-t_{2})}{\sqrt{(z-\Delta L)^{2}+r^{2}}} \right\}.$$
(6)

III. MODEL OF SINGLE CONDUCTOR LINE

We consider a cylindrical perfect conductor line with radius a and length L shown in Fig.3. In order to analyze the current propagation on the single conductor line, we divide the line into N_L equal cells, i.e., the cell length ΔL and the boundary of the cells ξ_n are

$$\Delta L = \frac{L}{N_L}, \quad \xi_n = n\Delta L \quad (n = 0, 1, \cdots, N_L).$$
(7)

We assume that the current propagates on the surface of the conductor line and that the current on the *n*th cell is represented by the forward propagating current $I_n^{f}(t)$ on $z = \xi_n$ and the backward propagating current $I_n^{b}(t)$ on $z = \xi_{n+1}$, where $n = 0, 1, \ldots, N_L - 1$.

In order to satisfy the conservation of charge on the boundary of the cell, we assume that charge $\hat{Q}_n(t)$ is accumulated on $z = \xi_n$, where $n = 1, 2, ..., N_L$. The accumulated charge is defined by

$$\hat{Q}_{n}(t) = \int_{0}^{t} \left\{ I_{n-1}^{\mathrm{f}} \left(t' - \frac{\Delta L}{c} \right) - I_{n}^{\mathrm{f}}(t') - I_{n-1}^{\mathrm{b}}(t') + I_{n}^{\mathrm{b}} \left(t' - \frac{\Delta L}{c} \right) \right\} \mathrm{d}t'. \tag{8}$$

We approximate the surface current I_n^f and I_n^b and the accumulated charge \hat{Q}_n respectively by a set of N_{φ} currents I_n^f/N_{φ} and I_n^b/N_{φ} on the line elements and a set of N_{φ} point charges \hat{Q}_n/N_{φ} as shown in Fig.4.

IV. ANALYSIS BY LAPLACE TRANSFORM

Let us consider the electric field E_z at a point P(z, r). From Eq.(6), the electric field \tilde{E}_{zn} produced by the propagating current I_n^f and I_n^b is represented by

$$\tilde{E}_{zn} = \frac{-1}{4\pi\varepsilon_0 c} \sum_{m=1}^{N_{\varphi}} \left\{ \frac{I_n^{\rm fr} \left(t - \frac{R_n}{c}\right)}{N_{\varphi} R_n} - \frac{I_n^{\rm fr} \left(t - \frac{\Delta L + R_{n+1}}{c}\right)}{N_{\varphi} R_{n+1}} + \frac{I_n^{\rm br} \left(t - \frac{\Delta L + R_n}{c}\right)}{N_{\varphi} R_n} - \frac{I_n^{\rm br} \left(t - \frac{R_{n+1}}{c}\right)}{N_{\varphi} R_{n+1}} \right\}, \qquad (9)$$

where

$$R_n = \sqrt{r^2 + a^2 - 2ra\cos\frac{2\pi m}{N_{\varphi}} + (z - \xi_n)^2}.$$
 (10)

Additionally, electric field \hat{E}_{zn} produced by the accumulated charge \hat{Q}_n is represented by

$$\hat{E}_{zn}(z,r,t) = -\frac{1}{4\pi\varepsilon_0} \frac{\partial}{\partial z} \sum_{m=1}^{N_{\varphi}} \frac{1}{R_n} \frac{\hat{Q}_n\left(t - \frac{R_n}{c}\right)}{N_{\varphi}}.$$
 (11)

We derive an integral equation based on the boundary condition on the surface of the perfect conductor:

$$E_z(z,r) = \sum_{n=0}^{N_L - 1} \tilde{E}_{zn}(z,r) + \sum_{n=1}^{N_L} \hat{E}_{zn}(z,r) = 0$$
(12)

When the current $I_0(s)$ is applied to the origin, the scaled integral equation in complex frequency domain is represented by

$$\sum_{n=1}^{N_{L}-1} \sum_{m=1}^{N_{\varphi}} \begin{cases} -\frac{e^{-R_{n}s}}{R_{n}} + \frac{e^{-(\Delta L + R_{n+1})s}}{R_{n+1}} \\ + & \left(\frac{e^{-R_{n}s}}{R_{n}}\right)'\frac{1}{s} - \left(\frac{e^{-R_{n+1}s}}{R_{n+1}}\right)'\frac{e^{-\Delta Ls}}{s} \end{cases} I_{n}^{f}(s)$$
$$+ \sum_{n=1}^{N_{L}-1} \sum_{m=1}^{N_{\varphi}} \begin{cases} -\frac{e^{-(\Delta L + R_{n})s}}{R_{n}} + \frac{e^{-R_{n+1}s}}{R_{n+1}} \\ - & \left(\frac{e^{-R_{n}s}}{R_{n}}\right)'\frac{e^{-\Delta Ls}}{s} + \left(\frac{e^{-R_{n+1}s}}{R_{n+1}}\right)'\frac{1}{s} \end{cases} I_{n}^{b}(s)$$
$$= -\sum_{m=1}^{N_{\varphi}} \begin{cases} -\frac{e^{-R_{0}s}}{R_{0}} + \frac{e^{-(\Delta L + R_{1})s}}{R_{1}} \\ - & \left(\frac{e^{-R_{1}s}}{R_{0}}\right)'\frac{e^{-\Delta Ls}}{s} \end{cases} I_{0}(s), \quad (13)$$

where

$$\left(\frac{\mathrm{e}^{-R_n s}}{R_n}\right)' = -\frac{(1+R_n s)(z-\xi_n)}{R_n^3} \mathrm{e}^{-R_n s}.$$
 (14)

We solved the integral equation numerically in the complex frequency domain and obtain the waveforms in time domain using the numerical inversion of the Laplace transform [7].

V. RESULTS OF ANALYSIS

A. Uniform single conductor line

We consider a single conductor line with radius a = 0.05mm and length L = 60mm. When a step current of magnitude 0.5A is applied to the conductor line, the calculated current I_n and charge Q_n defined by

$$I_{n}(t) = I_{n}^{f}(t) - I_{n}^{b}(t)$$
(15)

$$Q_n(t) = \{I_n^{t}(t) + I_n^{b}(t)\}\Delta L/c + \hat{Q}_{n+1}(t)$$
 (16)

are respectively shown in Fig.5 and Fig.6, where T = 200 ps. From these figures we can confirm that the step current with distortion propagates. The charge accumulation in the vicinity of the origin in Fig.6 is linked to the distortion of the current waveform. Since the accumulated charge \hat{Q}_n on the boundary is less than $10^{-2}pC$, almost all charge Q_n is the charge density in the cell.

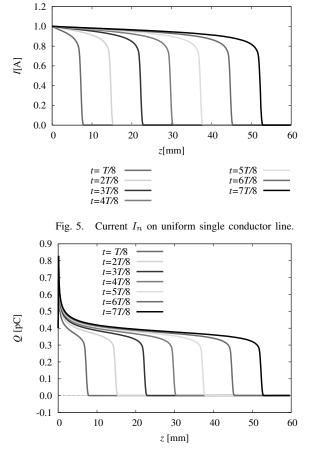


Fig. 6. Charge Q_n on uniform single conductor line.

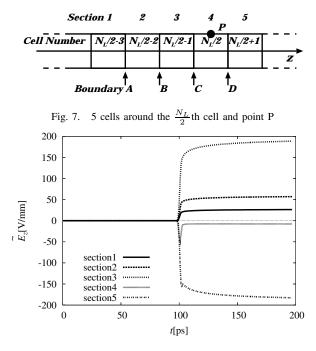


Fig. 8. Electric field caused by each the cell current on P.

In order to clarify the mechanism of the current propagation, we consider the electric filed E_z on the surface point P in the $\frac{N_L}{2}$ th cell shown in Fig.7. We pick up 5 cells which are named section 1 to 5 around the $\frac{N_L}{2}$ th cell, and show the electric field which each the cell current produces on the point P in Fig.8. The sections 1 to 3 generate positive E_z on P and the currents in the section 4 are induced by the positive E_z . After the rising edge comes in the section 5, the section 5 generate negative E_z on P. Thus, the positive electric field induces the current on the rising edge and the recursive induction causes the current propagation[5].

B. Connection of single conductor lines with different radius

We consider as second example that the radius of the single conductor line changes at L/2. From z = 0 to z = L/2, the radius a_1 is equal to 0.05mm. From z = L/2 to z = L, the radius a_2 is equal to 0.4mm. Fig.9 and Fig.10 represent the propagation of current and charge, respectively. We can confirm the reflection and transmission of current propagation at z = 30mm. Since the radius a_2 is larger than a_1 , large positive current is transmitted to the large radius line and large positive current is reflected to the small radius line.

In order to clarify the mechanism of the transmission, we consider the electric field E_z on the surface point P in the $\frac{N_L}{2}$ th cell shown in Fig.11. As we analyzed in the previous example, we pick up 5 cells and calculate the effect of the current and the charge. Fig.12 and Fig.13 represents the effect of each cell current and each charge on the boundary of the cell, respectively. From these figures, the charge on the boundary C causes strong electric field on P, and large current is transmitted to the line with large radius.

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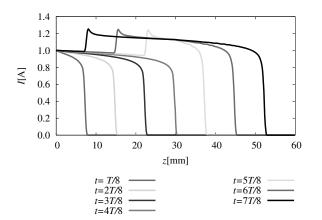


Fig. 9. Current I_n on the single conductor line. The radius of the line changes at z = 30mm.

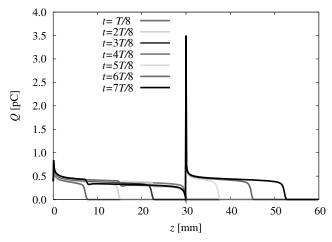


Fig. 10. Charge Q_n on the single conductor line. The radius of the line changes at z = 30mm.

VI. CONCLUSION

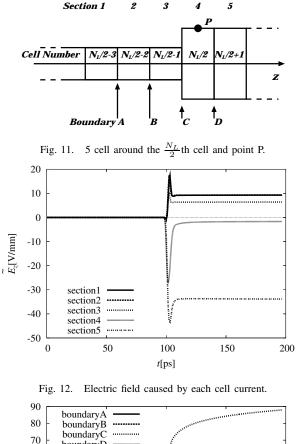
This paper proposed a method for analyzing transient phenomena on single conductor line which does not have explicit return path. Using the electric field caused by the current on the finite length line element which is analytically calculated, we derived the integral equation of the cell currents. Using the proposed method, we reveal the mechanism of the current propagation based on the model in which the current and charge causes the electric field and the electric field causes the induced current. Especially, we clarified that the reflection and transmission of current are generated at the connection of the single conductor lines with different radius and that the charge on the connection mainly induces the current transmission.

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REFERENCES

 C.R. Paul:" A Comparison of the Contributions of Common-Mode and Differential-Mode Currents in Radiated Emissions," IEEE Trans EMC, Vol.31, No.2, pp.189-193, 1989.



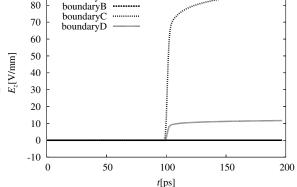


Fig. 13. Electric field caused by each charge on the boundary of the cell.

- [2] C. R. Paul:Analysis of Multiconductor Transmission Lines, (John Wiley & Sons, Inc., 1994).
- [3] A. Sommerfeld: Electrodynamics, (Academic Press, New York, 1964).
- [4] G.Goubau: "Surface Waves and Their Application to Transmission Lines," Journal of Applied Physics, Vol. 21, (1950), pp. 1119–1128.
- [5] T. Hisakado, K. Yoshimura, K. Okumura: "Analysis of Common Mode Propagation Based on Single Conductor Line," Proc. PIERS2006, 1P3, pp.76-80, 2006.
- [6] R.Lundholm, R.B.Finn and W.S.Price:"Calculation of Transmission Line Lightning Voltages by Field Concepts," AIEE Trans, Vol.77, No.3, pp.1271-1281, 1958.
- [7] A. Yonemoto, T. Hisakado and K. Okumura, Accuracy Improvement of the FFT-Based Numerical Inversion of Laplace Transforms, *IEE Proc.-Circuits Devices S yst.*, Vol.150, (2003), pp. 399–404.