A Novel Active Element Pattern Method for Calculation of Large Linear arrays

Shuai Zhang, Shuxi Gong, Huan Hu
Xidian University, Xi'an, Shaanxi 710071 China

***Abstract*-The standard active element pattern (AEP) method is presented to predict the radiation pattern of large finite arrays. In this method, the AEPs of a large array are deduced from those of a small subarray. Thus, it needs to compute the AEPs of the subarray element-by-element and costs much CPU time. Also, this paper proposes a simplified AEP (SAEP) method to eliminate this complicated procedure. In the expression of the SAEP method the radiation field of a large finite array is simply related to the total radiation field of two small subarrays. Compared with the standard AEP method, the SAEP method allows us to arrive at a more efficient calculation of the radiation from large arrays. Numerical results show that the SAEP method has the same accuracy as the simulator HFSS while maintaining the simplicity of the pattern multiplication method.**

I. INTRODUCTION

Consider a uniform linear array of N identical elements (Fig. 1), ordinary array theory ignores the mutual coupling effects between elements, and expresses the pattern of the array in the well-known pattern multiplication [1] form of an element factors timing an array factor. However, as demand for phased arrays that requires electronic scanning capabilities has increased, needs for accurate analysis is required, such as the method of moments [2], finite element methods [3], etc.. However, even the most efficient numerical method remains infeasible for large arrays. To solve this problem, a powerful and accurate method is presented, depending on the knowledge of the active element pattern (AEP) [4-6]. In this standard AEP method, the AEPs of a large array are deduced from a small array by ignoring the coupling beyond the small array size. Then the large array calculation problem is transformed into that of a small array. However, the standard AEP method needs to compute and store the AEPs of the small array element-by-element.

In this paper, the standard AEP method is modified to eliminate this fussy procedure. In the novel derived expression, the field radiated by a large array is only related to the total radiation field of two small arrays. Compared with the standard AEP method, the proposed one achieves a more efficient calculation of the radiation from large arrays. More importantly, the simplified AEP (SAEP) method significantly extends the range of problems that can be rigorously treated with limited resources. Numerical result shows that the pattern calculated using the SAEP method is in good agreement with the one simulated by the FEM-Based Simulator Ansoft HFSS.

II. STANDARD ACTIVE ELEMENT PATTERN METHOD

For the array in Fig. 1, the elements are equally placed along the x-axis with the first one at the origin of the coordinate system. The far field radiated by the array can be obtained by summation of the AEPs of all the elements [4-6]

$$E_N(\theta,\phi)=\sum_{n=1}^{N}E_{N,n}(\theta,\phi) \quad (1)$$

where $E_{N,n}(\theta,\phi)$ is the AEP [4] of the nth element in an N-element array.

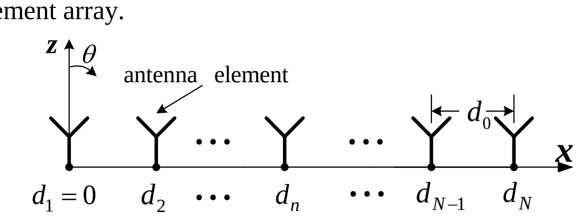


Figure 1. Geometry of a uniform linear array of N identical elements.

As the number of elements in such an array increases, the AEPs of the interior elements (those elements located away from both the ends of the array) become more and more alike [6]. In this case, these AEPs can be assumed to be identical, and the elements of the array can be divided into two *edge* element groups and an *interior* element group (such as the 8-element array shown in Fig. 2). Therefore, the AEPs of the interior elements can be approximated by that of the central element of a small array. Also, the AEP of each edge element of the large array can be approximated by that of this small array, respectively. We shall refer to such a small array as *subarray*. For simplicity, we could illustrate the above procedure for using a 5-element linear array to deduce a linear array of 8 elements as shown in Fig. 2. Then, the radiation pattern of a large array can be calculated by summation of the deduced AEPs of all the elements.

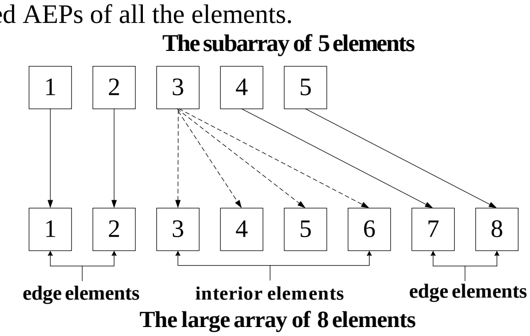


Figure 2. Linear array of 5 elements that is used to deduce an 8-element array

Assume that the number of the subarray elements is M (M is odd), the radiation field of it can be expressed as

$$E_{\mathrm{M}}(\theta,\phi)=\sum_{n=1}^{\mathrm{M}}E_{\mathrm{M},n}(\theta,\phi)=E_l+E_{ce}+E_r$$
$$=\sum_{n=1}^{(\mathrm{M}+1)/2-1}E_{\mathrm{M},n}(\theta,\phi)+E_{\mathrm{M},(\mathrm{M}+1)/2}(\theta,\phi)+\sum_{n=(\mathrm{M}+1)/2+1}^{\mathrm{M}}E_{\mathrm{M},n}(\theta,\phi) \quad (2)$$

where E_{ce} is defined as the AEP of the central element of the subarray; E_l and E_r are defined as the summation of the AEPs of the left and right edge elements, respectively.

According to the equivalence method shown in Fig. 2, the far field radiated by a large array of N (N>M) elements can be deduced from the subarray by

$$E_{\mathrm{N}}(\theta,\phi)=E_l+E_{ce}\sum_{n=1}^{\mathrm{N}-\mathrm{M}+1}\exp[\mathrm{j}k(n-1)d_0\sin\theta\cos\phi]+E_r\cdot\exp[\mathrm{j}k(\mathrm{N}-\mathrm{M})d_0\sin\theta\cos\phi] \quad (3)$$

where d_0 is the inter-element spacing; $\mathrm{k}=2\pi/\lambda$ is the free space propagation constant, λ is the wavelength.

III. Simplified AEP Method

Note that the standard AEP method of (3) converts the radiation calculation of a large array into that of a small subarray of M elements. However, it needs to compute and store the AEPs of the subarray element-by-element, thereby leading to much CPU time and fussy operational procedure. These limitations motivate the simplified active element pattern (SAEP) method to the solution of large finite arrays.

A. Derivation of the SAEP Method

Based upon (2) and (3), the difference of the total radiation field between an N-element array and an M-element array is

$$E_{\mathrm{N}}(\theta,\phi)-E_{\mathrm{M}}(\theta,\phi)=\sum_{n=1}^{\mathrm{N}}E_{\mathrm{N},n}(\theta,\phi)-\sum_{n=1}^{\mathrm{M}}E_{\mathrm{M},n}(\theta,\phi)$$
$$=E_{\mathrm{M},(\mathrm{M}+1)/2}(\theta,\phi)\sum_{n=1}^{\mathrm{N}-\mathrm{M}}\exp(\mathrm{j}2\mathrm{k}nd_0\sin\theta\cos\phi)+\sum_{n=\mathrm{M}-(\mathrm{M}+1)/2+2}^{\mathrm{M}}E_{\mathrm{M},n}(\theta,\phi)\{\exp[\mathrm{j}2\mathrm{k}(N-\mathrm{M})d_0\sin\theta\cos\phi]-1\} \quad (4)$$

According to (4), the difference of the radiation field between an (M+1)-element array and an array of M elements is

$$E_{\mathrm{M}+1}(\theta,\phi)-E_{\mathrm{M}}(\theta,\phi)=E_{\mathrm{M},(\mathrm{M}+1)/2}(\theta,\phi)\cdot\exp(\mathrm{j}2\mathrm{k}d_0\sin\theta\cos\phi)+\sum_{n=\mathrm{M}-(\mathrm{M}+1)/2+2}^{\mathrm{M}}E_{\mathrm{M},n}(\theta,\phi)\cdot[\exp(\mathrm{j}2\mathrm{k}d_0\sin\theta\cos\phi)-1] \quad (5)$$

Substituting (5) into (4), we get

$$E_N(\theta,\phi)-E_{\mathrm{M}}(\theta,\phi)=[E_{\mathrm{M}+1}(\theta,\phi)-E_{\mathrm{M}}(\theta,\phi)]\cdot\sum_{n=1}^{\mathrm{N}-\mathrm{M}}\exp[\mathrm{j}2\mathrm{k}(n-1)d_0\sin\theta\cos\phi] \quad (6)$$

Then, the field radiated by a uniform N-element linear array can be written in a simple form as

$$E_{\mathrm{N}}(\theta,\phi)=E_{\mathrm{M}}(\theta,\phi)+[E_{\mathrm{M}+1}(\theta,\phi)-E_{\mathrm{M}}(\theta,\phi)]\cdot\frac{1-\exp[\mathrm{j}2\mathrm{k}(\mathrm{N}-\mathrm{M})d_0\sin\theta\cos\phi]}{1-\exp(\mathrm{j}2\mathrm{k}d_0\sin\theta\cos\phi)} \quad (7)$$

Observe that the radiation from the array is simply related to the radiation fields of an M-element and an (M+1)-element small array. Compared with the standard AEP method, the SAEP method eliminates the complicated operation of individual element pattern computation and storage for the subarray. Meanwhile, the calculation results of the two methods are identical since they have the same equivalence method of AEPs as shown in Fig. 2. Thus, the SAEP method allows us to arrive at a fast and accurate calculation of the radiation pattern of a large array when numerical methods and simulation software are infeasible.

B. Verification of the SAEP method

The configuration of the antenna element is shown in Fig. 4. The rectangular patch is printed on the surface of the dielectric substrate with a thickness of 1 mm and a relative permittivity of 2.65. The length L and the width W of the patch are chosen to be 8.7 and 8.4 mm, respectively. The central operating frequency is 10 GHz. The simulator HFSS is employed to compute the radiation field of the subarray and the results of it are taken as reference. In the examples, the pattern calculated using the standard AEP method is not given since it is identical to that of the SAEP method.

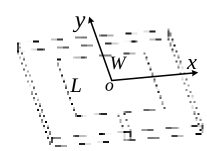
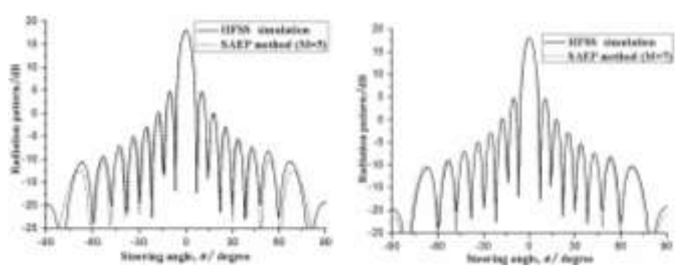


Figure 3. Configuration of the microstrip patch antenna

a. Linear array of 16 elements with $d_0=0.5\lambda$

Given a uniform linear array composed of 16 microstrip patch antennas, we will discuss how many elements are needed in a subarray for satisfied accuracy. The inter-element spacing is $d_0=0.5\lambda$.



(a) M=5 (b) M=7

Figure 4. Comparison of the radiation pattern simulated by HFSS and calculated by the SAEP method

Fig. 4 (a) shows the comparison of the radiation patterns simulated by HFSS and calculated by the SAEP method while M=5. Observe that the results calculated by the SAEP method are consistent with those simulated by HFSS near the main radiation direction (at boresight of $\theta = 0°$). Away from the main radiation direction, the SAEP method fails since the mutual coupling and the array edge effects cause significant variation in AEPs across the array. In order to improve the accuracy of the SAEP method, more elements should be considered as edge elements. Fig. 4 (b) shows the comparison between the radiation pattern calculated by the SAEP method while M=7 and that simulated by HFSS. Observe that the calculated results closely match the simulated ones in the whole region of the θ -angle.

Through the above comparison, we can assume that the SAEP method can obtain satisfactory accuracy while M=7. In this situation, the accuracy of the SAEP method is similar with that of the HFSS simulation software.

b. Large array of 100 elements with $d_0 = 0.3\lambda$

To demonstrate the superiority of the SAEP method, a linear array consists of 100 microstrip patch antennas (Fig. 3) is considered. The inter-element spacing of the array is chosen as 0.3 λ for stronger mutual coupling effects.

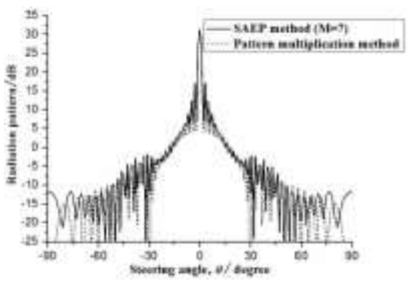


Figure 5. Comparison of the radiation pattern calculated by the SAEP method and by the pattern multiplication method.

For this array, the simulation software and the numerical methods are infeasible with our PC. However, the SAEP method can still fast and accurately calculate the radiation from the array. The radiation pattern of the array calculated by the SAEP method with M=7 is compared to that computed using the pattern multiplication method (in which the radiation from an array is expressed as the product of an element factor and an array factor) in Fig. 5. Note that the results computed using the pattern multiplication method are consistent with those calculated by the SAEP method near the main radiation direction. Away from the main radiation direction, especially when the steering angle approaching to the array edge, the difference between two methods becomes larger and larger since the pattern multiplication method cannot efficiently consider the effects of the mutual coupling between elements and the edge diffraction. The comparisons demonstrate that the SAEP method does lead to a more accurate representation of the radiation from array than the pattern multiplication method.

IV. CONCLUSIONS

This paper proposes a SAEP method for calculating the radiation pattern of large finite arrays. With this method, the radiation field of a large finite array is simply related to the total radiation field of two small subarrays. The examples demonstrate that the SAEP method has the similar accuracy as the HFSS simulation software while maintaining the simplicity of the pattern multiplication method, and its computational cost increases slightly with an increase of the number of array elements. Thus, the SAEP method can fast and accurately solve complex and large array problems using a desktop PC when numerical methods and simulation software are infeasible.

ACKNOWLEDGMENT

This work was supported by the Fundamental Research Funds for the Central Universities of China under Grant K5051302031, and the New Teacher Innovation Foundation of Xidian University under K5051302037.

REFERENCES

[1] C.A. Balanis 2002, *Antenna theory*, 2nd, New York, John Wiley&Sons, Inc. Press.

[2] Ma J., Gong S.-X., and Wang X. (2012), ‘Efficient Wide-Band Analysis of Antennas Around a Conducting Platform Using MoM-PO Hybrid Method and Asymptotic Waveform Evaluation Technique ’, *IEEE Transactions on Antennas and Propagation*, 60, 6048-6052.

[3] Zhang P.F., Gong S.-X., and Zhao S.F. (2009), ‘Fast Hybrid FEM/CRE-UTD Method to Compute the Radiation Pattern of Antennas on Large Carriers’, *Progress In Electromag-netics Research*, 89, 75-84.

[4] Plzar D.M. (1994), ‘The Active Element Pattern’, *IEEE Transactions on Antennas and Propagation*, 42, 1176-1178.

[5] Toh B.Y., Fusco V.F., and Buchanan N.B. (2001), ‘Retrodirective Array Tracking Prediction Using Active Element Characterization’, *Electronics Letters*, 37, 727-728.

[6] Kelly D.F., and Stutzman W.L. (1993), ‘Array Antenna Pattern Modeling Methods that Include Mutual Coupling Effects’, *IEEE Transactions on Antennas and Propagation*, 41, 1625-1632.