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On Logic Functions of Complementary Sets of Polyphase Sequences

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Abstract—**A set consisting of subsets of sequences is called as a complementary sequence sets, if the sum of autocorrelation functions of the sequences in a subset is zero except for the zero-shift, and the sum of the cross-correlation functions of the corresponding sequences in these two subsets is zero everywhere. This paper formulates and investigates the logic functions of the generalized polyphase complementary sequence sets of N subsets of N sequences of length N including biphase and quadriphase ones, which have been proposed by Suehiro.**

I. Introduction

Complementary sequences are a pair of bi-phase sequences having an ideal impulse property in which the sum of a periodic autofunctions of two two-phase sequences is zero except for the zero-phase shift [1]. Many have been discussed, such as their extensions and related sequences, codes [2]-[4], and logic functions generating them [5]-[7]. Furthermore complementary sets of sequences has been discussed [8]- [9], such that the sum of autocorrelation functions of the sequences in a subset is zero except for the zero-shift. and the sum of the cross-correlation functions of the corresponding sequences in these two subsets is zero any shift. Teng and Liu have shown the construction method of complementary sets of bi-phase sequences, and Suehiro has given that for complementary sets of $K = N$ subsets (or maites) of $M = N$ polyphase sequences of length $L = N^l$ expressed as $CS(L = N^l, K = N, M = K)$. They will be applied for radar and synchronization sequences, spreading sequences for CDMA, digital watermarks, and so on [10].

In this paper, the complementary sets of polyphase sequences proposed by Suehiro are discussed, and their logic functions mapping integers to integer rings are derived. In section 2, the basic matters required in this paper such as vectors and the Hadamard matrix are explained, and complementary sets are defined. In section 3, first the Suehiro's construction method of the complementary sets are outlined, and it is represented by matrices. Next their logic functions are derived and extended to the complementary sets of q-phase sequences with $N = q^r$. In section 4, as an example, several complementary sets are discussed.

II. Definitions

First, a vector over finite ring and an unitary matrix of elements with unit magunitude called a complex Hadamard matrix are introduced. Next a complementary sequence set is defined.

A. A compelex Hadamard matrix

Let Z_q be the ring of integers modulo q. Let $\boldsymbol{x}$ be a vector of order n on Z_q written as

$$\boldsymbol{x} = (x_0, x_1, \cdots, x_{n-1}) \in V_q^n, \tag{1}$$

which is a coefficient vector of integer $x (0 \leq x \leq q^n - 1)$, denoted by

$$x = x_0 q^0 + x_1 q^1 + \cdots + x_{n-1} q^{n-1}. \tag{2}$$

The complex Sylvester-type Hadamard matrix of order $N = q^n$ is given as

$$H = [h_{y,x}]_{0 \leq x,y < N} = [\omega_q^{\boldsymbol{x}\cdot\boldsymbol{y}}]_{\boldsymbol{x},\boldsymbol{y} \in V_q^n} \tag{3}$$

satsfying

$$HH^* = H^*H = NE, \tag{4}$$

where $\boldsymbol{x} \cdot \boldsymbol{y}$ denotes the inner product of vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, $\omega_q = \exp(j2\pi/q)$, and E the unit matrix of order N,where $j = \sqrt{-1}$. If $\boldsymbol{x}$ and $\boldsymbol{y}$ are integer x and y, and $N = q$, Eq. (6) denotes the well-known Fourier transform matrix . In general, a complex Hadamard matrix is corresponding to a unitary matrix and even if each row or each column is replaced, it is also one.

Let a be polyphase sequences of length $N = q^n$ written as

$$\left.\begin{array}{lcl} a & = & (a_0, a_1, \cdots, a_x, \cdots, a_{N-1}), a_x = \omega_q^{f_a(\boldsymbol{x})} \\ b & = & (b_0, b_1, \cdots, b_x, \cdots, b_{N-1}), b_x \in C \end{array}\right\} \tag{5}$$

where $f_a(\cdot)$ is an adequate generating (logic) function. The Hadamard transform is defined by

$$b_v = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} a_x \omega_q^{vx}, \tag{6}$$

where $0 \leq v < N$. Note that in the case of $N = q$, the above equation denotes the well-known Fourier transform. The Hadamard transform is expressed by

$$\boldsymbol{b} = \frac{1}{\sqrt{N}} H\boldsymbol{a}, \tag{7}$$

where $\boldsymbol{b}$ and $\boldsymbol{b}$ mean vertical vectors in the sequence of Eq. (5). If $|b_v| = 1$ for all v, $f_a(\cdot)$ is called a bent function, or a is collectively called bent.

In the future, to discuss the property of bent, a complex Hadamard matrix based on the Sylvester-type Hadamard matrix will be used here.

B. Complementary sets

Let A be a set of K subsets A^z of M complex sequences a^z_y with length L defined as

$$\left.\begin{array}{rcl} A &=& \{A^0, \cdots, A^z, \cdots, A^{K-1}\}, \\ A^z &=& \{a^z_0, \cdots, a^z_y, \cdots, a^z_{M-1}\}, \\ a^z_y &=& (a^z_{y,0}, \cdots, a^z_{y,x}, \cdots, a^z_{y,L-1}), \end{array}\right\} \quad (8)$$

where $a^z_{y,x}$ denotes an complex element with the unit magnitude, e.g., $|a^z_{y,x}| = 1$. The aperiodic correlation function between a^z and $a^{z'}$ is defined by

$$C_{a^z_y, a^{z'}_y}(\tau) = \begin{cases} \sum_{x=0}^{L-1-\tau} a^{z'}_{y,x}(a^z_{y,x+\tau})^* & (0 \le \tau \le L-1), \\ \sum_{x=0}^{L-1+\tau} a^{z'}_{y,x-\tau}(a^z_{y,x})^* & (1-L \le \tau < 0), \\ 0 & (|\tau| \ge N), \end{cases} \quad (9)$$

where $*$ denotes complex conjugate or complex conjugate transpose of a matrix described later. The periodic correlation function can be defined by

$$\begin{array}{rcl} R_{a^z_y, a^{z'}_{y'}}(\tau) &=& \sum_{x=0}^{L-1} a^{z'}_{y,x}(a^z_{y,x+\tau \bmod L})^* \\ &=& C_{a^z_y, a^{z'}_y}(\tau) + C_{a^z_y, a^{z'}_y}(\tau - L). \end{array} \quad (10)$$

Consider the aperiodic correlation function between the subsets of A^z and $A^{z'}$ defined by

$$C_A(z, z', \tau) = \sum_{y=0}^{M-1} C_{a^z_y, a^{z'}_y}(\tau). \quad (11)$$

If the set A possesses the correlation properties

$$C_A(z, z', \tau) = \begin{cases} ML & (\tau = 0, z = z'), \\ 0 & (\tau = 0, z \ne z'), \\ 0 & (\tau \ne 0), \end{cases} \quad (12)$$

it is called a complementary sequence set which is expressed by $CS(L, K, M)$.

Note that a set of binary sequence pairs, A^1 and A^2, is well known as a perfect binary complementary pairs.

III. Construction of complementary sets

In this section the construction method of generalized complementary sequence sets $CS(L = N^m, K = N, M = N)$ presented by Suehiro [9] are outlined, and their logic functions mapping from V^n_q to Z_q or R_q are formulated.

A. Suehiro's construction method

Let B be a complex Hadamard matrix of order N as

$$B = [b_{yx}]_{0 \le y,x < N} = HG, \quad (13)$$

where H is the Sylvester-type Hadamard matrix of order N of (3), and G is a diagonal matrix whose diagonal elements are absolute values.

Let $\hat{b}$ be a sequence of length N^2 given by arranging each row of the matrix B in order, written as

$$\left.\begin{array}{rcl} \hat{b} &=& (\hat{b}_0, \cdots, \hat{b}_{\hat{x}}, \cdots, \hat{b}_{N^2-1}), \\ b_{\hat{x}} &=& \hat{b}_{x_1 N + x_0} = b_{x_1, x_0} \end{array}\right\} \quad (14)$$

where $\hat{x} = x_1 N + x_0 \quad (0 \le \hat{x} < N^2, 0 \le x_0, x_1 < N)$. It is called an N-shift orthogonal sequence, because the aperiodic autocorrelation takes zero at N times shift, i.e, $R_{\hat{b},\hat{b}}(kN) = 0$ for $1 \le k < N$.

Let C be a complex Hadamard matrix of order N written as

$$C = [c_{yx}]_{0 \le y,x < N}. \quad (15)$$

A set D of N mates of order N, whose aperiodic crosscorrelation function takes zero for any N times shifts, is represented as an $N \times N^2$ matrix.

$$\left.\begin{array}{rcl} D &=& [d_{yx}]_{0 \le y < N, 0 \le \hat{x} < N^2}, \\ d_{y\hat{x}} &=& d_{y, x_1 N + x_0} = c_{yx_1} \hat{b}_{x_1 N + x_0}, \\ &=& c_{yx_1} b_{x_1 x_0}. \end{array}\right\} \quad (16)$$

Similarly, let E be a complex Hadamard matrix of order N written as

$$E = [e_{yx}]_{0 \le y,x < N}, \quad (17)$$

Then, a complementary sets $CS(L = N^2, K = N, M = N)$ can be expressed as

$$\left.\begin{array}{rcl} F &=& \{F^0, \cdots, F^z, \cdots, F^{N-1}\}, \\ F^z &=& \{F^z_0, \cdots, F^z_y, \cdots, F^z_{N-1}\}, \\ F^z_y &=& (f^z_{y,0}, f^z_{y,1}, \cdots, f^z_{y,\hat{x}}, \cdots, f^z_{y,N^2-1}), \\ f^z_{y,\hat{x}} &=& f^z_{y, x_1 N + x_0} = e_{z, x_0} d_{y\hat{x}}. \end{array}\right\} \quad (18)$$

Next, an N-shift orthogonal sequence in which the sequence length is expanded by N times is given as

$$G^z_y = \begin{array}{llll} (\ g^z_{y,0}, & g^z_{y,1}, & \cdots, & g^z_{y,N-1} \\ g^z_{y,N}, & g^z_{y,N+1}, & \cdots, & g^z_{y,2N-1} \\ \cdots, & g^y_{x'} & ,\cdots & \\ g^z_{y,N^3-N}, & g^z_{y,N^3-N+1}, & \cdots, & g^z_{y,N^3-1}), \end{array} \quad (19)$$

and

$$g^z_{y,x'} = f^z_{y, x_2 N + x_1} = e_{z,x_1} d_{x_0, x_2 N + x_1} \quad (20)$$

by interleaving in order from the head element of F^z_0 to F^z_{N-1} for all z, where $x' = \hat{x}N + x_0 = x_2 N^2 + x_1 N + x_0$ and $y = x_0$.

Let P be a complex Hadamard matrix of order N given as

$$P = [p_{yx}]_{0 \le y,x < N}. \quad (21)$$

Similar to (18), a commplementary set $CS(N^3, N, N)$ is given as

$$\left.\begin{array}{rcl} A^z &=& [A^z_0, \cdots, A^z_y, \cdots, A^z_{N-1}], \\ A^z_y &=& (a^z_{y,0}, a^z_{y,1}, \cdots, a^z_{y,x'}, \cdots, a^z_{y,N^3-1}), \\ a^z_{yx'} &=& p_{zx_0} g_{yx'}. \end{array}\right\} \quad (22)$$

By repeating in order from Eq. (17), a complementary set of extended length, $CS(N^n, N, N)$, can be generated.

IV. Formalization of complementary sets

First the logic functions of polyphase complementary sets $CS(N^m, N, N)$ are formalized, and ones of q or more phase elements in the case of $N = q^n$ are derived.

A. Representation of complementary sets

The Hadamard matrix of (13) is expressed as

$$\left.\begin{array}{rcl} B &=& [b_{yx} = \omega_q^{h^B(y,x)}], \\ h^B(y,x) &=& \boldsymbol{x} \cdot \boldsymbol{y} + g^B(\boldsymbol{x}), \end{array}\right\} \quad (23)$$

where $g^B(\cdot)$ is corresponding to the diagonal elements of G. The sequence $\hat{b}$ of (14) can be expressed by

$$\hat{b}_{\hat{x}=x_1N+x_0} = \omega_q^{h^B(x_1,x_0)}. \quad (24)$$

As described above, if a scalar x and a vector $\boldsymbol{x}$ clearly correspond to each other, the vector will also be expressed as the scalar, that is, $h^B(y,x) = h^B(\boldsymbol{y},\boldsymbol{x})$.

The Hadamard matrices of (15), (17) and (21) are respectively expressed as

$$\left.\begin{array}{l} C = [c_{yx} = \omega_q^{h^C(y,x)}], \\ h^C(y,x) = \boldsymbol{x} \cdot \boldsymbol{y} + g^C(\boldsymbol{x}), \end{array}\right\} \quad (25)$$

$$\left.\begin{array}{l} E = [e_{yx} = \omega_q^{h^E(y,x)}], \\ h^E(y,x) = \boldsymbol{x} \cdot \boldsymbol{y} + g^E(\boldsymbol{x}), \end{array}\right\} \quad (26)$$

and

$$\left.\begin{array}{l} P = [p_{yx} = \omega_q^{h^P(y,x)}], \\ h^P(y,x) = \boldsymbol{x} \cdot \boldsymbol{y} + g^P(\boldsymbol{x}). \end{array}\right\} \quad (27)$$

The set D of (16) can be written as

$$\left.\begin{array}{rcl} D &=& [d_{y\hat{x}} = \omega_q^{h^D(y,\hat{x})}], \\ h^D(y,\hat{x}) &=& h^C(y,x_1) + h^B(x_1,x_0), \end{array}\right\} \quad (28)$$

where $x' = x_1N + x_0$.

The set F of (18) is given as

$$\left.\begin{array}{rcl} F_y^z &=& [f_{y\hat{x}}^z = \omega_q^{h^F(z,y,\hat{x})}], \\ h^F(z,y,\hat{x}) &=& h^C(y,x_1) + h^B(x_1,x_0) + h^E(z,x_0), \end{array}\right\} \quad (29)$$

where z indicates each subset.

Similarly the set G of (19) is written as

$$\left.\begin{array}{rcl} G^y &=& [g_{x'}^y = \omega_q^{h^G(y,x')}], \\ h^G(y,x') &=& h^E(y,x_1) + h^D(x_0, x_2N + x_1) \\ &=& h^E(y,x_1) + h^C(x_0, x_2N + x_1) \\ && + h^B(x_2,x_1), \end{array}\right\} \quad (30)$$

where $x' = x_2N^2 + x_1N + x_0$.

Therefore the complementary set $CS(N^3, N, N)$ of (22) can be expressed as

$$\left.\begin{array}{rcl} a_{yx'}^z &=& \omega_N^{h^A(z,y,x')}, \\ h^A(z,y,x') &=& h^P(z,x_0) + h^G(y,x') \\ &=& h^P(z,x_0) + h^E(y,x_1) \\ && + h^C(x_0, x_2N + x_1) + h^B(x_2,x_1). \end{array}\right\} \quad (31)$$

By changing D to A and repeating the equation (29), the complementary set $CS(N^l, N, N)$ is given.

B. Derivation of logic functions

Here is a simple example represented by $0 \le x_0 < N$ as the above matrix expression in the form of a logical function. The logic functions of (23) - (30) can be respectively expressed as

$$h^B(y_0,x_0) = yx_0 + g^B(x_0), \quad (32)$$

$$h^{\hat{B}}(\hat{x}) = h^B(x_1x_0), \quad (33)$$

$$h^C(y_0,x_0) = yx_0 + g^C(x_0), \quad (34)$$

$$\begin{array}{rcl} h^D(y_0,\hat{x}) &=& h^{\hat{B}}(\hat{x}) + h^C(y,x_1) \\ &=& h^B(x_1,x_0) + h^C(y,x_1) \\ &=& x_1x_0 + g^B(x_0) + yx_1 + g^C(x_1), \end{array} \quad (35)$$

$$h^E(y,x_0) = yx_0 + h^E(x_0), \quad (36)$$

$$\begin{array}{rl} h^F(z,y,\hat{x}) &= h^D(y,x_1) + h^E(z,x_0) \\ &= x_1x_0 + yx_1 + zx_0 \\ &\quad + g^B(x_0) + g^C(x_1) + g^E(x_0), \end{array} \quad (37)$$

and

$$\begin{array}{rcl} h^G(y,x') &=& h^F(y,x_0,\hat{x}), \\ &=& x_2x_1 + x_0x_2 + zx_1 \\ && + g^B(x_1) + g^C(x_2) + g^E(x_1), \end{array} \quad (38)$$

where $\hat{x} = x_1N + x_0$ and $x' = \hat{x}N + x_0$.

Therefore the logic function of $CS(N^2, N, N)$ of (31) can be written as

$$\begin{array}{rl} h^A(z,y,\hat{x}) &= h^G(y,x') + h^P(z,x_0) \\ &= x_2x_1 + x_2x_0 + zx_1 + yx_1 \\ &\quad + g^B(x_1) + g^C(x_2) + g^E(x_1) + g^P(x_0). \end{array} \quad (39)$$

By repeating from (36) the logic function of $CS(N^3, N, N)$ is given as

$$\begin{array}{rl} h^A(z,y,x') &= h^G(z,x_0,x') + h^{E_2}(y,x_0) \\ &= x_3x_2 + x_3x_1 + x_1x_0 + zx_2 + yx_0 \\ &\quad + g^C(x_3) + g^E(x_2) + g^P(x_1) + g^B(x_0). \end{array} \quad (40)$$

Similarly, $CS(N^4, N, N)$ is given by

$$\begin{array}{rl} h^A(z,y,x'') &= h^A(z,x_0,x') + h_3^P(y,x_0) \\ &= x_4x_3 + x_4x_2 + x_2x_1 + zx_3 + x_0x_1 \\ &\quad + y_0x_0 + g^3(x_3) + g^2(x_2) + g^1(x_1) + g^0(x_0). \end{array} \quad (41)$$

Repeating this in order gives the following theorem.

[Theorem 1] A complementary sequence set of $CS(N^n, N, N)$ is generated by

$$\begin{array}{rl} h^n(z,y,x) &= x_{n-2}x_{n-1} + x_{n-1}x_{n-3} + \\ &\quad + x_{n-3}x_{n-4} + x_{n-4}x_{n-5} + \cdots + x_1x_0 \\ &\quad + yx_{n-2} + zx_0 + g^{n-1}(x_{n-1}) + g^{n-2}(x_{n-2}) \\ &\quad + \cdots + g^1(x_1) + g^0(x_0), \end{array} \quad (42)$$

where

$$\left.\begin{array}{l} x = x_{n-1}N^{n-1} + x_{n-2}N^{n-2} + \cdots + x_1 N + x_0, \\ \boldsymbol{x} = (x_0, x_1, \cdots, x_{n-1}). \end{array}\right\} \quad (43)$$

If $q = p^m$, the logic function is given by replacing $\boldsymbol{x}_k = (x_0^k, x_1^k, \cdots, x_i^k, \cdots, x_m^k)$ as a vector and the product as an inner product, where each of x_i^k's is a different element of $\boldsymbol{x}$.

Note that Theorem 1 can be rigorously proved using induction.

C. Examamples

Logic functions of complementary sequence sets in Theorem 1 are illustrated with examples.

Example 1 Consider a bi-phase complementary set $CS(2^4, 2, 2)$ generated by a logic function given as

$$h^4(z, y, x) = x_2 x_3 + x_3 x_1 + x_1 x_0 + y x_2 + z x_0 \quad (44)$$

with $g^1(x_1) = 0$ and $g^0(x_0) = 0$. The truth table is given as

TABLE I
TRUTH TABLE OF EQ.(44).

	$\boldsymbol{x}$				(z, y)			
x	x_3	x_2	x_1	x_0	$(0, 0)$	$(0, 1)$	$(1, 0)$	$(1, 1)$
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0
2	0	0	1	0	0	0	0	0
3	0	0	1	1	0	0	0	0
4	0	1	0	0	0	1	0	1
5	0	1	0	1	0	1	0	1
6	0	1	1	0	0	1	0	1
7	0	1	1	1	0	1	0	1
8	1	0	0	0	0	0	1	1
9	1	0	0	1	0	0	1	1
10	1	0	1	0	1	1	0	0
11	1	0	1	1	1	1	0	0
12	1	1	0	0	0	1	1	0
13	1	1	0	1	0	1	1	0
14	1	1	1	0	1	0	0	1
15	1	1	1	1	1	0	0	1

Therefore the binary complementary set is written as

$$\begin{array}{rcl} A & = & \{A^0, A^1\} \\ A^0 & = & \left\{\begin{array}{l} a_0^0 = (0001000100101101), \\ a_1^0 = (0001111000100010), \end{array}\right. \\ A^1 & = & \left\{\begin{array}{l} a_0^1 = (0100010001111000), \\ a_1^1 = (0100101101110111), \end{array}\right. \end{array} \quad (45)$$

Note that this set denotes well-discussed binary complementary pairs [1].In fact, all elements of the vector $\boldsymbol{x}$ can be interchangeable. For examaple

$$h^4(z, y, x) = x_1 x_2 + x_2 x_3 + x_3 x_0 + y x_1 + z x_0, \quad (46)$$

Note that if $g^1(x_1) = x_1/2$ or $g^0(x_0) = x_0/2$, it indecates a quadriphase complementary set.

Example 2 Consider a quadriphase complementary sets $CS(4^2, 4, 4)$ generated by

$$h^2(y, z, x) = x_1 x_0 + y x_1 + z x_0. \quad (47)$$

The truth table is written as where 0, 1, 2, and 3 1, denotes 1, j, -1 and $-j$, respectively. Therefore the quadriphase complementary set is given as

$$\begin{array}{rcl} A & = & \{A^0, A^1, A^2, A^3\} \\ A^0 & = & \left\{\begin{array}{l} a_0^0 = (0000012302020321) \\ a_1^0 = (0000123020203210) \\ a_2^0 = (0000230102022103) \\ a_3^0 = (0000301220201032) \end{array}\right. \\ A^1 & = & \left\{\begin{array}{l} a_0^1 = (0123020203210000) \\ a_1^1 = (0123131321033333) \\ a_2^1 = (0123202003212222) \\ a_3^1 = (0123313121031111) \end{array}\right. \\ A^2 & = & \left\{\begin{array}{l} a_0^2 = (0202032100000123) \\ a_1^2 = (0202103222223012) \\ a_2^2 = (0202210300002301) \\ a_3^2 = (0202321022221230) \end{array}\right. \\ A^3 & = & \left\{\begin{array}{l} a_0^3 = (0321000001230202) \\ a_1^3 = (0321111123013131) \\ a_2^3 = (0321222201232020) \\ a_3^3 = (0321333323011313) \end{array}\right. \end{array} \quad (48)$$

Example 3 Consider a bi-phase complementary set $CS(2^4, 4, 4)$. Let $x_0 = (x'_0, x'_1)$, $x_1 = (x'_2, x'_3)$, $z = (z'_0, z'_1)$ and $y = (y'_0, y'_1)$. Substituting them into (47) gives the logic function

$$\begin{array}{rcl} h^4(z, y, x) & = & {x_0}'{x_2}' + {x_1}'{x_3}' \\ & & + {y_1}'{x_3}' + {y_0}'{x_2}' + {z_1}'{x_1}' + {z_0}'{x_0}'. \end{array} \quad (49)$$

TABLE II
TRUTH TABLE OF EQ.(47).

	$\boldsymbol{x}$			$A^0\ (z = 0, y)$				$A^1\ (z = 1, y)$			
x	x_1	x_0	$x_0 x_1$	0	1	2	3	0	1	2	3
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	1	1	1	1
2	0	2	0	0	0	0	0	2	2	2	2
3	0	3	0	0	0	0	0	3	3	3	3
4	1	0	0	0	1	2	3	0	1	2	3
5	1	1	1	1	2	3	0	2	3	0	1
6	1	2	2	2	3	0	1	0	1	2	3
7	1	3	3	3	0	1	2	2	3	0	1
8	2	0	0	0	2	0	2	0	2	0	2
9	2	1	2	2	0	2	0	3	1	3	1
10	2	2	0	0	2	0	2	2	0	2	0
11	2	3	2	2	0	2	0	1	3	1	3
12	3	0	0	0	3	2	1	0	3	2	1
13	3	1	3	3	2	1	0	0	3	2	1
14	3	2	2	2	1	0	3	0	3	2	1
15	3	3	1	1	0	3	2	0	3	2	1

The bi-phase complementary set A is written as

$$\begin{aligned}
A &= \{A^0, A^1, A^2, A^3\} \\
A^0 &= \begin{cases} a_0^0 = (0000010100110110) \\ a_1^0 = (0000101011001001) \\ a_2^0 = (0000010111001001) \\ a_3^0 = (0000101011000110) \end{cases} \\
A^1 &= \begin{cases} a_0^1 = (0101000001100011) \\ a_1^1 = (0101111101101100) \\ a_2^1 = (0101000010011100) \\ a_3^1 = (0101111110010011) \end{cases} \\
A^2 &= \begin{cases} a_0^2 = (0011011000000101) \\ a_1^2 = (0011100100001010) \\ a_2^2 = (0011011011111010) \\ a_3^2 = (0011100111110101) \end{cases} \\
A^3 &= \begin{cases} a_0^3 = (0110001101010000) \\ a_1^3 = (0110110001011111) \\ a_2^3 = (0110001110101111) \\ a_3^3 = (0110110010100000) \end{cases}
\end{aligned} \tag{50}$$

where a superscript and a subscript of a_y^z means $z = z_0 + z_1 2$ and $y = y_0 + y_1 2$, respectively.

As in Example 1, all elements of the vector $\boldsymbol{x'}$ seem to be interchangeable.

V. Conclusions

The logic functions of complementary sets of polyphase sequences proposed by Suehiro have been formulated. Actually, it has been confirmed that there are cases where new complementary sets can be generated even if the variables of the given logic function are replaced. This means that Suehiro's construction method can be further generalized.

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