# Methods and Results of Calculating Voltage Characteristics at Equipment Inputs for Non-Contact Injection of Pulse Interference into a Power Cable 

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#### Abstract

The effective methods of calculating pulse interference characteristics in power cables are described in the paper. Calculations of voltage pulses for non-contact injection of electrical pulse interference into power lines are executed. Key words: pulse interference, telegraph equation, non-contact injection, power line.


## I. Introduction

International Electrotechnical Commission (IEC) standard 61000-4-5 describes the characteristics of electrical pulse interference into communication and power lines. This standard presents experimental data indicating that microsecond pulse interference can deliver substantial energy to electronic equipment located within buildings and thus put it out of service.

The contributions in this paper complement the IEC standard. This paper examines the results of a theoretical and experimental study of the generation of induced nanosecond pulses by non-contact means occurring in exposed power lines. These pulses will be a serious threat to connected electronic equipment.
II. Methods of calculating voltage characteristics AT EQUIPMENT INPUTS FOR NON-CONTACT INJECTION OF PULSE INTERFERENCE INTO A POWER CABLE

## A. General Considerations



Fig. 1 Calculation diagram
The methods described in this paper are intended for the numerical assessment of voltages inductively injected into a
power line. Injection is carried out by means of an affecting circuit that is parallel to the power cable.

The calculation diagram is shown in Fig. 1. The same figure also shows the adopted current directions in the circuits. Radii of all wires in both the affecting and power circuits have been selected equal and are identified as $r_{0}$. Where $U_{g}$ generator voltage; $Z_{g}=i \omega L_{g}+\frac{1}{i \omega C_{g}}$ - impedance of the generator; $L_{g}, C_{g}$ - inductance of the connecting wires and generator capacitance; $Z_{n}$ - generator load impedance; $L$ length of the affecting circuit; $H$ - width of the affecting circuit; $d$ - distance from the affecting circuit to the phase wire; $l$ - length of the power circuit wire; $a$ - distance between the phase and the neutral wires in the power circuit; $h$ - distance from the neutral wire to the ground; $Z_{1}, Z_{2}, Z_{3}, Z_{4}, Z_{5}, Z_{6}$ - possible loads in the power circuit.

The methods described here make it possible to determine the voltage induced between the phase wire and the ground, the voltage induced between the neutral wire and the ground, the voltage across load $Z_{2}$, which imitates input resistance of equipment connected to the power cable, as well as currents in different sections of the circuit.

## B. Calculated Ratios Used to Determine Voltages and Currents in the Affecting Circuit

To determine the values of currents and voltages in the affecting circuit, let us record the system of telegraph equations in a frequency-domain representation (inverse Fourier transform is used to obtain time functions) [1, 2]:
$\left.\begin{array}{l}\left.\begin{array}{l}\frac{\partial U(x)}{\partial x}=-Z I_{0}(x) \\ U_{0}(x)=-\frac{1}{Y} \frac{\partial I_{0}(x)}{\partial x}\end{array}\right\}\end{array}\right\}$

Where $U_{0}(x), I_{0}(x)$ - voltage and current; $Z, Y$ resistance and conductivity per unit length in the affecting circuit.

The solution of this system is well known and can be recorded as:
$I_{0}(x)=K_{1} e^{-\gamma x}+K_{2} e^{\gamma x}$
$\left.U_{0}(x)=Z_{0}\left(K_{1} e^{-\gamma x}-K_{2} e^{\gamma x}\right)\right\}$
Where $Z_{0}=\sqrt{\frac{Z}{Y}}$ - wave resistance of the line; $\gamma=\sqrt{Z Y}$ phase constant of the line; $K_{1}, K_{2}$ - coefficients derived from boundary conditions.

In the case of zero-loss lines, for the adopted designations of $Z_{0}=120 \ln \left(\frac{H}{r_{0}}\right), \tilde{a}=\frac{i \grave{u}}{c}, c$ - speed of light.

Boundary conditions for the adopted designations of Fig. 1 can be formulated as follows:

$$
\begin{equation*}
U_{0}(0)=U_{g}=Z_{g} I_{0}(0), U_{0}(L)=Z_{n} I_{0}(L) . \tag{3}
\end{equation*}
$$

By omitting arithmetical calculations, we obtain:

$$
\left.\begin{array}{l}
K_{l}=\frac{-U_{g} e^{\gamma L}\left(Z_{0}+Z_{n}\right)}{e^{-\gamma L}\left(Z_{0}-Z_{g}\right)\left(Z_{0}-Z_{n}\right)-e^{\gamma L}\left(Z_{0}+Z_{g}\right)\left(Z_{0}+Z_{n}\right)} \\
K_{2}=\frac{-U_{g} e^{-\gamma L}\left(Z_{0}-Z_{n}\right)}{e^{-\gamma L}\left(Z_{0}-Z_{g}\right)\left(Z_{0}-Z_{n}\right)-e^{\gamma L}\left(Z_{0}+Z_{g}\right)\left(Z_{0}+Z_{n}\right)} \tag{4}
\end{array}\right\}
$$

For the time characteristic of $U_{g}$ we will use a step waveform with a voltage rise time of 1-10 ns.

## C. Calculated Ratios Used to Determine Voltages and Currents in the Power Circuit

To determine the parameters of currents and voltages in the power line, let us record the system of telegraph equations as follows [2]:
$\frac{\partial U_{l}(x)}{\partial(x)}=-Z_{11} I_{l}(x)-Z_{12} I_{2}(x)+{\stackrel{\circ}{a_{l}}(x)}$
$U_{1}(x)=-\frac{1}{Y_{11}} \frac{\partial I_{l}(x)}{\partial x}-\frac{1}{Y_{12}} \frac{\partial I_{2}(x)}{\partial x}$
$\left.\frac{\partial U_{2}(x)}{\partial x}=-Z_{22} I_{2}(x)-Z_{12} I_{1}(x)+\stackrel{\circ}{a}_{2}(x)\right\}$
$U_{2}(x)=-\frac{1}{Y_{21}} \frac{\partial I_{l}(x)}{\partial x}-\frac{1}{Y_{22}} \frac{\partial I_{2}(x)}{\partial x}+$
$+\left(\frac{1}{Y_{23}}-\frac{1}{Y_{24}}\right) \frac{\partial I_{0}(x)}{\partial x}$
Where $U_{1}(x), I_{1}(x), U_{2}(x), I_{2}(x)$ - voltages and currents in phase and neutral wires; $Z_{11}, Z_{12}, Z_{22}, Y_{11}, Y_{12}, Y_{22}$ - own and mutual per unit length resistances and conductivities of the phase wire and the neutral wire; $\grave{a}_{l}(x), \circ_{2}(x)$ - distributed electromotive forces created by the affecting circuit in wires of the power circuit.

In the case of a zero-loss line, per unit length parameters for the designations adopted in Fig. 1 can be determined using the following expressions $[2,3]$ :
$Z_{1 I}=\frac{i \grave{u} \grave{l}_{0}}{2 \partial} \ln \left(\frac{2(h+a)}{r_{0}}\right), Z_{22}=\frac{i \grave{u} \grave{l}_{0}}{2 \partial} \ln \left(\frac{2 h}{r_{0}}\right)$
$Z_{12}=\frac{i \grave{u} \grave{l}_{0}}{2 \partial} \ln \left(\frac{2 h+a}{a}\right), Y_{i j}=\frac{\tilde{a}^{2}}{Z_{i j}}, \tilde{a}=\frac{i \grave{u}}{c}$
For distributed electromotive forces created by the affecting circuit, it is possible to record:
$\grave{a}_{1}(x)=i \grave{u} M_{1} I_{0}(x), \stackrel{\circ}{a}_{2}(x)=i \grave{u} M_{2} I_{0}(x)$,
Where $\quad M_{1}=\frac{\grave{i}_{0}}{2 \partial} \ln \left(1+\frac{H}{d}\right), M_{2}=\frac{\grave{i}_{0}}{2 \partial} \ln \left(1+\frac{H}{d+a}\right)$.
It should be noted that system (5) ignores the inverse effect of currents in the power circuit on currents formed in the affecting circuit.

In the adopted assumptions, system (4) can be solved analytically. After simple arithmetic operations, system (5) is reduced to separate differential equations for currents:
$\left.\begin{array}{l}\frac{\partial^{2} I_{l}(x)}{\partial x^{2}}-\tilde{a}^{2} I_{l}(x)=\frac{\left(Y_{12} \circ_{l}(x)-Y_{22} \circ_{2}(x)\right) Y_{11} Y_{12}}{Y_{22} Y_{l 1}-Y_{l 2}^{2}}=-A_{l}(x) \\ \frac{\partial^{2} I_{2}(x)}{\partial x^{2}}-\tilde{a}^{2} I_{2}(x)=\frac{\left(Y_{12} \circ_{2}(x)-Y_{l 1} \circ_{l}(x)\right) Y_{22} Y_{12}}{Y_{22} Y_{l 1}-Y_{12}^{2}}=-A_{2}(x)\end{array}\right\}$
The solution of equations can be recorded as follows:
$I_{l}(x)=C_{1} e^{-\tilde{\alpha} \tilde{x}}+C_{2} e^{\tilde{\alpha} \tilde{x}}+\frac{e^{-\tilde{\alpha} x} x}{2 \tilde{a}} \int_{0}^{x} A_{l}\left(x^{\prime}\right) e^{\tilde{\tilde{x}^{\prime}}} d x^{\prime}+$
$+\frac{e^{\tilde{a} x}}{2 \tilde{a}} \int_{x}^{L} A_{l}\left(x^{\prime}\right) e^{-\tilde{a^{\prime}}} d x^{\prime}$

$+\frac{e^{\tilde{\alpha}}}{2 \tilde{a}} \int_{x}^{L} A_{2}\left(x^{\prime}\right) e^{-\tilde{\tilde{x}^{\prime}}} d x^{\prime}$
It should be noted that during integration, the top limit of integration may not exceed the value of $L$, i.e., the length of the affecting circuit.

Using (9) and (4), we can obtain a solution for voltages:
$U_{1}(x)=C_{1} \frac{\tilde{a}}{Y_{11}} e^{-\tilde{\alpha} x}-C_{2} \frac{\tilde{a}}{Y_{11}} e^{\tilde{\alpha} x}+C_{3} \frac{\tilde{a}}{Y_{12}} e^{-\tilde{\alpha} x}-$
$-C_{4} \frac{\tilde{a}}{Y_{l 2}} e^{\tilde{\alpha} \tilde{x}}+\frac{e^{-\tilde{\alpha} x}}{2} \int_{0}^{x} \grave{a}_{l}\left(x^{\prime}\right) e^{\tilde{\alpha^{\prime}}} d x^{\prime}-\frac{e^{\tilde{\alpha} x}}{2} \int_{x}^{L} \grave{a}_{l}\left(x^{\prime}\right) e^{\tilde{\alpha^{\prime}}} d x^{\prime}$
$U_{2}(x)=C_{1} \frac{\tilde{a}}{Y_{12}} e^{-\tilde{a} x}-C_{2} \frac{\tilde{a}}{Y_{12}} e^{\tilde{a} x}+C_{3} \frac{\tilde{a}}{Y_{22}} e^{-\tilde{a} x}-$
$-C_{4} \frac{\tilde{a}}{Y_{22}} e^{\tilde{\alpha} \tilde{x}}++\frac{e^{-\tilde{\alpha} x} x}{2} \int_{0}^{\overbrace{2}}\left(x^{\prime}\right) e^{\tilde{\alpha^{\prime}}} d x^{\prime}-\frac{e^{\tilde{\alpha} x} L}{2} \int_{x}^{\AA_{2}}\left(x^{\prime}\right) e^{\tilde{\tilde{x}^{\prime}}} d x^{\prime}]$
The coefficients $C_{1}, C_{2}, C_{3}, C_{4}$ - are derived from the boundary conditions that can be formulated as follows using the designations of Fig. 1:

## on the left

$I_{4}-I_{7}-I_{2}(0)=0, I_{5}+I_{7}-I_{1}(0)=0$
$\left.U_{2}(0)=-Z_{4} I_{4}, U_{1}(0)=-Z_{5} I_{5}, U_{1}(0)=U_{2}(0)-Z_{1} I_{7}\right\}$
on the right
$I_{1}(l)-I_{6}-I_{8}=0, I_{2}(l)+I_{8}-I_{3}=0$
$\left.U_{1}(l)=Z_{2} I_{3}, U_{1}(l)=Z_{6} I_{6}, U_{2}(l)=U_{1}(l)-Z_{2} I_{8}\right\}$
Leaving the variables $U_{1}, U_{2}, I_{1}, I_{2}$, in the boundary conditions, we ultimately obtain the following expressions:
on the left
$\left.-\left(Z_{1}+Z_{4}\right) U_{2}(0)+Z_{4} U_{1}(0)-Z_{1} Z_{4} I_{2}(0)=0\right\}$
$\left.-\left(Z_{1}+Z_{5}\right) U_{1}(0)+Z_{5} U_{2}(0)-Z_{1} Z_{5} I_{1}(0)=0\right\}$
on the right
$\left.-\left(Z_{2}+Z_{6}\right) U_{1}(l)+Z_{6} U_{2}(l)-Z_{2} Z_{6} I_{1}(l)=0,\right\}$
$\left.-\left(Z_{2}+Z_{3}\right) U_{2}(l)+Z_{3} U_{1}^{2}(l)-Z_{2}^{2} Z_{3} I_{2}(l)=0.\right\}$

Using boundary conditions (13), (14) and solutions (9), (10), we obtain a system of linear algebraic equations to determine the unknown constants. This system is solved using standard analytical and numerical methods.

The methods presented above have been implemented in the form of a computer application written using FORTRAN. For basic data it uses specific values of parameters presented in the diagram in Fig. 1, as well as a number of calculated harmonics, a time step, and calculation point coordinate $x$. The computer calculation time was approximately 10 s .

## D. Verification of Methods

To verify the accuracy of the analytical methods, we compared the results obtained using such methods with the results of calculation using the aforementioned methods based on the numerical solution of Pocklington's integral equation $[4,5]$. As a test situation we examined a situation where the affecting circuit and the power circuit are located in unconfined space (corresponds to the situation of $h=\infty$ ). For the time characteristic $U_{g}$ we used a step function with voltage buildup time of 10 ns and an amplitude of 10 kV . Generator capacity was $C_{g}=40 \mathrm{pF}$, the inductance of connecting wires $-L_{g}=5 \mu \mathrm{H}$, and the generator load $Z_{n}=552 \mathrm{ohm}$. The influencing circuit had a length of $L=5 \mathrm{~m}$, width of $H=0.1 \mathrm{~m}$, and a conductor radius of $r_{0}=0.001 \mathrm{~m}$.

The following values were adopted as parameters of the power circuit: cable length $l=20 \mathrm{~m}$, distance between phase and neutral wires $a=0.005 \mathrm{~m}$, distance from the neutral wire to the ground $h=1 \mathrm{~m}$, distance from the affecting circuit to the phase wire $d=0.02 \mathrm{~m}$, radius of conductors $r_{0}=0.001 \mathrm{~m}$, resistance values $Z_{1}=50 \mathrm{ohm}, Z_{2}=50 \mathrm{ohm}$, and $Z_{3}=Z_{4}=Z_{5}=Z_{6}=\infty$.

The results of the calculation of voltage between the phase and neutral wires for points with coordinates 0 and 20 m are presented in Fig. 2. As the presented graphs indicate, the results of the calculations performed using different methods correlate very well.


Fig. 2 Voltage between the phase and neutral wires for points with coordinates $x=0$ (on the left) and $x=20 \mathrm{~m}$ (on the right) integral equations - ; telegraph equations .........

## III. RESULTS OF THE CALCULATION OF VOLTAGE

 CHARACTERISTICS AT THE EQUIPMENT INPUTS FOR NONCONTACT INJECTION OF PULSE INTERFERENCE INTO A POWER CABLEThe developed methods were used to calculate current and voltage waveforms at different points of the calculation model (Fig. 1). Calculations were performed for the following source data: $U_{g}=10^{5} \mathrm{~V}, t_{f g}=1 \mathrm{~ns}, C_{g}=40 \cdot 10^{-12} \mathrm{~F}, L=5 \mathrm{~m}$, $Z_{3}=Z_{5}=Z_{6}=\infty, H=0.5 \mathrm{~m}, \quad d=0.01 \mathrm{~m}, \quad a=0.005 \mathrm{~m}$, $h=1 \mathrm{~m}$. Values of the parameters $Z_{n}, Z_{1}, Z_{2}, Z_{3}, Z_{4}$ varied within the range of their possible variation, with $Z_{1}=Z_{2}$. The result of current calculation in the affecting circuit is presented in Fig. 3a. Subsequent Fig. 3b shows the result of voltage calculation on the power cable load at $Z_{4}=0$. Table 1 presents a summary of the results shown in fig. 1 at $Z_{4}=0$ and shows the results of the same calculations at $Z_{4}=\infty$. It should be noted that the first of the named values of the parameter $Z_{4}$ is closer to a typical value.

Thus, the completed calculations have shown that for the non-contact injection of nanosecond pulse interference into a power cable, the levels of voltage at equipment inputs may reach nine kilovolts.


Fig. 3 Current in the affecting circuit (a) and voltage on load $Z_{2}$ (b)

TABLE I
Summary of Calculation Results

| $l, \mathrm{~m}$ | $\begin{gathered} Z_{1}=Z_{2}, \\ \text { ohm } \end{gathered}$ | $\begin{gathered} Z_{n}, \\ \text { ohm } \end{gathered}$ | $\begin{gathered} L_{g}, \\ \mu \mathbf{H} \end{gathered}$ | Current in the affecting circuit, A | Voltage on load $Z_{2}$, V |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | $Z_{4}=0$ | $Z_{4}=\infty$ |
| 100 | 1000 | 50 | 5 | 100 | 8500 | 250 |
|  |  |  | 1 | 120 | 9000 | 300 |
|  |  | 600 | 5 | 100 | 6000 | 600 |
|  | 600 | 600 | 1 | 120 | 6000 | 700 |
|  |  |  | 5 | 100 | 4700 | 360 |
|  | 50 | 600 | 5 | 100 | 320 | 180 |
| 20 | 1000 | 50 | 1 | 120 | 9000 | 300 |
|  | 192 | 745 | 5 | 100 | - | 11 |
|  |  |  | 1 | 120 | - | 13 |
|  | 50 | 50 | 5 | 150 | 800 | 450 |
|  |  |  | 1 | 120 | 900 | 520 |
|  |  | 600 | 5 | 100 | 320 | 180 |
|  |  | 100 | 5 | 100 | 240 | 120 |
| 5 | 50 | 600 | 5 | 100 | 420 | 180 |

IV. Conclusions

Calculations have shown that the magnitude of pulse voltages at equipment inputs for non-contact injection of nanosecond pulse interference into a power cable using a voltage generator of 100 kV may reach 9 kV , which may significantly (by almost three orders of magnitude) exceed the immunity level of connected equipment. Therefore, it is necessary to use protective devices that can reduce the peak level of this type of voltage pulse by about 1,000 times.

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