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A Multi-solver Domain Decomposition Method for Rigorous Conductor Modeling of Signal Integrity in Integrated Circuits

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***Abstract*—A frequency-domain multi-solver domain decomposition method is presented to accurately analyze signal integrity problems in multi-scale integrated circuits. In particular, we propose a novel formulation to rigorously account for the conductor loss due to finite conductivities in metals. The proposed nonconformal domain decomposition method follows a hierarchical domain partitioning strategy of the original problem. The most suitable computational electromagnetic technique is employed for each of the subregions. To improve the convergence in the DDM iterations, an optimized Robin type transmission condition is introduced to enforce the field continuities across sub-domain interfaces. Moreover, a hierarchical multi-level fast multiple method is adopted to address the low frequency issues particularly in IC applications. Rigorous numerical experiments validate the potential efficiency and benefits offered by the proposed method.**

I. INTRODUCTION

With the increase of clock frequency and edge rates as well as the continuously downscaling of feature size and 3-D interconnect technologies in high-speed systems, signal integrity (SI) effects such as signal delay, reflection, attenuation, dispersion and crosstalk have become one of the dominant factors in current deep submicrometer CMOS technologies limiting overall performance of high-speed systems.

The goal of this work is to perform full wave analysis to study signal integrity problems of multi-scale integrated circuits (ICs), which consists of complex geometries and various materials properties. To elucidate the issues involved, we consider a product-level IC package model shown in Fig. 1. To accurate simulate the electrically small yet geometrically complicated ICs is of vital importance for engineers.

The first challenge comes from the complexity of the geometry. The package model contains more than 40,000 entities, such as 3-D interconnects including through silicon vias, buried vias, multiple trace lines, and metallic power/ground layers. It will be extremely daunting to generate a good quality finite element mesh for the entire package with all the small features, let alone solving it efficiently. Secondly, we should also mention that due to the multi-scale nature of the model, the resulting mesh will have wide range of element sizes. As a consequence, the application of the finite element methods to such a discretization with wide range of element sizes usually leads to an extremely ill conditioned system matrix equation. Furthermore, it is well know that at high frequencies, the electric current in a finite conductor decreases exponentially and is restricted to the conductor surface known as skin depth. The complete numeral analysis of finite thickness conductor with the skin depth effect in electromagnetic analysis will require excessive computer resources, which is the major motivation of this work.

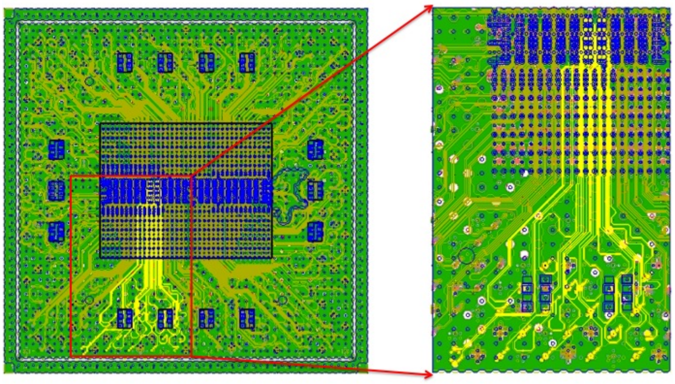

Fig. 1. Top view of the simulated IBM package geometry and 20 signal lines.

II. MULTI-SOLVER DOMAIN DECOMPOSITION METHOD

The fundamental strategy of the multi-solver domain decomposition method (MS-DDM) [1] is to decompose the entire computational domain into many subregions based on the local material properties and geometrical features. Subsequently, we employ the most suitable computational electromagnetic (CEM) technique for each of the subregions. The proposed multi-solver domain decomposition scheme follows a hierarchical domain partitioning strategy. Take a stripling model as an example, the two-level domain partition is illustrated in Fig. 2.

For the IC application, it is natural to decompose the model into dielectric subregion and conductor region. Also, we treat the unbounded space exterior to the IC model as an additional domain instead of using absorbing boundary condition (ABC) to avoid the unwanted spurious reflection from the truncation boundary. So at Level 1 the entire computational domain is decoupled to three regions, interior dielectric region, interior conductor region and the exterior region.

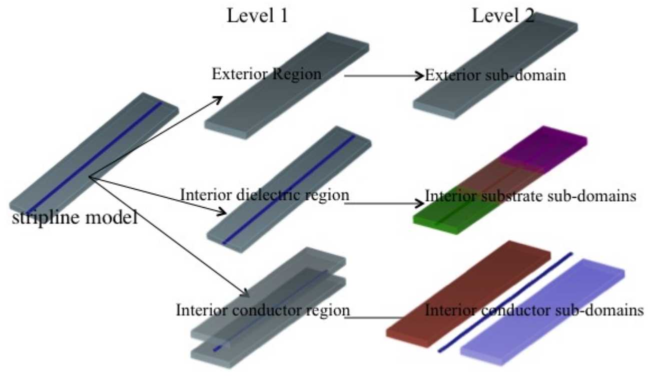

Fig. 2. Two-Level hierarchical domain partition strategy of a stripling model.

At domain partition Level 2, for the dielectric subregion, a finite element based non-overlapping domain decomposition method (FE-DDM) [2] is employed to efficiently model the geometrically complex multi-scale structures and account for the non-uniform material properties. Thus the dielectric region of the stripling model is further partitioned into three sub-domains. Another advantage of using FE-DDM is that it can utilize the sub-domain repetitions to drastically reduce the computational resources. For the conductor region, the skin and proximity effects are significant at high frequencies and the FE method shall require considerable dense volume mesh for the high-density subregion. When the frequency goes higher, the effect to generate a good volumetric mesh could be a daunting task. Therefore, a surface integral equation domain decomposition method (SIE-DDM) [3] is desirable to solve the conductor sub-domains with homogeneous material properties. For the stripling case, three local conductor sub-domains are enclosed by closed surfaces and solved individually through the generalized combined field integral equation (GCFIE).

The system matrix equation of the proposed MS-DDM can be written explicitly as the following:

$$\begin{bmatrix} \mathcal{A}_{\text{FEM}} & \mathcal{M}_{\Gamma\Gamma}^{01} & \mathcal{M}_{\Gamma\Gamma}^{02} & \mathcal{D}_{\Gamma\Gamma}^{0,ext} \\ \mathcal{M}_{\Gamma\Gamma}^{10} & \mathcal{T}_{\text{GCFIE}}^{1} & \mathcal{C}_{\Gamma\Gamma}^{12} & \mathcal{D}_{\Gamma\Gamma}^{1,ext} \\ \mathcal{M}_{\Gamma\Gamma}^{20} & \mathcal{C}_{\Gamma\Gamma}^{21} & \mathcal{T}_{\text{GCFIE}}^{2} & \mathcal{D}_{\Gamma\Gamma}^{2,ext} \\ \mathcal{D}_{\Gamma\Gamma}^{ext,0} & \mathcal{D}_{\Gamma\Gamma}^{ext,1} & \mathcal{D}_{\Gamma\Gamma}^{ext,2} & \mathcal{F}_{\text{GCFIE}}^{ext} \end{bmatrix} \begin{bmatrix} u_{\text{FEM}} \\ u_{\text{GCFIE}}^{1} \\ u_{\text{GCFIE}}^{2} \\ u_{\text{GCFIE}}^{ext} \end{bmatrix} = \begin{bmatrix} y_{\text{FEM}}^{inc} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

Sub-domain matrix $\mathcal{A}_{\text{FEM}} = \begin{bmatrix} \mathcal{A}_{\text{FEM}}^{1} & \mathcal{P}_{\Gamma\Gamma}^{12} \\ \mathcal{P}_{\Gamma\Gamma}^{21} & \mathcal{A}_{\text{FEM}}^{2} \end{bmatrix}$ denotes the FE-DDM matrix at Level 1. Here $\mathcal{P}_{\Gamma\Gamma}^{12}$ and $\mathcal{P}_{\Gamma\Gamma}^{21}$ is the FEM sub-domain coupling matrices by second order transmission condition [4]. Subdomain matrices $\mathcal{T}_{\text{GCFIE}}^{1}$, $\mathcal{T}_{\text{GCFIE}}^{2}$ and $\mathcal{F}_{\text{GCFIE}}^{ext}$ is the interior conductor sub-domain matrices at Level 2 and exterior enclosed surface sub-domain matrix at Level 1. $\mathcal{C}_{\Gamma\Gamma}^{12}$ and $\mathcal{C}_{\Gamma\Gamma}^{21}$ is the conductor sub-domain coupling matrices. $\mathcal{M}_{\Gamma\Gamma}^{01}$, $\mathcal{M}_{\Gamma\Gamma}^{02}$, $\mathcal{M}_{\Gamma\Gamma}^{10}$ and $\mathcal{M}_{\Gamma\Gamma}^{20}$ indicates the coupling matrices between dielectric and conductor sub-domains through the use of an optimized Robin type transmission condition. Finally, $\mathcal{D}_{\Gamma\Gamma}^{i,ext}$ and $\mathcal{D}_{\Gamma\Gamma}^{ext,i}$ $(i \in \{0,1,2\})$ is the coupling matrices between interior sub-domain and exterior regions. Here the $\mathcal{A}$, $\mathcal{T}$ and $\mathcal{F}$ block matrices only talk to their neighbor sub-domains when they are touching. Therefore we end up have a sparse type matrix equation.

The MS-DDM can efficiently model the signal integrity problem in IC models by combining FEMs versatility and robustness to model geometrically complex structures and spatially varying materials and also the SIEs ability to efficiently and accurately solve unbounded domains. MS-DDM iteratively solves each of the three regions with excitations that include radiations coming from all the other sub-domains. The continuities of tangential electric and magnetic fields on the interfaces between touching sub-domains are implicitly enforced via the optimized Robin type transmission conditions. The coefficient in the TC between dielectric-conductor sub-domain is optimized with respect to the high density material property in the conductor region to facilitate fast convergence in the MS-DDM iterations.

To further accelerate the performance of the GCFIE method, a hierarchical multi-level fast multipole method (H-MLFMM) [5] is adopted to accelerate the matrix vector-multiplication (MVP) and reduce memory consumption for the surface IE methods, especially for the low frequency and multi-scale problems.

III. CONCLUSION

We present herein a MS-DDM to accurately analyze signal integrity in multi-scale integrated circuits (ICs) to rigorously account for the conductor loss due to finite conductivities in metals. The most efficient CEM technique is employed for each of the subregions. We shall present numerical results for stripling model and IC package problems, and demonstrate the accuracy and flexibility of the proposed MS-DDM.

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