Transient Analysis of Electromagnetic Fields of Nano Structures by Integral Solvers with FILT

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***Abstract*— A novel computational technique is proposed for analysis of electromagnetic problems with nano structures. Our method is based on integral solvers in the complex frequency domain and fast inverse Laplace transform. The wavelength and time responses obtained by the proposed method are investigated.**

I. Introduction

For designing plasmonic devices, analysis of electromagnetic problems with nanoscale objects plays an important role [1]–[3]. We propose a novel computational technique for analysis of the nanoscale electromagnetic problems. In our method, solutions in the complex frequency domain can be obtained by using integral solvers, which is based on the PMCHWT (Poggio–Miller–Chang–Harrigton–Wu–Tsai) method [4] or boundary integral equation method (BIEM) using the static approximation [5], [6]. The function in the time domain can be obtained by using fast inverse Laplace transform (FILT) [7]. Compared with the exact solution, the computational accuracy is verified.

II. Formulation

We assume that the scatterer is a metallic nano object as shown in Figure 1. The permittivity and the permeability of the object are ε_2 and μ_2, respectivelly.

In our method, the conventional PMCHWT method is extended in the complex frequency domain. We sum the external and internal EFIE, such as

$$-\mathbf{E}^i(\mathbf{r})\hat{f}(s) = [L_1(\mathbf{r},\mathbf{r}') + L_2(\mathbf{r},\mathbf{r}')]\cdot\mathbf{J}_\Omega(\mathbf{r}') - [K_1(\mathbf{r},\mathbf{r}') + K_2(\mathbf{r},\mathbf{r}')]\cdot\mathbf{M}_\Omega(\mathbf{r}'). \quad (1)$$

Similarly, we sum the external and internal MFIE, such as

$$-\mathbf{H}^i(\mathbf{r})\hat{f}(s) = [K_1(\mathbf{r},\mathbf{r}') + K_2(\mathbf{r},\mathbf{r}')]\cdot\mathbf{J}_\Omega(\mathbf{r}') + [\eta_1^{-2}L_1(\mathbf{r},\mathbf{r}') + \eta_2^{-2}L_2(\mathbf{r},\mathbf{r}')]\cdot\mathbf{M}_\Omega(\mathbf{r}'), \quad (2)$$

where L_m and K_m are integral operators defined by

$$L_m(\mathbf{r},\mathbf{r}')\cdot\mathbf{X}(\mathbf{r}') = -s\mu_m\int_\Omega\left[\mathbf{X}(\mathbf{r}') - \frac{\nabla\nabla}{S_m^2}\mathbf{X}(\mathbf{r}')\right]g_m(\mathbf{r},\mathbf{r}')d\Omega', \quad (3)$$

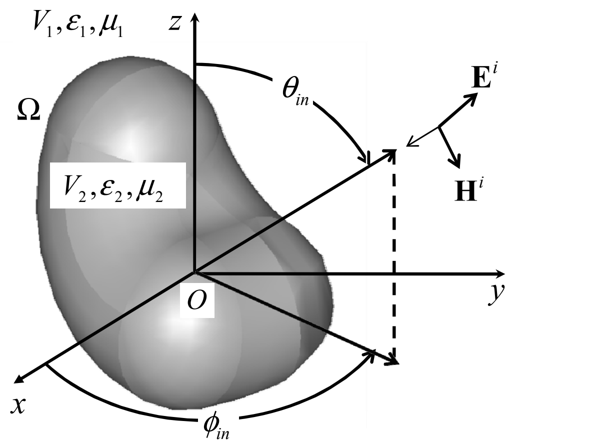

Fig. 1. Geometry and coordinate system.

$$K_m(\mathbf{r},\mathbf{r}')\cdot\mathbf{X}(\mathbf{r}') = \int_\Omega \mathbf{X}(\mathbf{r}')\times\nabla g_m(\mathbf{r},\mathbf{r}')d\Omega', \quad (4)$$

$$g_m(\mathbf{r},\mathbf{r}') = \frac{\exp(-S_m|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|}, \quad (5)$$

s is the complex frequency, $S_m = s\sqrt{\varepsilon_m\mu_m}$, $\hat{f}(s)$ is the spectrum of the incident wave, $\mathbf{J}_\Omega$ is the unknown electric currents distribution, and $\mathbf{M}_\Omega$ is the unknown magnetic currents distribution.

For the BIEM by the static approximation, we assume that the object size is much smaller than the incident wavelength. Then, the boundary integral equation in the complex frequency domain can be expressed as

$$\sigma(\mathbf{r}) - \frac{\hat{\lambda}}{2\pi}\int_\Omega \sigma(\mathbf{r}')\frac{\mathbf{n}\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}d\Omega' = 2\varepsilon_1\hat{\lambda}\mathbf{n}\cdot\mathbf{E}^i\hat{f}(s), \quad (6)$$

where

$$\hat{\lambda} = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1}, \quad (7)$$

σ is the unknown electric charge density and $\mathbf{n}$ is the unit normal vector.

The solution obtained by using integral solvers can be transformed into the time domain by FILT such as

$$f(t) := \frac{1}{2\pi j}\int_{\gamma-j\infty}^{\gamma+j\infty}\hat{F}(s)E_{st}(st,\alpha)ds, \approx \frac{e^{\alpha}}{t}\sum_{m=1}^{K}F_m, \quad (8)$$

where $E_{st}(\bullet)$ is the approximate exponential function, α is the approximate parameter, K is the truncation number, and

$$F_m := (-1)^m \operatorname{Im}\left[\hat{F}\left\{\frac{\alpha + j(m-0.5)\pi}{t}\right\}\right], \quad (9)$$

which is computed by the integral solvers in the complex frequency domain.

III. Computational Results

We investigate the wavelength and time responses of the electric field near a gold nanosphere. The radius of the sphere is 10.0 nm. The permittivity of gold is expressed as the Lorents-Drude model [8]. The incident wave is a sinusoidal plane wave and comes from $\theta_{in} = 0^\circ$ and $\phi_{in} = 0^\circ$. The observation point is at x = 12.0 nm on the x axis. We can confirm that all wavelength responses are in good agreement as shown in Figure 2.

Figure 3 shows the time response of the electric field. Here, the incident wavelength is 525 nm. The computational result corresponds to the exact solution. To validate the number of digit of computational accuracy, we show the convergence process of the relative error for varying the truncation number of FILT K in Eq. (9). The error shown in Figure 4 converges when the truncation number K increases. The number of digit of the accuracy corresponds to the approximate parameter α for each case.

IV. Conclusion

We proposed the novel computational technique for analysis of electromagnetic problems of nano structures. The wavelength and time responses for a gold nanosphere are investigated. Compared with the exact solution, we verify that our proposed method is error controllable.

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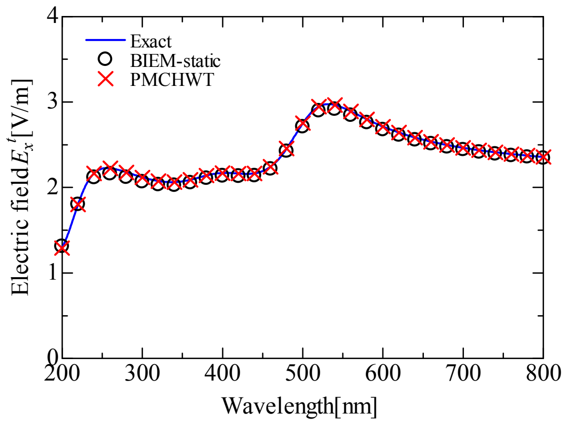

Fig. 2. Wavelength response of the electric field for the gold nanosphere with the radius 10.0nm. The observation point is at x = 12.0 nm on the x axis.

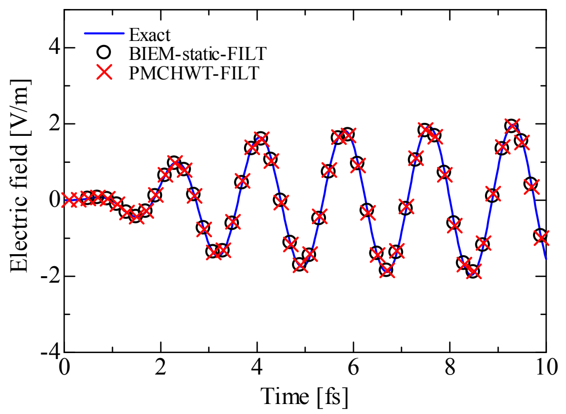

Fig. 3. Time response of the electric field for the gold nanosphere with the radius 10.0nm. The observation point is at x = 12.0 nm on the x axis.

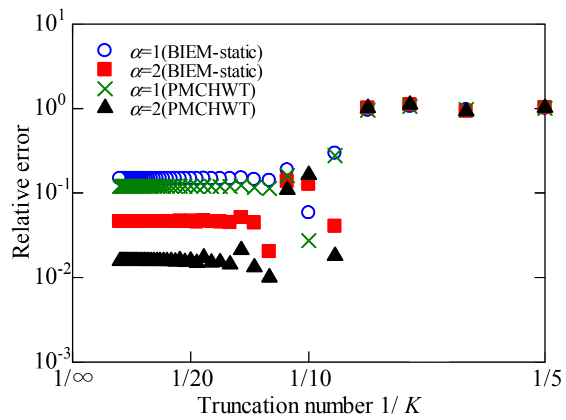

Fig. 4. Convergence process of the relative error for varying the truncation number K in Eq. (9).

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