

Accuracy of PO Diffraction Components given by MER Peripheral Integration for Various Surface Curvature

Pengfei Lu^{#1}, Makoto Ando^{#2}

[#] *Electrical and Electronic Engineering Department, Tokyo Institute of Technology
2-12-1-S3-19 O-okayama, Meguro-ku, Tokyo, 152-8552, Japan*

¹ Lui@antenna.ee.titech.ac.jp

² mando@antenna.ee.titech.ac.jp

Abstract—The accuracy of Modified Edge Representation in surface-to-line integral reduction of Physical Optics is investigated in total field. Only the diffraction components is separated and compared between two results given by MER line integration and PO surface integration as the changing of the scatterer surface curvature.

I. INTRODUCTION

The Modified Edge Representation (MER) [1] is the concept to be used in the surface-to-line integral reduction [2], [3], [4] for computing the scattered field given by PO surface radiation [5] integrals which is one of the high frequency asymptotic techniques in diffraction analysis. The surface-to-line integral reduction is important for reducing the computation and extracting the diffraction components. It has been verified that the surface integral of PO currents is uniformly reduced into a MER line integration in higher frequency case on the flat scatterer, if no inner SPP [6]. If SPP exists inside, PO surface integrals will be reduced into two MER line integrals by using high frequency approximation and Stokes' theorem; one is the integration along the periphery of the scatterer generating the diffracted field while another is the infinitesimally small indentation around inner Stationary Phase Point (SPP) producing the reflected field as shown in Fig 1 [7]. The latter one from SPP equals to Scattering Geometrical Optics (SGO) if the reflected wave is spherical has been shown in [8], and the correction term in analytical form has been derived if the reflection has the aberration [9]. In this paper, the accuracy of diffraction components calculated by MER and PO has been investigated depending on surface curvature of the scatterer.

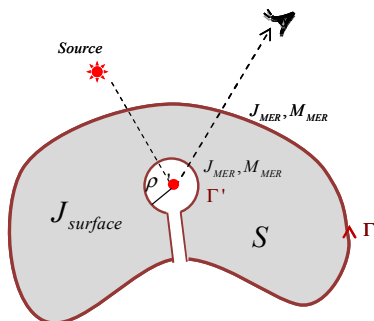


Fig. 1. Surface to line integral reduction by EECs defined in MER.

II. MODIFIED EDGE REPRESENTATION AND PHYSICAL OPTICS

A. Modified Edge Representation Line Integration

MER line integration is an alternative methodology for the PO radiation pattern calculation of surfaces as shown in Fig. 2.

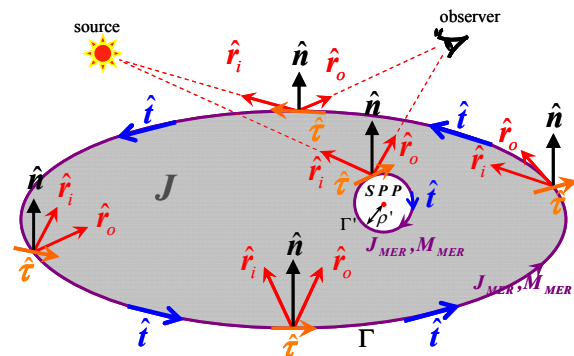


Fig 2 Parameters used in MER line integration.

The waves reflected from inner SPP and diffracted from periphery of the scatterer are defined by the MER line integration as,

$$E_{MER}^{SPP} = \lim_{\rho' \rightarrow 0} \left\{ j \frac{k}{4\pi} \oint_{\Gamma'} \hat{r}_o \times [\hat{r}_o \times \eta J_{MER} + M_{MER}] \frac{e^{-jk r_o}}{r_o} dl \right\} \quad (1)$$

$$E_{MER}^{per} = j \frac{k}{4\pi} \oint_{\Gamma} \hat{r}_o \times [\hat{r}_o \times \eta J_{MER} + M_{MER}] \frac{e^{-jk r_o}}{r_o} dl$$

the MER unit vector \hat{t} is defined as,

$$(\hat{r}_i + \hat{r}_o) \cdot \hat{t} = 0 ; \hat{n} \cdot \hat{t} = 0 \quad (2)$$

equivalent electric and magnetic line currents along the actual edge \hat{t} are defined using the modified edge vector \hat{t} as,

$$\begin{aligned}
 J_{MER} &= \frac{\{\hat{r}_o \times (\hat{r}_o \times J_{PO})\} \cdot \hat{t}}{j(1 - (\hat{r}_o \cdot \hat{t})^2)(\hat{r}_i + \hat{r}_o) \cdot (\hat{n} \times \hat{t})} \hat{t} \\
 M_{MER} &= \frac{\eta(\hat{r}_o \times -J_{PO}) \cdot \hat{t}}{j(1 - (\hat{r}_o \cdot \hat{t})^2)(\hat{r}_i + \hat{r}_o) \cdot (\hat{n} \times \hat{t})} \hat{t} \quad (3) \\
 J_{PO} &= 2\hat{n} \times H^i
 \end{aligned}$$

It was also confirmed that PO surface integration can be well approximated to MER line integration if only the radiation term is used in the definition of H^i in (3).

B. Physical Optics

In PO, the scattering wave E_{PO}^S is expressed as the surface radiation integral of the currents on the scatterer as,

$$\begin{aligned}
 E_{PO}^{total} &= E^i + E_{PO}^S \\
 E_{PO}^S &= j \frac{k\eta}{4\pi} \int_S \hat{r}_o \times \hat{r}_o \times J_{PO} \frac{e^{-jkr_o}}{r_o} dS \quad (4) \\
 J_{PO} &= 2\hat{n} \times H^i
 \end{aligned}$$

where r_o is the distance from the integration point to the observer and J_{PO} is PO surface current, \hat{n} is the unit vector normal to the surface. The diffracted wave given by PO can be calculated by $E_{PO}^S - E_{SGO}$, where E_{SGO} gives the reflection components expressed as,

$$E_{SGO} = E_{GO} - E^i = \begin{cases} E_{GO}^{SPP} & (\text{incident, reflected ray}) \\ 0 & (\text{incident ray only}) \\ -E^i & (\text{shadow}) \end{cases} \quad (5)$$

where GO reflected contribution is given by [10],

$$E_{GO}^{SPP} = E^i(SPP) \cdot \bar{R} \sqrt{\frac{\rho_1^r \rho_2^r}{(\rho_1^r + r_o)(\rho_2^r + r_o)}} e^{-jkr_o} \quad (6)$$

GO considers an astigmatic ray which leaves from the reflection point SPP, as shown in Fig. 3 and Fig. 4. r_o measured from the SPP. The spreading factor of the ray depends on principal radii of curvature ρ_1^r and ρ_2^r of the reflected ray can be expressed as,

$$\begin{aligned}
 \frac{1}{\rho_1^r} &= \frac{1}{2} \left(\frac{1}{\rho_1^i} + \frac{1}{\rho_2^i} \right) + \frac{1}{f_1} \\
 \frac{1}{\rho_2^r} &= \frac{1}{2} \left(\frac{1}{\rho_1^i} + \frac{1}{\rho_2^i} \right) + \frac{1}{f_2} \quad (7)
 \end{aligned}$$

where ρ_1^i, ρ_2^i is principal radii of curvature of incident wave front,

$$\begin{aligned}
 \frac{1}{f_1} &= \frac{1}{\cos \phi} \left(\frac{\sin^2 \theta_2}{a_1} + \frac{\sin^2 \theta_1}{a_2} \right) \\
 &+ \sqrt{\frac{1}{\cos^2 \phi} \left(\frac{\sin^2 \theta_2}{a_1} + \frac{\sin^2 \theta_1}{a_2} \right)^2 - \frac{4}{a_1 a_2}} \quad (8) \\
 \frac{1}{f_2} &= \frac{1}{\cos \phi} \left(\frac{\sin^2 \theta_2}{a_1} + \frac{\sin^2 \theta_1}{a_2} \right) \\
 &- \sqrt{\frac{1}{\cos^2 \phi} \left(\frac{\sin^2 \theta_2}{a_1} + \frac{\sin^2 \theta_1}{a_2} \right)^2 - \frac{4}{a_1 a_2}}
 \end{aligned}$$

where a_1, a_2 is radii of curvature of the reflecting surface, θ_1, θ_2 is angle between the direction of the incident rays r_i and u_1, u_2 which is unit vector in the principal direction of scatterer at SPP with principal radius of curvature a_1, a_2 .

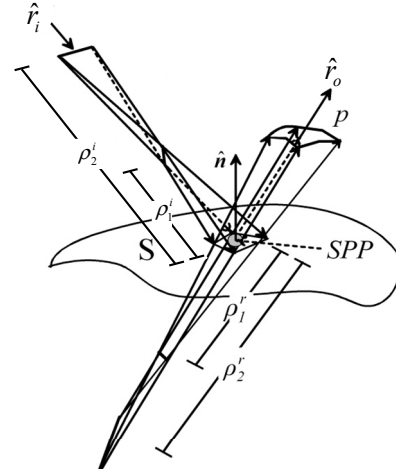


Fig. 3. GO Reflected ray tube.

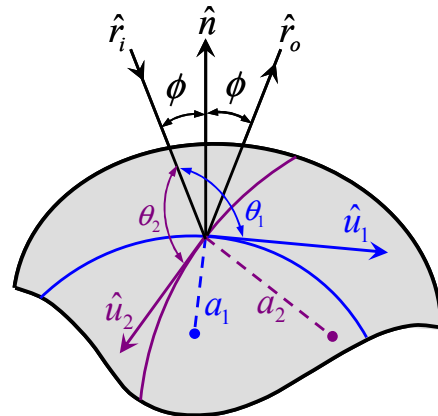


Fig. 4. Local geometry of the surface.

III. AGREEMENT BETWEEN MER AND PO IN DIFFRACTION AND ITS DEPENDENCE UPON SURFACE CURVATURE

E_{MER}^{per} and E_{MER}^{SPP} in (1) express the diffracted (from the periphery) and reflected (from the inner SPP) terms respectively. The diffraction components calculated by E_{MER}^{per} and $E_{PO}^S - E_{SGO}$ are compared in Fig. 5 and Fig. 6 respectively comes from the convex scatterer model as shown in Fig. 7 and concave model in Fig. 8.

It has been found that the difference appears obviously if the observer located at RB in convex scatterer case, but is extraordinarily small in concave scatterer one. Then the amplitude of diffraction observed at RB for various surface curvature is drawn in Fig. 9, where the curvature with minus

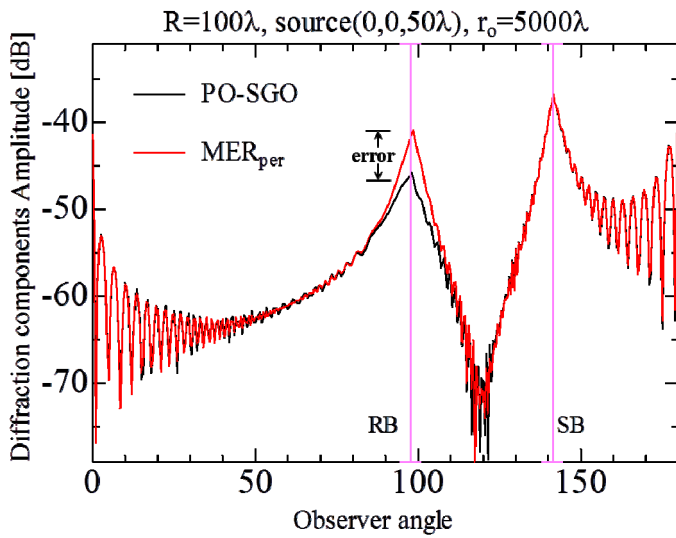


Fig. 5. Diffraction field generated from convex scatterer.

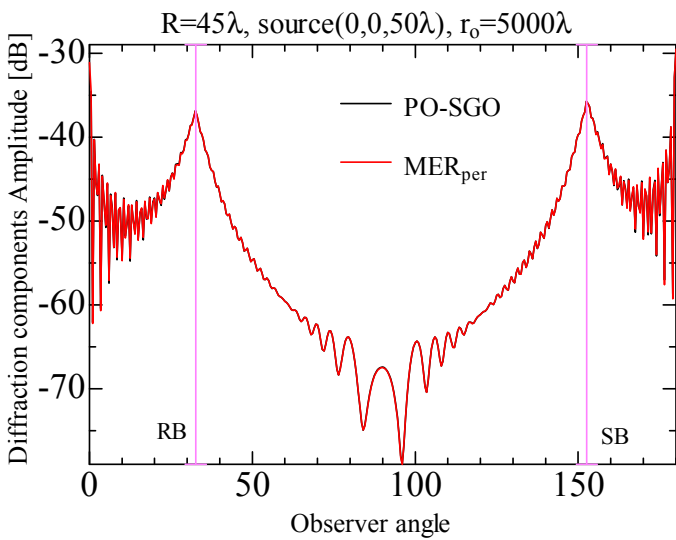


Fig. 6. Diffraction field generated from convex scatterer.

sign implies that the surface is concave as Fig. 10, and convex scatterer if curvature is the plus value as Fig. 11.

We found that the agreement of diffractions given by MER and PO is better in concave scatterer case than in convex one, as it gives us the same conclusions from Fig. 5 and Fig. 6. With the increasing of the surface curvature, the error between MER and PO is also arising in convex scatterer case.

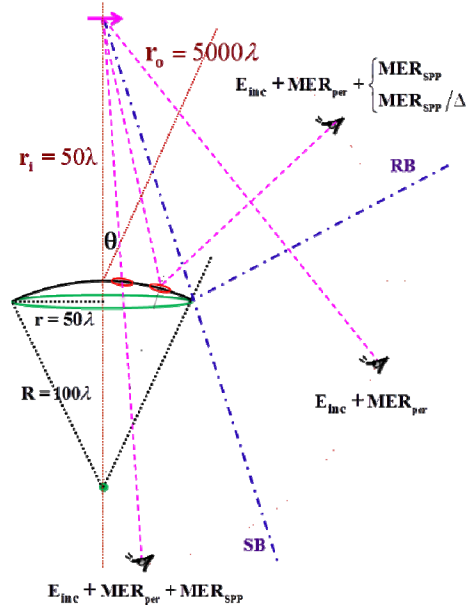


Fig. 7. Convex scatterer form partial sphere.

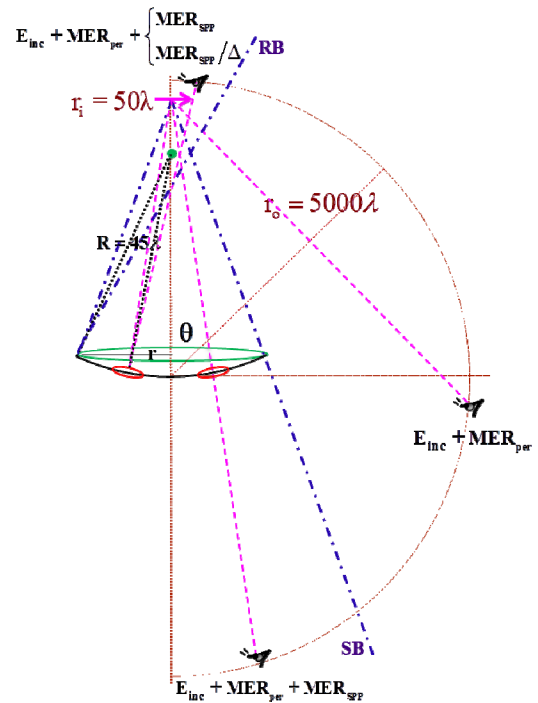


Fig. 8. Concave scatterer form partial sphere.

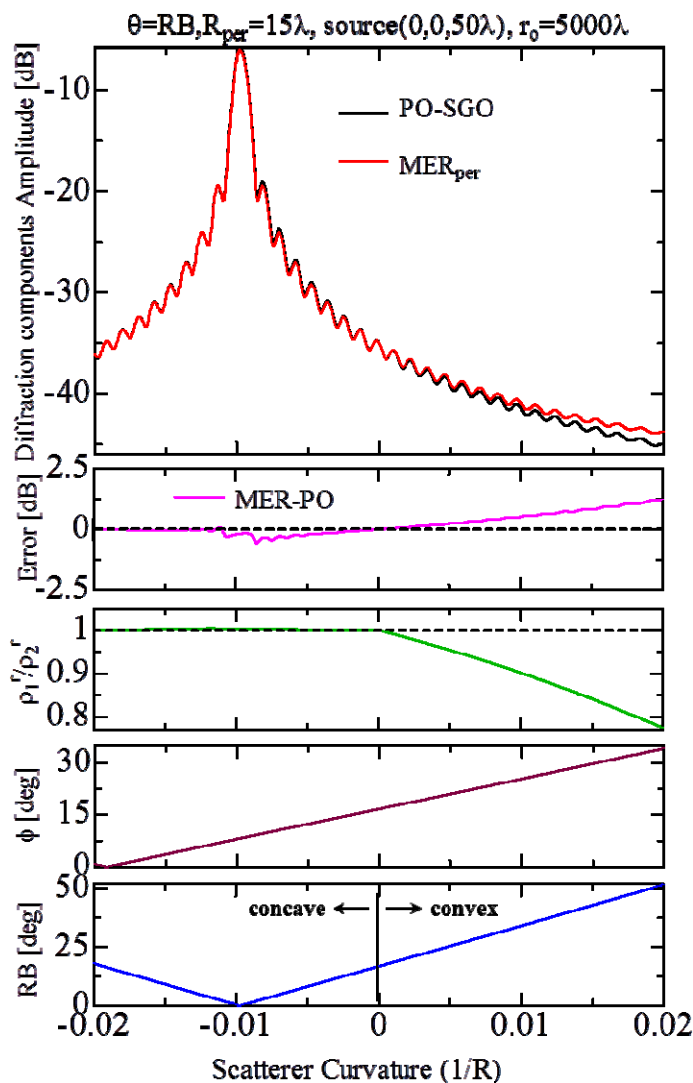


Fig. 9. Diffraction components amplitude at RB.

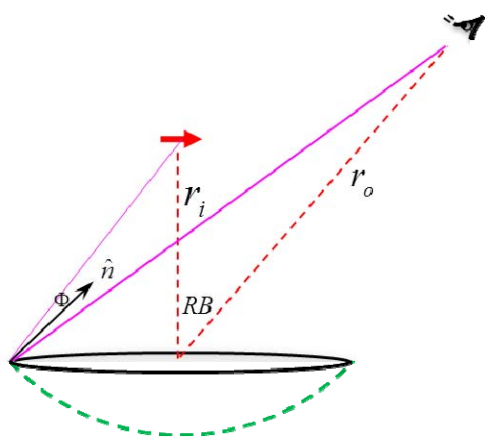


Fig. 10. Observer located at RB of concave scatterer.

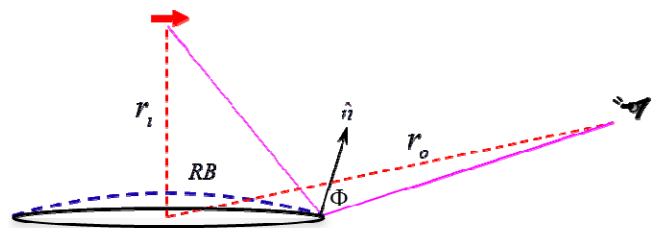


Fig. 11. Observer located at RB of convex scatterer.

IV. CONCLUSION

The diffraction components calculated by PO and MER have been compared for two types of curved scatterer, concave and convex. It is investigated, the agreement of diffractions given by MER and PO is better in concave scatterer case than in convex one. It has been confirmed that the error coming from RB location related with the surface curvature of the scatterer.

ACKNOWLEDGMENT

This work was conducted in part as the Research and Development for Expansion of Radio Wave Resources, the Ministry of Internal Affairs and Communications.

REFERENCES

- [1] T. Gokan, M. Ando and T. Kinoshita, "A new definition of equivalent edge currents in a diffraction analysis," IEICE Technical Report, AP89-64, Dec. 1989.
- [2] Michaeli A., "Equivalent edge currents for arbitrary aspects of observation", IEEE Trans. Antennas Propag., 32, 252-258, 1984.
- [3] Johansen P. M. and O. Breinbjerg, "An exact line integral representation of the physical optics scattered field: The case of a perfectly conducting polyhedral structure illuminated by electric Herizian dipoles", IEEE Trans. Antennas Propag., 43(7), 689-696, 1995.
- [4] Albani M. and S. Maci, "An exact line integral representation of the PO radiation integral from a flat perfectly conducting surfaces illuminated by elementary electric or magnetic dipoles", Turk. J. Electr. Eng., 10(2), 291-305T, 2002.
- [5] Silver S., *Microwave Antenna Theory and Design*, McGraw- Hill, New York, 1949.
- [6] T. Murasaki and M. Ando, "Equivalent edge currents by the modified edge representation: physical optics components," IEICE Trans. Electron, vol.E75-C, No.5, pp.617-626, May, 1992.
- [7] K. Sakina and M. Ando, "Line Integral Representation for Diffracted Fields in Physical Optics Approximation Based on Field Equivalence Principle and Maggi-Rubinowicz Transformation," IEICE Trans. Communication, vol.E84-C, No.10, pp1521-1527, Oct. 2001.
- [8] L. Rodriguez, K. Yukimasa, T. Shijo and M. Ando, "Inner stationary phase point contribution of physical optic in terms of the modified edge representation line integrals (curved surfaces)", Radio Sci., vol.42, No.6, pp.1-8, 2007.
- [9] Pengfei Lu, Makoto Ando, "Difference of Scattering Geometrical Optics Components and Line Integrals of Currents in Modified Edge Representation", AGU Journals, Radio Sci., doi:10.1029/2011RS004899, 2012.
- [10] C. A. Balance, *Advanced Engineering Electromagnetics*, Wiley, Canada, pp.753-757, 1989.