

# A Novel Reciprocity Based Development of the Green's Dyadic for Canonical Impedance Cylinders and Spheres- An Angularly Guided Wave Representation

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**Abstract**—A novel approach based on the electromagnetic (EM) reciprocity theorem is described for developing the Green's dyadics pertaining to canonical impedance cylinders and spheres. These Green's dyadics are chosen here to directly emphasize waves guided in the angular direction. The latter constitute canonical solutions useful in the construction of ray fields, within the UTD framework, that are excited by antennas on arbitrary convex surfaces with an impedance boundary condition (IBC). It is noted that the IBC can be utilized to model highly metallic, smooth, convex surfaces with a sufficiently thin material coating.

## I. INTRODUCTION

The construction of EM dyadic Green's functions, for wave problems conforming to coordinate systems in which the solution becomes separable, is commonly achieved via the use of vector wave functions (VWFs) expressed in those coordinates [1]. Such solutions are quite cumbersome when expressed explicitly in terms of VWFs; furthermore, they do not easily lend themselves to arrive at representations of waves guided along circumferential or angular directions. The latter type of angularly guided wave representations emphasize the creeping wave nature of the fields that are required in obtaining asymptotic high frequency uniform geometrical theory of diffraction (UTD) ray solutions, which impart a direct and vivid picture for the radiation by EM sources in the presence of such structures. The transformation of the standard VWF solutions, which generally emphasize waves guided in a direction away from the radiating object, into a form which emphasizes angularly guided waves has then to be achieved by a long, tedious function-theoretic process referred to as the Watson transform. On the other hand, the present approach utilizes potential theory and reciprocity to systematically arrive at the Green's dyadics. A major advantage of the present approach is that it provides Green's dyadics directly in terms of any choice of guided wave directions; i.e., in terms of any one of the three orthogonal coordinate directions in which

the solution is separable. The angular (circumferential for cylinders or polar for spheres) is just one choice that is useful in emphasizing the creeping wave behavior and is selected here to be specific. The details of the approach are summarized below.

## II. TECHNICAL APPROACH

As indicated above, the canonical problems of EM source excited circular cylinder and sphere geometries with an IBC are considered. Green's dyadics are constructed for these problems via the use of potential theory and EM reciprocity theorem. Basically, the potential theory is first utilized to obtain field solutions to special source directions (that become directly obvious for any given geometry). These special potentials can be obtained in terms of scalar Green's functions. The scalar Green's functions can be constructed in terms of convolution integrals as in [2,3], based on the well developed theory of linear operators in Hilbert space.

The process for the construction of Green's dyadic proceeds via the following steps:

### Step 1

For the sake of being specific, consider a tiny electric current moment of strength  $d\vec{p}_e(\vec{r}')$  placed at  $\vec{r}'$ , as in Fig. 1, which radiates in the presence of an IBC cylinder or sphere. The observer is located at  $\vec{r}$ . The source and observer points are  $P'$  and  $P$ , respectively. Let the EM fields generated by  $d\vec{p}_e(\vec{r}')$  be denoted by  $(\vec{E}_e, \vec{H}_e)$ . The objective is to find  $(\vec{E}_e, \vec{H}_e)$  which can be expressed as

$$\vec{E}_e(\vec{r}) = d\vec{p}_e(\vec{r}') \cdot \overline{\overline{G}}_{ee}(\vec{r}|\vec{r}') ; \quad \vec{H}_e(\vec{r}) = d\vec{p}_e(\vec{r}') \cdot \overline{\overline{G}}_{me}(\vec{r}|\vec{r}')$$

Once the  $(\vec{E}_e, \vec{H}_e)$  are known and expressed in the above format, the Green's dyadics  $\overline{\overline{G}}_{ee}$  and  $\overline{\overline{G}}_{me}$  can be identified directly and simply by inspection.

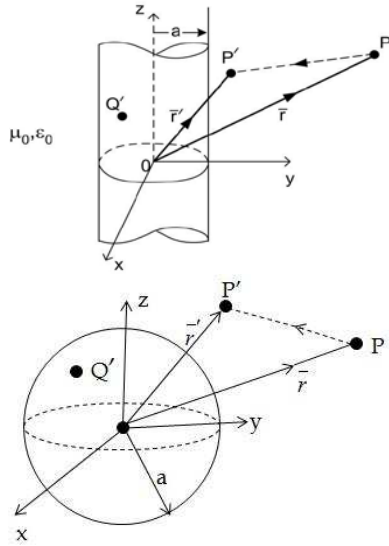


Fig. 1. The cylinder and sphere geometries with an IBC. The source  $d\bar{p}_e$  is at  $\bar{r}'$ . The observer is at  $\bar{r}$ .

### Step 2

Introduce two test sources at  $P$  (i.e., at  $\bar{r}$ ), namely a tiny electric test current moment  $d\bar{p}_{et}(\bar{r})$  and a tiny magnetic test current moment  $d\bar{p}_{mt}(\bar{r})$ , respectively, which radiate in the presence of cylinder/sphere. These test sources are chosen to be pointing in a special direction,  $\hat{u}$ , where  $\hat{u} = \hat{z}$  (axial direction) for a cylinder, and  $\hat{u} = \hat{r}$  (radial direction) for a sphere. Basically,  $\hat{u}$  is in a direction such that it is the generator for cylinders and is orthogonal to the circular cylinder cross section; the same can be true for a wedge geometry. Likewise,  $\hat{u}$  is orthogonal to the spherical surface. In the case of planar stratified media,  $\hat{u}$  will have to be chosen to be orthogonal to the planar stratification. Thus,  $d\bar{p}_{et}(\bar{r}) = \hat{u}dp_{et}(\bar{r})$ , and it generates the test potentials  $\bar{A}_{et}(\bar{r}')$  and  $\bar{F}_{et}(\bar{r}')$  at  $\bar{r}'$  which are  $\hat{u}$  directed. Likewise,  $d\bar{p}_{mt}(\bar{r}) = \hat{u}dp_{mt}(\bar{r})$  generates the test potentials  $\bar{A}_{mt}(\bar{r}')$  and  $\bar{F}_{mt}(\bar{r}')$  at  $\bar{r}'$  that are  $\hat{u}$  directed. The advantage of the special direction  $\hat{u}$  selected for the test sources will become clear in the subsequent steps. It is noted that for the  $d\bar{p}_{et}(\bar{r})$  electric test source,

$$\begin{aligned}\bar{A}_{et}(\bar{r}') &= \mu_0 d\bar{p}_{et}(\bar{r}) G_{aet}(\bar{r}'|\bar{r}) ; \\ \bar{F}_{et}(\bar{r}') &= \epsilon_0 d\bar{p}_{et}(\bar{r}) G_{fet}(\bar{r}'|\bar{r}).\end{aligned}$$

Likewise for the  $d\bar{p}_{mt}(\bar{r})$  magnetic test source

$$\begin{aligned}\bar{A}_{mt}(\bar{r}') &= \mu_0 d\bar{p}_{mt}(\bar{r}) G_{amt}(\bar{r}'|\bar{r}) ; \\ \bar{F}_{mt}(\bar{r}') &= \epsilon_0 d\bar{p}_{mt}(\bar{r}) G_{fnt}(\bar{r}'|\bar{r}).\end{aligned}$$

The scalar Green's functions  $G_{aet}$ ,  $G_{fet}$ ,  $G_{amt}$  and  $G_{fnt}$  can be found via the techniques in [2,3]. Unlike the procedure in [1], the scalar Green's functions above can be directly obtained to emphasize waves guided along any of the different coordinate directions at the very outset [2,3]. These test potentials generate the test fields. In particular,  $d\bar{p}_{et}(\bar{r})$  generates the electric field  $\bar{E}_{et}(\bar{r}') = \bar{E}_{aet}(\bar{r}') + \bar{E}_{fet}(\bar{r}')$ , where  $\bar{E}_{aet}$

comes from  $\bar{A}_{et}$ , and  $\bar{E}_{fet}$  comes from  $\bar{F}_{et}$ . Likewise  $d\bar{p}_{mt}(\bar{r})$  generates the electric field  $\bar{E}_{mt}(\bar{r}') = \bar{E}_{amt}(\bar{r}') + \bar{E}_{fnt}(\bar{r}')$ , where  $\bar{E}_{amt}$  comes from  $\bar{A}_{mt}$  and  $\bar{E}_{fnt}$  comes from  $\bar{F}_{mt}$ .

### Step 3

The original fields  $(\bar{E}_e, \bar{H}_e)$  which are of interest can be related to the test fields of step 2 via the reciprocity theorem, namely

$$dp_{et}(\bar{r}) [\hat{u} \cdot \bar{E}_e(\bar{r})] = d\bar{p}_e(\bar{r}') \cdot \bar{E}_{et}(\bar{r}')$$

and

$$dp_{mt}(\bar{r}) [\hat{u} \cdot \bar{H}_e(\bar{r})] = -d\bar{p}_e(\bar{r}') \cdot \bar{E}_{mt}(\bar{r}').$$

Since the right hand side of the last two equations are known (via scalar Green's function mentioned above), the  $\hat{u}$  components of the desired fields  $(\bar{E}_e, \bar{H}_e)$  on the left side become known also. If one lets  $(\bar{E}_e, \bar{H}_e)$  to be generated by the potentials  $\bar{A}_e = A_e \hat{u}$  and  $\bar{F}_e = F_e \hat{u}$ , respectively, then it can be shown always that this special choice  $\hat{u}$  yields

$$\begin{bmatrix} \hat{u} \cdot \bar{E}_e \\ \hat{u} \cdot \bar{H}_e \end{bmatrix} = \frac{1}{j\omega} \left[ \frac{\partial^2}{\partial u^2} + k^2 \right] \begin{bmatrix} A_e/\epsilon_0 \\ F_e/\mu_0 \end{bmatrix} \propto \begin{bmatrix} A_e \\ F_e \end{bmatrix}$$

where the left side of the above is known. Hence the right side becomes known. Once  $\bar{A}_e = A_e \hat{u}$  and  $\bar{F}_e = F_e \hat{u}$  become known,  $(\bar{E}_e, \bar{H}_e)$  can be found as usual via differentiating  $\bar{A}_e$  and  $\bar{F}_e$ .

### Step 4

Finally, from the relations in step 1, the Green's dyadics  $\bar{\bar{G}}_{ee}$  and  $\bar{\bar{G}}_{me}$  can be found via inspection as indicated in that step. The source point correction (required for completeness) can be immediately and trivially added, as a dyadic delta function term, as discussed in [4]. If one has a magnetic current element  $d\bar{p}_m(\bar{r}')$  at  $P'$  instead of the electric current, the procedure to obtain the Green's dyadic for this case follows the same way as for the electric case.

## III. CONCLUSIONS

A procedure based on reciprocity and potentials is presented for constructing the Green's dyadics pertaining to canonical geometries for which the relevant solutions become separable. This novel approach does not need the use of the more involved VWFs [1]; furthermore, unlike the approach involving the use of VWFs, the present method provides alternative representations for the Green's dyadics that emphasize guided wave propagation in any desired coordinate direction without the need for any Watson transformation.

## REFERENCES

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