Time Reversal Imaging without Information of Incident Field

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***Abstract*—A time-reversal approach to reconstructing the electrical parameters of an inhomogeneous object from measured total field data is considered. It is shown that the approach does not require the information of the incident field in the region of interest enclosing the object when the primary source is placed outside of the region. Numerical simulations are performed to validate the effectiveness of the approach.**

I. INTRODUCTION

Electromagnetic imaging methods have attracted increasing interests with many applications [1-7]. Imaging methods based on inverse scattering can estimate not only the location and size but also the shape and internal structure of a scattering object of interest by reconstructing the electrical parameter distributions of the object. Most inverse scattering methods assume the knowledge of incident fields as well as the total field measured on an observation surface. Recently, we have proposed an inverse scattering method which does not require the knowledge of incident field in the region of interest enclosing an unknown scattering object [8]. Time reversal imaging methods have also received considerable attention since they yield accurate location estimate of point scatterers. Up to now, most time reversal methods have focused on detection and localization of point-like or small targets [9-12] and not dealt with estimation of electrical parameter profile of scatterers. We have proposed a time-reversal imaging which can reconstruct electrical parameter distributions of the unknown object [13]. This imaging technique requires the knowledge of incident field.

In this paper, we present a time-reversal imaging method without information of incident fields. In earlier work [13], a primary source as well as an unknown scatterer is located in the region enclosed by an observation surface. Placing the primary source external to the observation surface in the same manner as [8] yields reconstruction method without the information of the incident field produced by the primary source. In order to assess the effectiveness of the approach, numerical simulations are carried out.

II. FORMULATION

A. Total field

Let us consider scattering by an inhomogeneous object placed in free space with the permittivity ε_0, permeability μ_0 as shown in Fig.1(a). For simplicity, two-dimensional case is examined, i.e., the electric parameters of the object depend only on the x and y directions, and a short pulsed wave is generated by an electric current source $\mathbf{J}(\mathbf{r},t) = J_z(\mathbf{r},t)\hat{z}$ flowing along the z-direction, where $\mathbf{r} = (x, y)$ and $\hat{z}$ is the unit vector in the z-direction. We assume that the source is turned on at time $t = 0$ and there is no electromagnetic fields before time $t = 0$. The total field $\mathbf{v}(\mathbf{r},t)$ satisfy Maxwell's equations in the following form:

$$\left[\bar{A}\frac{\partial}{\partial x} + \bar{B}\frac{\partial}{\partial y} - \bar{C}\frac{\partial}{\partial(ct)} - \bar{D}\right]\mathbf{v} = \mathbf{j}\,, \tag{1}$$

where

$$\mathbf{v} = \begin{pmatrix} E_z \\ \eta H_x \\ \eta H_y \end{pmatrix}, \quad \mathbf{j} = \begin{pmatrix} \eta J_z \\ 0 \\ 0 \end{pmatrix}, \tag{2}$$

under the initial condition of zero fields:

$$\mathbf{v}(\mathbf{r},t) = 0\,, \quad t < 0. \tag{3}$$

$\eta (= \sqrt{\mu_0/\varepsilon_0})$ is the characteristic impedance of free space and

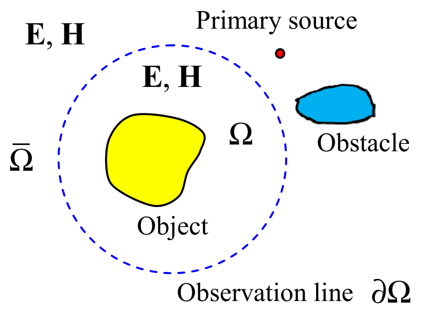
$$\bar{A} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \bar{B} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\bar{C} = \begin{pmatrix} \varepsilon_r(\mathbf{r}) & 0 & 0 \\ 0 & \mu_r(\mathbf{r}) & 0 \\ 0 & 0 & \mu_r(\mathbf{r}) \end{pmatrix}, \bar{D} = \begin{pmatrix} \eta\sigma(\mathbf{r}) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4}$$

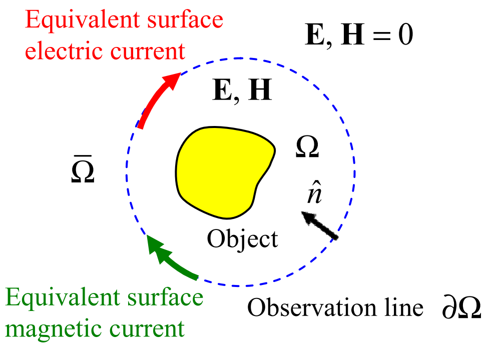
and where

$$\varepsilon_r(\mathbf{r}) = \begin{cases} \varepsilon_s(\mathbf{r}) & \mathbf{r} \in S \\ 1 & \mathbf{r} \notin S \end{cases}, \quad \mu_r(\mathbf{r}) = \begin{cases} \mu_s(\mathbf{r}) & \mathbf{r} \in S \\ 1 & \mathbf{r} \notin S \end{cases},$$

$$\sigma(\mathbf{r}) = \begin{cases} \sigma_s(\mathbf{r}) & \mathbf{r} \in S \\ 0 & \mathbf{r} \notin S \end{cases}. \tag{5}$$



(a) Original problem



(b) Equivalent problem

Fig. 1. (a) Original and (b) equivalent problems in the interior region Ω.

In (5), S denotes the support of the object with the relative permittivity ε_s , the relative permeability μ_s , and conductivity σ_s .

B. Equivalent field

The total field is measured on an observation line ∂Ω during a time interval [0, T]. We apply the field equivalence principle [14] to the whole space which is divided into two regions; denoted by Ω and $\bar{\Omega}$ and separated by the observation line ∂Ω as shown in Fig.1(b). The impressed primary source $\mathbf{J}$ is assumed to be in the region $\bar{\Omega}$, while the scattering object is in the region Ω . The equivalent surface currents are given by the tangential components of the measured total fields over the surface ∂Ω as

$$\mathbf{s} = \begin{pmatrix} \eta\left[n_x H_y - n_y H_x\right] \\ -n_y E_z \\ n_x E_z \end{pmatrix} \delta_{\partial\Omega} , \tag{6}$$

where $\delta_{\partial\Omega}$ is a delta function representing a source concentrated on the surface ∂Ω , and n_x and n_y are the components of the unit vector $\hat{n}$ inward normal to ∂Ω . The electromagnetic fields $\mathbf{u}(\mathbf{r}, t)$ of the equivalent problem are the solution of Maxwell's equations,

$$\left[\bar{A}\frac{\partial}{\partial x} + \bar{B}\frac{\partial}{\partial y} - \bar{C}\frac{\partial}{\partial(ct)} - \bar{D}\right]\mathbf{u} = \mathbf{s} , \tag{7}$$

under the initial condition of zero fields,

$$\mathbf{u}(\mathbf{r},t) = 0 , \quad t < 0 , \tag{8}$$

where

$$\mathbf{u} = \begin{pmatrix} E_z^{eq} \\ \eta H_x^{eq} \\ \eta H_y^{eq} \end{pmatrix} . \tag{9}$$

In the equivalent problem, the primary source and obstacles yielding unwanted clutter located in the exterior region $\bar{\Omega}$ are removed and the original object is remained. The equivalent surface current $\mathbf{s}$ produces the same total field in the interior region Ω as does the primary source $\mathbf{J}$, while null field in the exterior region $\bar{\Omega}$, i.e.,

$$\mathbf{u}(\mathbf{r},t) = \begin{cases} \mathbf{v}(\mathbf{r},t) & \mathbf{r}\in\Omega \\ 0 & \mathbf{r}\in\bar{\Omega} \end{cases} \quad \text{(for the original object)} \tag{10}$$

If we replace the object by a different object,

$$\mathbf{u}(\mathbf{r},t) \neq \begin{cases} \mathbf{v}(\mathbf{r},t) & \mathbf{r}\in\Omega \\ 0 & \mathbf{r}\in\bar{\Omega} \end{cases} \quad \text{(for a different object).} \tag{11}$$

C. Time-reversed field

Let us consider the time-reversed field which is generated by the time-revered surface current source $\mathbf{s}$ (6). We assume the total field $\mathbf{v}$ essentially vanishes after time $t > T$. Then, the time-reversed field $\mathbf{w}(\mathbf{r}, t)$ is the solution of the following equation:

$$\left[\bar{A}\frac{\partial}{\partial x} + \bar{B}\frac{\partial}{\partial y} - \bar{C}\frac{\partial}{\partial(ct)} - \bar{D}\right]\mathbf{w} = \mathbf{s} , \tag{12}$$

under the final condition of zero fields,

$$\mathbf{w}(\mathbf{r},t) = 0 , \quad T < t , \tag{13}$$

where

$$\mathbf{w} = \begin{pmatrix} E_z^{tr} \\ \eta H_x^{tr} \\ \eta H_y^{tr} \end{pmatrix} . \tag{14}$$

The time-reversed field is calculated backward in time from $t = T$ to 0. The equivalent surface current $\mathbf{s}$ produces the same total field in the interior region Ω as does the primary source $\mathbf{J}$, while null field in the exterior region $\bar{\Omega}$, i.e.,

$$\mathbf{w}(\mathbf{r},t) = \begin{cases} \mathbf{v}(\mathbf{r},t) & \mathbf{r}\in\Omega \\ 0 & \mathbf{r}\in\bar{\Omega} \end{cases} \quad \text{(for the original object)} \tag{15}$$

If we replace the object by a different object,

$$\mathbf{w}(\mathbf{r},t) \neq \begin{cases} \mathbf{v}(\mathbf{r},t) & \mathbf{r} \in \Omega \\ 0 & \mathbf{r} \in \Omega \end{cases} \quad \text{(for a different object).} \tag{16}$$

D. Inverse problem

The time-domain inverse scattering problem under consideration here is the estimation of $\varepsilon_r(\mathbf{r})$, $\mu_r(\mathbf{r})$, and $\sigma(\mathbf{r})$ with the knowledge of the measured total field data $\mathbf{v}$. If the estimated electrical parameters is identical with the true ones, the equivalent fields produced in the exterior region $\bar{\Omega}$ by the equivalent surface current $\mathbf{s}$ is null, while an incorrect estimated electrical parameters, however, does not give null fields there (see (10) and (11)). This observation cast the inverse scattering problem considered here as an optimization problem where the following functional of the electrical parameters to be minimized [7]:

$$F^{eq}(\mathbf{p}) = \sum_{n=1}^{N} \int_0^T \int_{\bar{\Omega}} \left| \mathbf{u}_n(\mathbf{r},t) \right|^2 d\mathbf{r}dt \tag{17}$$

where $\mathbf{p} = (\varepsilon, \mu)$ is the parameter vector to be estimated. The vectors $\mathbf{u}_n(\mathbf{r},t)$ represents the equivalent fields for an estimated parameter $\mathbf{p}$ due to the nth equivalent surface current $\mathbf{s}_n$, n = 1, …., N. Note that the surface current $\mathbf{s}_n$ is given by the total fields $\mathbf{v}_n(\mathbf{r},t)$ measured on $\partial\Omega$ for the object with true parameter $\mathbf{p}^{true}$ due to the nth illuminating source $\mathbf{j}_n$.

Since the same observation is true for the time-reversed field $\mathbf{w}_n(\mathbf{r},t)$, we can use the following functional instead of (17):

$$F^{tr}(\mathbf{p}) = \sum_{n=1}^{N} \int_0^T \int_{\bar{\Omega}} \left| \mathbf{w}_n(\mathbf{r},t) \right|^2 d\mathbf{r}dt\,. \tag{18}$$

III. NUMERICAL RESULTS

Numerical simulation is carried out for a nondestructive testing problem of retrieving the location and size of two unknown defects in a known host medium, as shown in Fig.2. The background is assumed to be free space. The total field $\mathbf{v}(\mathbf{r},t)$ is calculated with the finite-difference time-domain method. The homogeneous host medium, with relative permittivity ε_s = 4.0, is a 2λ×2λ square whose center is located at the origin, (x,y) = (0,0). Here λ is the wavelength in free space corresponding to the highest frequency contained in the primary source:

$$\mathbf{j}_n = I(t)\,\delta(\mathbf{r}-\mathbf{r}_n)\,\hat{\mathbf{z}}\,, \tag{19}$$

where $\mathbf{r} = \mathbf{r}_n$ is the location of the nth primary source and the time factor

$$I(t) = \frac{d^3}{dt^3}\exp\left[-\alpha^2(t-\tau)^2\right], \tag{20}$$

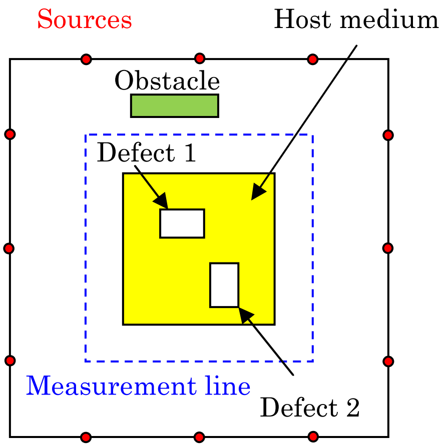


Fig. 2. Detection of two defects in a square cylinder.

TABLE I. THE DEFECT PARAMETERS ESTIMATED BY GA.

Case1 (based on (18))

	x_{c1}	y_{c1}	l_{x1}	L_{y1}	x_{c2}	y_{c2}	l_{x2}	L_{y2}
True	-0.25	0.38	0.50	0.25	0.38	-0.50	0.25	0.50
GA	-0.25	0.38	0.50	0.25	0.38	-0.50	0.25	0.50

Case2 (based on (17))

	x_{c1}	y_{c1}	l_{x1}	L_{y1}	x_{c2}	y_{c2}	l_{x2}	L_{y2}
True	-0.25	0.38	0.50	0.25	0.38	-0.50	0.25	0.50
GA	-0.25	0.38	0.50	0.25	0.38	-0.50	0.25	0.50

and where $\tau = \beta\Delta t$, $\alpha = 5.6/\tau$, and β = 132 with time step Δt = 0.98($\Delta x/c\sqrt{2}$) and cell size $\Delta x = \Delta y = \lambda/10\sqrt{4}$. Defect 1 is a 0.50λ×0.25λ square of ε_r = 1.0 with its center at (−0.25λ, 0.38λ), and defect 2 is a 0.25λ×0.50λ square of ε_r = 1.0 with its center at (0.38λ, −0.50λ). The measurements are carried out along the periphery $\partial\Omega$ of square of side 3λ. The primary sources are located λ away from the measurement line $\partial\Omega$. A genetic algorithm (GA), which is a global optimization technique, is applied to the minimization of the functional (18). The parameters characterizing the host medium (the location, size and dielectric constant) and the number and the dielectric constants of two defects are assumed to be known, while the location (x_{c1},y_{c1}) of the center and the size $l_{x1} \times l_{y1}$ of defect 1 and the location (x_{c2},y_{c2}) of the center and the size $l_{x2} \times l_{y2}$ of defect 2 are searched for. In order to show that the estimate is not affected by a clutter, a λ×0.3λ dielectric square obstacle with ε_r = 2.0 is placed between the primary source and the observation surface. Using 12 successive illuminations, the estimated values of the characterizing parameters in one trial of the GA search after 200 generations are shown in the case 1 of Table I. For comparison, the result based on the functional (17) is shown in the case 2. Both results are the same as the true ones.

IV. Conclusion

We have presented an inverse scattering approach using time reversal of the forward-propagation total field measured on a closed surface enclosing unknown scatterers. Without the knowledge of the primary source generating the incident field and unwanted obstacles near the source, the electrical parameter distributions of the scatterers of interest can be reconstructed from only measured total field data. The numerical simulation results for a nondestructive testing problem showed the effectiveness of the approach. The extension of the approach to three-dimensional case and combination of the time reversal and equivalent fields will be considered in future works.

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