Imaging Buried Objects within the Second-Order Born Approximation through a Multiresolution-Regularized Inexact-Newton Method

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***Abstract*—The inspection of 2-D scatterers buried in a lossy half-space from field measurements is formulated within the framework of the second-order Born approximation (*SOBA*) of the inverse scattering problem. An iterative multi-scaling approach (*IMSA*) is combined with a two-step inexact-Newton (*IN*) algorithm to solve the arising problem. A set of preliminary numerical results is presented to assess the features and potentialities of the considered approach also in comparison with state of the art *IN-SOBA* techniques.**

I. INTRODUCTION AND MOTIVATION

Subsurface imaging problems arise in several diagnostics applications related to ground penetrating radar, including civil engineering, cultural heritage monitoring, and non-destructive testing and evaluation [1]-[6]. Unfortunately, the complexity of the associated non-linear and ill-posed/ill-conditioned formulations has prevented the development of simple, computationally efficient, reliable, and effective solution techniques [2][7][8]. Indeed, the theoretical difficulties in subsurface imaging, which arise also in free-space inverse scattering formulations [8], yield to local minima and non-existence/non-uniqueness issues, which pose unique challenges in the development of microwave imaging algorithms [2][7][8].

In this framework, linearized models have been applied to subsurface scenarios comprising weak scatterers [5]. Furthermore, to avoid the use of fully nonlinear models, often requiring numerically expensive inversion techniques based on global optimizers [9]-[11], intermediate solutions based for instance on the Second-Order Born Approximation (*SOBA*) have been proposed when strong scatterers are at hand [12]. More specifically, the *SOBA* [13] has been recently used by exploiting its variational formulation [14], which has been then solved by means of an inexact-Newton (*IN*) method [15]. The arising technique has been numerically shown to provide good regularization capabilities in different scenarios [15]. Nevertheless, it is known that local minima issues can arise also in *IN*-based approximated inverse scattering solvers, owing to the intrinsic non-linearity of the imaging problem [16]-[18].

To overcome these issues, the combination of *IN* techniques [19][20] with iterative multi-scaling (*IMSA*) approaches [21] has been recently proposed in free-space scenarios [16]-[18]. Thanks to the joint features of these two techniques, microwave imaging problems derived from different formulations (e.g., contrast field/contrast source) have effectively been solved through *IMSA−IN* [16]-[18]. However, the application of this approach to subsurface inspection has never been considered in the literature.

Accordingly, the aim of this work is to combine the regularization features of the *IN-SOBA* solver with the capabilities of the *IMSA* to mitigate local minima issues in a subsurface imaging problem. The arising *IMSA-IN-SOBA* is then assessed also in comparison with state-of-the-art "bare" *IN-SOBA* techniques [15] through a set of preliminary numerical results.

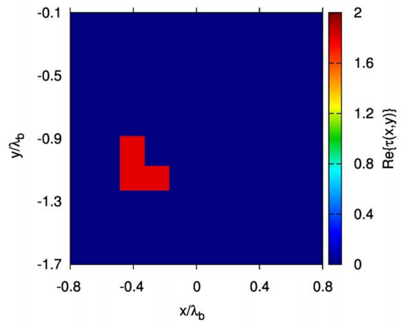


Fig. 1. *Homogeneous L-Shaped Cylinder* – Actual dielectric profile.

II. MATHEMATICAL FORMULATION AND *IMSA-IN-SOBA* SOLUTION

Consider V TM time-harmonic waves $E_{inc}^{v}(\mathbf{r})$ (z component, $v=1,\ldots,V$) that impinge on a cylindrical unknown scatterer buried inside a homogeneous half-space domain Ω characterized by the complex dielectric permittivity ε_b and magnetic permeability μ_b and whose upper medium is supposed to be free space. Moreover, assume that the investigation domain is discretized by *N* pixel-based functions (with barycenters $\mathbf{r}_n$, *n=1,..,N*), and that *M* field measurement points (located in $\mathbf{r}_m$, *m=1,..,N*) outside Ω are at hand. Under such circumstances, the electric field scattered by the object, $E^v_{scatt}(\mathbf{r})$ complies with [15]

$$\mathbf{y}^v = \mathbf{\Lambda}^v\left(\mathbf{x}^v\right) \qquad (1)$$

where the known and unknown terms are $\mathbf{y}^v = \left[E^v_{scatt}(\mathbf{r}_m), m = 1,...,M\right]^t$ and $\mathbf{x}^v = \left[\tau(\mathbf{r}_n), n = 1,...,N\right]^t$, respectively, being $\tau(\mathbf{r}) = \varepsilon(\mathbf{r})/\varepsilon_b - 1$ the contrast function, $\varepsilon(\mathbf{r})$ the dielectric permittivity of the scatterer. In (1), the operator $\mathbf{\Lambda}^v(\mathbf{x})$ is computed according to the *SOBA* as [15]

$$\Lambda^v(\mathbf{x}) = B^v[\tau(\mathbf{r})] - k_b^2 \int_\Omega \tau(\mathbf{r}') B^v[\tau(\mathbf{r}')] g(\mathbf{r},\mathbf{r}') d\mathbf{r}', \mathbf{r} \in \Omega$$

where $B^v[\tau(\mathbf{r}')] = -k_b^2 \int_\Omega \tau(\mathbf{r}') E^v_{inc}(\mathbf{r}') g(\mathbf{r},\mathbf{r}') d\mathbf{r}', \mathbf{r} \in \Omega$ is the first-order Born operator, *g* is the half-space Green's function, k_b is the background wavenumber, and $E^v_{inc}(\mathbf{r})$ is the *v*-th incident field.

To solve such problem, an *IMSA-IN* technique is applied by setting a different pulse-basis expansion at each *IMSA* step yielding a set of *S* discrete inverse problems $\mathbf{y}^{(s)} = \mathbf{\Lambda}^{(s)}\left(\mathbf{x}^{(s)}\right)$, *s* = 1, …, *S* [21]. Accordingly, the s-th discrete problem is solved by

a) setting *l*=0 and using the previous-step reconstruction (at the *s*=0, the null solution is assumed) to initialize the unknown $\mathbf{x}^s_l$;

b) linearizing Eq. (1) to yield $\mathbf{\Lambda}'_{\mathbf{x}^s_l} \delta\mathbf{x}^s_l = \mathbf{y}^s - \mathbf{\Lambda}\left(\mathbf{x}^s_l\right)$, being $\mathbf{\Lambda}'_{\mathbf{x}^s_n}$ the Frèchet derivative of $\mathbf{\Lambda}$ at point $\mathbf{x}^s_l$ [15];

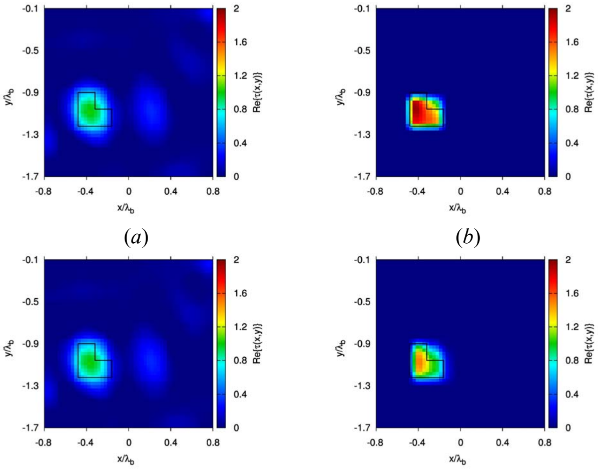
c) computing through *I* steps of a truncated Landweber algorithm the regularized solution $\delta\mathbf{x}^s_l$ of the linearized equation [21];

d) updating the current solution with $\mathbf{x}^s_{l+1} = \mathbf{x}^s_l + \delta\mathbf{x}^s_l$, and increasing *l* to *l*+1.

The steps (*b*)-(*d*) are then repeated until *l*=*L* to yield the s-th scale reconstructed dielectric profile $\mathbf{x}^s_L$, which is then filtered and clustered to compute the Regions of Interest (*RoI*s) [21]. The *IMSA-IN-SOBA* is then iterated on the updated Eq. (1) at the *s+1* step [21]. The entire process is terminated when predefined criteria (e.g., on the size of the *RoI* or on the functional value) are met [16]-[18][21].

III. PRELIMINARY NUMERICAL RESULTS

To preliminary validate the features of the proposed *IMSA−IN-CSI* technique, an investigation domain (with $\varepsilon_b=4\varepsilon_0$ and $\mu_b=\mu_0$) of size $1.6\lambda_b$ (being λ_b the wavelength in the background) starting $0.1\lambda_b$ below the air-soil interface is imaged by considering *M*=*V*=16 field sources/probes equally spaced in a cross-well setup [22] (with ±*0.2* λ_b offset along *x* with respect to the investigation domain). Moreover, the following parameters have been assumed for the inversion: *I*=*L*=30, *S*=4.



(*a*) (*b*)

(*c*) (*d*)

Fig. 2. *Homogeneous L-Shaped Cylinder* – Retrieved dielectric profiles by (*a*)(*c*) *BARE-IN-SOBA* [15] and (*b*)(*d*) *IMSA-IN-SOBA* in correspondence with (*a*)(*b*) noiseless data, and (*c*)(*d*) *SNR*=20 dB.

To preliminary validate the procedure, a homogeneous L-shaped cylinder (*τ=1.8* – Fig. 1) has been firstly imaged assuming noiseless data [Figs. 2(*a*), 2(*b*)]. The plots of the retrieved contrasts by the state-of-the-art *BARE-IN-SOBA* [*N=1600* cells - Fig. 2(*a*)] and *IMSA-IN-SOBA* [*N=400* at each IMSA step - Fig. 2(*b*)] indicate that both methods are able to retrieve the presence and position of the unknown cylinder, but the BARE technique provides a lower accuracy in terms of retrieved τ, and it also yields some artifacts [Fig. 2(*a*) vs. 2(*b*)]. Similar conclusions can be drawn when analysing noisy data [e.g., *SNR*=20 dB - Figs. 2(*c*), 2(*d*)]. Indeed, the *IMSA-IN-SOBA* retrieved profile remarks the capability of the multi-resolution strategy to refine the reconstruction around the region of interest [e.g., sharper details can be observed – Fig. 2(*d*)] while allocating very few unknowns in the background, as expected from the considered multi-focusing strategy [21].

IV. CONCLUSION

The second-order Born approximation (*SOBA*) is considered to formulate the problem of retrieving two-dimensional scatterers buried in a lossy half-space. The combination of the *IMSA* and a two-step inexact-Newton (*IN*) algorithm is proposed to solve the arising problem. A preliminary assessment of the performance and robustness of the proposed technique show it advantages in terms of accuracy and effectiveness with respect to state of the art *IN-SOBA* techniques.

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