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Quasi-Analytical Models for Metamaterial Homogenization

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Abstract — **This work presents a quasi-analytical approach to the homogenization of metamaterials realized by the superposition of identical periodic planar surfaces. It is based on the definition of an equivalent admittance matrix for the single layer, combined with the application of the Bloch theory for the analysis of the periodically loaded equivalent transmission line modeling the layered structure. Different theoretical definitions of equivalence are discussed in connection with practical applications of the homogenization concept. It is shown that the uniqueness of the equivalent homogeneous medium depends on the final objective of the homogenization procedure, i.e. on the specific metamaterial behavior to be mimicked.**

I. INTRODUCTION

The continuous interest on metamaterials, started more than a decade ago, has in parallel inspired a lot of work about their modeling, and the most proper and efficient methods of analysis and design [1]-[2]. The development of a reliable and accurate approach for the modeling of the electromagnetic behavior of artificial materials is a key point to fully exploit their potential. Considerable effort has been made to represent these artificial structures as effective homogeneous media, described by a set of equivalent constitutive parameters [3]-[7]. This approach may reveal very useful in the design of metamaterial-based devices. However, the procedure for the retrieval of the equivalent parameters is not univocally defined; in fact, different techniques can be used depending on the metamaterial characteristics and on the goal of the homogenization process. In this framework, different models can be set up relying on the definition of different "homogenization equivalences".

Volumetric metamaterials can be realized by cascading a number of periodic surfaces made of patch- or slot-type elements, as shown in Fig. 1. This kind of surfaces is widely known and is indicated with different denominations depending on the application, as for instance Frequency Selective Surfaces (FSS) [8], Partially Reflecting Surfaces (PRS) and, more recently, with the new exotic denomination of MetaSurfaces [9]. This class of volumetric metamaterials is particularly interesting due to the ease of fabrication, and the wide variety of realizable electromagnetic behaviors. In this work we describe a general approach for the homogenization of these artificial materials, and we show how it can be applied to satisfy different homogenization equivalences.

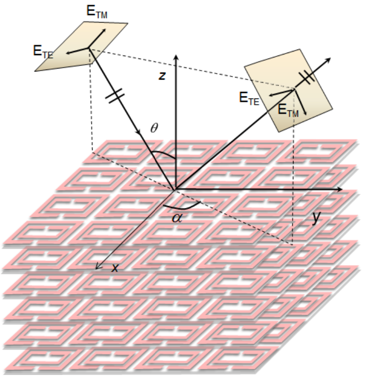


Fig. 1. Geometry for the multilayer structure under analysis.

II. DEFINITION OF THE EQUIVALENCE MODELS

The definition of "homogenization equivalence" actually depends on the final objective of the homogenization procedure, since this latter determines which characteristics of the artificial media must be also exhibited by the effective homogeneous medium. In most of the cases, the definition of the equivalent parameters is based on a scattering analysis; in some other cases, the effective homogeneous medium is required to match the dispersion properties of the metamaterial. Moreover, the capability of the homogenization model to correctly represent the structure of the supported fields could also be important.

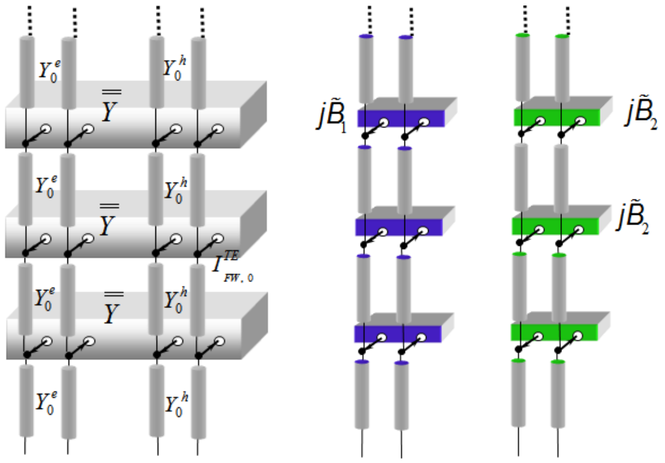
Starting from these considerations, we can define three different homogenization equivalences, namely:

- External Equivalence: slabs of the artificial multilayer material and of the equivalent homogeneous medium possess the same scattering and transmission matrices as a function of frequency and wavenumber.
- Dispersion Equivalence: the artificial multilayer material and the equivalent homogeneous medium admit the same solutions of the dispersion equation for the two dominant eigenmodes.
- Modal equivalence: the field structure of the two modes supported by the equivalent homogeneous medium matches the one of the two dominant eigenmodes of the artificial multilayer material.

In the following, a general homogenization procedure that can be adapted to match the different types of equivalence is presented.

III. HOMOGENIZATION PROCEDURE

The class of artificial media described above can be conveniently analyzed resorting to an equivalent periodically loaded transmission line model, in which each layer is modeled via the equivalent admittance [10], Fig. 2. Applying the Bloch theory for periodic structures [11] it is possible to define an equivalent uniform transmission line, which satisfies both the External equivalence and the Dispersion equivalence.



(a) (b)

Fig. 2. (a) Equivalent transmission line model of the multilayer artificial material; (b) eigenmode transmission line for the definition of the homogenization equivalences.

A. External Equivalence

The "External Equivalence" approach leads to the definition of a set of equivalent constitutive parameters capable of providing the correct prediction of the scattering from a metamaterial slab consisting of an arbitrary number of periodic layers [6]. The Bloch theory provides the propagation constant and characteristic impedance of an equivalent uniform transmission line with the same scattering coefficients of the periodically loaded transmission line. Hence, this transmission line parameters can be used to define a set of constitutive parameters satisfying the external equivalence.

This approach to homogenization is intrinsically ambiguous, due to the fact that the scattering is only determined by the components of the fields that are tangential to the interface. This ambiguity can also be explained in terms of volumetric equivalent currents, since different equivalent media are associated with different sets of equivalent currents, but any set of non-radiating currents do not affect the scattering properties of the slab.

B. Dispersion Equivalence

The "Dispersion Equivalence" leads to the definition of an equivalent homogeneous medium which admits the same solutions of the dispersion equations (i.e. the same relationship between the propagation vector components and the frequency) as for the two dominant modes of the artificial material. The "Dispersion Equivalence" can provide useful results in applications where the focus is on the characterization of the propagation inside the unbounded material

As a general rule, the scattering coefficients from a given material slab do not univocally identify the dispersion relation of the material, since there is an ambiguity in the definition of the longitudinal component of the propagation constant. This means that the "External Equivalence" does not automatically guarantees the "Dispersion Equivalence". It is noted however that by imposing that the homogeneous medium is described by the same equivalent transmission line model of the multilayer material it is possible to simultaneously satisfy both the "External Equivalence" and the "Dispersion Equivalence". The homogeneous medium defined through this procedure is not univocally defined, but it possesses all the properties requires for some particular applications, like for instance the design of planar lenses, filters and polarizers. In fact, in these cases, the knowledge of the transfer function of a metamaterial slab is the key point for the analysis and/or design processes.

C. Modal Equivalence

The "External Equivalence" and the "Dispersion Equivalence" leave an ambiguity in the definition of some entries of the equivalent tensors.

In order to complete the definition of the equivalent medium, a "Modal Equivalence" can be set by considering also the longitudinal field components. In this respect, we can distinguish two cases: the "Single Mode Equivalence" and the "Full Modal Equivalence". The first one consists in matching all the field components for one of the two dominant eigenmodes, and it is appropriate when only one mode is of interest, e.g. when only one dominant mode is propagating. The second one consists in matching all the field components of the two dominant eigenmodes with a unique couple of tensors. This latter equivalence can always be rigorously established when TE and TM modes are decoupled in the artificial medium. In the most general case, it is approximately achievable under the assumption that the metamaterial is not bianisotropic and presents a low degree of spatial dispersion.

In the proposed homogenization approach, the definition of the implementation of the Modal equivalence requires the determination of the average longitudinal fields in the metamaterial. This step can be done in quasi analytical form if a spectral MoM is used for the analysis of the planar periodic layers.

Fig. 3 shows the set representation of the homogenization equivalences.

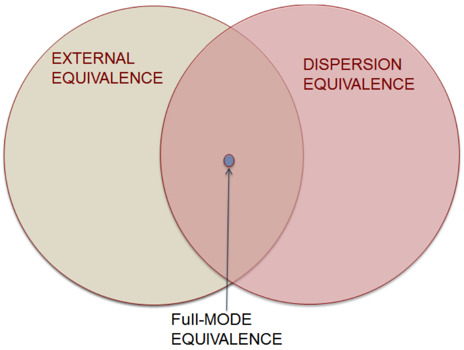


Fig. 3. Set theory representation of the relationship between the three different homogenization equivalences and corresponding equivalent parameters.

IV. CONCLUSIONS

A quasi-analytical homogenization procedure has been presented for volumetric metamaterials consisting of planar periodic layers. Different approaches to the definition of equivalent constitutive parameters can be applied depending on which characteristics of the metamaterial must be also exhibited by the effective homogeneous medium. A first methodology defines an equivalent medium by matching the scattering properties of an artificial medium slab. A second one obtains the equivalent parameters by matching the solutions of the dispersion equation for the two dominant eigenmodes (Bloch modes). A third approach also considers the average longitudinal field and uses this information to identify a unique couple of equivalent tensors that match the fields of the two dominant Bloch modes.

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