Proceedings of the "2013 International Symposium on Electromagnetic Theory"

21AM1D-02

Eigenmode Analysis of Transmission Line-Based Phase-Nonreciprocal Metamaterials

Andrey Porokhnyuk[#1], Tetsuya Ueda[#2], Yuichi Kado[#3], Tatsuo Itoh[*1]

[#] *Department of Electronics, Kyoto Institute of Technology*
Kyoto 606-8585, Japan.
[1] d1821008@edu.kit.ac.jp

[*] *Electrical Engineering Department, University of California, Los Angeles*
Los Angeles, CA 90095-1594 USA.

***Abstract* — Eigenmode analysis combining elements of field theory and transmission line model was applied to ferrite-embedded phase-nonreciprocal transmission line metamaterial. The dispersion of dominant TE-modes is described analytically from magnetic properties of the ferrite and asymmetry of the structure. The results were proved with full-wave numerical simulation and experiment.**

I. Introduction

In the latest decade, composite right/left handed (CRLH) metamaterials [1]-[4] received a great interest of researchers. New types of nonreciprocal CRLH metamaterials were proposed. Earlier work was concentrated on nonreciprocity in the amplitude of transmission coefficients caused by dominance of CRLH or damping modes in opposite directions [5]-[8]. Later, pure phase-nonreciprocal CRLH metamaterials were introduced in [9], [10]. These structures support right-handed (RH), and left-handed (LH) mode propagation for opposite directions at the same frequency. The resulting phenomenon of unidirectional phase flow [9] was implemented in advanced leaky wave antennas [10], [11], and pseudo-traveling-wave resonators [10], [12], [13]. The latter are similar to zeroth-order resonators [14], [15], but show linear phase gradient that can be tuned in the resonator-based beam scanning antennas [10], [13], [16]. The CRLH properties of the structures were explained by equivalent circuit model, while the nonreciprocal difference in phase constants was estimated using numerical simulation [9].

In order to avoid relying on numerical simulations, we proposed a new analysis approach combining a simplified field analysis and transmission line model. It is known [17] that CRLH structure with series capacitors inserted demonstrate the same magnitude of phase nonreciprocity as

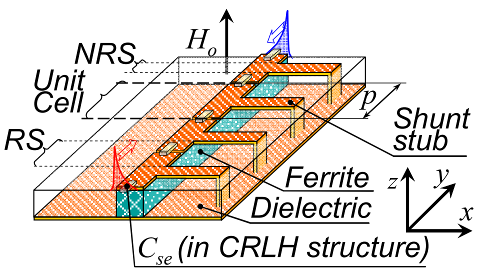

Fig. 1 Transmission line based nonreciprocal metamaterial. In ε-negative structure serial capacitors C_{SE} are not inserted.

analogous ε-negative structure without series capacitors. In the present work, we perform mode analysis of such ε-negative structure, giving us intelligible dispersion, impedance characteristics as well as simplified intrinsic relations for magnitude of phase nonreciprocity revealing its mechanisms [18].

II. Metamaterial Structure

The phase-nonreciprocal ε-negative metamaterial structure under consideration is composed of a central microstrip line with a ferrite rod embedded under the strip, as shown in Fig. 1. An internal dc magnetic field H_0 and induced saturation magnetization M_S in the ferrite rod are directed normally to the microstrip surface. Microstrip shunt stubs are inserted into central microstrip with a period p. It is well-known that the quasi-TE edge-guided modes [19] are dominant along ferrite rod-embedded microstrip line. If boundary conditions on side walls of the central microstrip line are equal, the line supports reciprocal propagation. Different boundary conditions result in nonreciprocal propagation. Shunt stubs provide asymmetry of boundary conditions. It was shown in [18], [20], [21] that such metamaterials are characterized by nonreciprocal propagation and ε-negative cutoff modes.

In our approach, the structure is analyzed as a sequence of unit cells that are decomposed into several sections on the basis of boundary conditions. We distinguish a reciprocal section (RS) with both magnetic wall boundary conditions and a nonreciprocal section (NRS) where one boundary is of magnetic wall type and other boundary is not. Boundary conditions can be described in the form of admittances that are ratios of magnetic to electric field components on the side walls of the central microstrip: $Y_1 = H_y / E_z$ at $x = 0$ and $Y_2 = -H_y / E_z$ at $x = w$, as shown in Fig. 2. Therefore for RS, $Y_1 = Y_2$, and for NRS, $Y_1 \neq Y_2$. It is convenient to treat the microstrip stub as simple admittance transformers, so its input admittance can be converted to Y_2 after normalizing it by effective cross section of the stub. In this case, stubs should be separated enough to avoid their electromagnetic coupling.

Mode analysis is applied to each section giving dispersion relation and transmission ABCD matrices. The dispersion and Bloch impedance characteristics of the periodic structure are

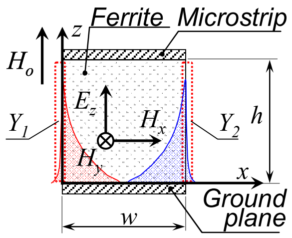

Fig.2 Electromagnetic model of ferrite rod-embedded microstrip line in cross-section. After [18] © 2012 IEEE

analyzed using ABCD matrix of the unit cell built from the sections. The ε - negative structure can be easily expanded to CRLH metamaterial by introducing series capacitances C_{se} between unit cells.

III. Field Analysis in Separate Sections

The simplified field analysis is performed at frequencies far above the ferromagnetic resonance. The dominant propagating electromagnetic waves in the ferrite-embedded microstrip line are simplified to TE modes, assuming that the substrate thickness is much smaller than the wavelength. The problem is reduced to analysis of transverse field distribution that satisfies Helmholtz equation:

$$\frac{\partial^2 E_z}{\partial x^2} + k_x^2 E_z = 0 \qquad (1)$$

with

$$k_x^2 = \varepsilon_r \frac{(\mu^2 - \mu_a^2)}{\mu} \left(\frac{\omega}{c}\right)^2 + \gamma^2$$

Here $\gamma = \alpha + j\beta$ is propagation constant in the longitudinal y-direction; α and β are attenuation and phase constants; ε_r is dielectric constant of the ferrite; μ and μ_a are diagonal and off-diagonal components of Polder's tensor.

The dispersion relation can be found using eigenmode analysis with boundary conditions Y_1 and Y_2, written down as ratio of field components. The component E_z is a solution of (1), and functional relation between E_z and H_y is found from Maxwell equations. In RS, magnetic wall boundaries give $Y_1 = Y_2 = 0$. Under influence of the shunt stub, Y_2 in NRS changes to finite imaginary value. In NRS, the dispersion is described by transcendental equation:

$$\cot(wk_x) = \frac{1}{jY_2}\sqrt{\frac{\varepsilon_o}{\mu_o}}\left(\frac{\gamma^2}{(\omega/c)\,k_x\mu} + \frac{\varepsilon_r(\omega/c)}{k_x}\right) + j\frac{\gamma\mu_a}{\mu\,k_x} \qquad (2)$$

where only the last term on the right-hand side is an odd function of γ, thus determining nonreciprocity. In RS, the solution is reduced to well-known dispersion of edge-guided mode $\gamma = \pm j\sqrt{\mu\varepsilon_r}\,\omega/c$ [19]. The analysis of (2) showed that in case of lossless propagation, nonreciprocity is observed purely in phase constant β.

With propagation constants known for both directions of propagation, the characteristic impedance can be estimated from the transverse profiles of E_z and H_x. The characteristic impedance appears to be reciprocal for lossless case.

IV. Transmission Line Analysis of The Structure.

Circuit parameters in the nonreciprocal metamaterial structure in Fig. 1 are acquired from ABCD matrix of its unit cell using transmission line analysis for infinitely periodic structure [22]. The unitary ABCD matrix F_{RS} of RS is a well-known transmission line matrix, and for NRS a nonunitary matrix F_{NRS} is obtained from conventional voltage-current analysis [22].

Two RS and one NRS are combined into the unit cell, so total ABCD matrix becomes $F_{Cell} = F_{RS}F_{NRS}F_{RS}$. By imposing periodic boundary condition on the unit cell, the dispersion of propagation constant γ_{MM} in the periodic ε - negative structure is obtained from equation $\det\left[F_{Cell} - \hat{I}e^{\gamma_{MM}p}\right] = 0$ as

$$\cosh(\gamma_{MM}p - \Delta\gamma_{NR}l_N) = \cos m_N \cos m_R + R^+ \sin m_N \sin m_R \qquad (3)$$

Here $m_R = j\gamma_R l_R$, $m_N = j\bar{\gamma}_N l_N$, subscripts "*R*" and "*N*" correspond to parameters of RS and NRS respectively. Quantities l_N and l_R are the lengths of sections, $\bar{\gamma}_N$ is an algebraic average of phase constants for opposite propagation directions in NRS. The Bloch characteristic impedance can be obtained from F_{Cell} using voltage-current analysis, and it is reciprocal in lossless case.

The CRLH structure can be analysed the same way by introducing a matrix F_{2Cse} of double series capacitance $2C_{se}$ into matrix of the unit cell: $F_{Cell} = F_{2Cse}F_{RS}F_{NRS}F_{RS}F_{2Cse}$.

V. Conclusions

The present approach gives a good analytic insight to dispersion of dominant TE-modes in one-dimensional nonreciprocal metamaterials. The derived dispersion relation proves pure phase constant-related nonreciprocity in case of lossless propagation. The approach also showed a good agreement with finite element full-wave numerical simulations and experiments.

Acknowledgment

This work was supported in part by KAKENHI #23560454 and #22109002.

References

[1] C. Caloz, T. Itoh, Electromagnetic metamaterials, John Wiley and Sons,

[2] G. Eleftheriades, K. Balmain, *Negative refraction metamaterials*, John Wiley and Sons, 2005.

[3] A. Lai, C. Caloz, and T. Itoh, "Composite right/left-handed transmission line metamaterials," *IEEE Microw. Mag.*, vol. 5, no. 3, pp. 34–50, Sep. 2004.

[4] Elefteriades, G. V., A. K. Iyer, and P. C. Kremer, "Planar negative refractive index media using planar L-C loaded transmission line," *IEEE Trans. Microw. Theory Tech.*, vol. 50, no. 12, 2702–2712, 2002.

[5] T. Ueda, M. Tsutsumi, "Nonreciprocal left-handed transmission characteristics of microstrip lines on the ferrite substrate," *IET Proc. Microw. Antennas Propag.*, vol. 1, no. 2, pp. 349-354, April 2007.

[6] T. Ueda, M. Tsutsumi, "Left-handed transmission characteristics of ferrite microstrip lines without series capacitive load," *IEICE Trans. Electron.*, vol. E89-C, pp. 1318-1323, Sep. 2006.

[7] M. Abdalla, Z. Hu, "On the study of nonreciprocal left handed coplanar waveguide over ferrite substrate with only shunt inductive load," *Microw. and Opt. Tech. Lett.*, vol. 49, no. 11, 2810–2814, 2007.

[8] T. Kodera, C. Caloz, “Uniform ferrite-loaded open waveguide structure with CRLH response and its application to a novel backfire-to-endfire leaky-wave antenna,” *IEEE Trans. Microw. Theory Tech.*, vol. 57, no. 4, pp. 784-795, April 2009.

[9] T. Ueda, K. Horikawa, M. Akiyama, M. Tsutsumi, “Nonreciprocal phase-shift composite right/left handed transmission lines and their application to leaky wave antennas,” *IEEE Trans. Antennas Propag.*, vol. 57, no. 7, pp. 1995-2005, July 2009.

[10] T. Ueda, “Transmission line microwave apparatus including at least one non-reciprocal transmission line part between two parts,” *U. S. Patent App. Pub*., US2010/0060388 A1, Mar. 28, 2010.

[11] K. Horikawa, T. Ueda, M. Akiyama, “Influence of reflected waves at a terminal of nonreciprocal phase-shift CRLH transmission lines on the leaky wave radiation,” *in Proc. 2009 Asia-Pacific Microw. Conf.*, Dec., 2009, pp. 151-154.

[12] T. Ueda, H. Kishimoto, “Pseudo-traveling wave resonator based on nonreciprocal phase-shift composite right/left handed transmission lines,” *2010 IEEE MTT-S Int. Microw. Symp. Digest*, May 2010, pp. 41-44.

[13] T. Ueda, S. Yamamoto, Y. Kado, and T. Itoh, "Pseudo-traveling wave resonator with magnetically tunable phase gradient of fields and its applications to beam steering antennas," *IEEE Trans. on Microw. Theory Tech.*, vol. 60, no. 10, pp. 3043-3054, Oct. 2012.

[14] A. Sanada, C. Caloz, and T. Itoh, “Zeroth-order resonance in composite right/left handed transmission line resonators,” *in Proc. 2003 Asia-Pacific Microw. Conf.*, Nov. 2003, pp. 1588-1592.

[15] M. Abdalla, Z. Hu, “Design and analysis of tunable left handed zeroth-order resonator on ferrite substrate,” *IEEE Trans. Magn.*, vol. 44, pp. 3095-3098, Nov. 2008.

[16] T. Ueda, S. Yamamoto, and Y. Kado, “Beam-scanning traveling-wave-resonator antenna based on nonreciprocal phase-shift CRLH transmission lines,” *in 2011 IEEE Int. Symp. Antennas Propag. Proc.*, July 2011, pp. 1058-1061.

[17] T. Ueda, J. Fukuda, Y. Kado, T. Itoh, “Nonreciprocal phase-shift CRLH transmission lines using geometrical asymmetry with periodically inserted double shunt stubs,” *Proc. 42nd Eur. Microw. Conf.*, Oct. 2012, pp. 570-573.

[18] A. Porokhnyuk, T. Ueda, Y. Kado, T. Itoh, “Mode analysis of nonreciprocal metamaterials using a combination of field theory and transmission line model,” *in 2012 IEEE MTT-S Int. Microw. Symp. Digest*, WE4J-5, June 2012, pp. 1-3.

[19] M.E. Hines, “Reciprocal and nonreciprocal modes of propagation in ferrite striopline and microstip devices,” *IEEE Trans. Microw. Theory Tech.*, vol. MTT-19, no.5, pp. 442-451, May 1971.

[20] T. Ueda, M. Akiyama, “Nonreciprocal phase-shift composite right/left handed transmission lines using ferrite-rod-embedded substrate,” *IEEE Trans. on Magnetics*, vol. 45, no. 10, pp. 4203-4206, Oct. 2009.

[21] H. Kishimoto, T. Ueda, and Y. Kado, “Experimental demonstration of nonreciprocal phase-shift composite right/left handed transmission lines using a ferrite-rod-embedded substrate,” *IEEE Trans. on Magnetics*, vol. 47, no. 10, pp. 3724-3727, Oct. 2011.

[22] R.E. Collin, *Foundations for Microwave Engineering*, 2nd ed., IEEE Press - Wiley, 2001