# Circuit Analysis of Electromagnetic Band Gap (EBG) Structures

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***Abstract*—This paper presents a circuit analysis of electromagnetic band gap structures (EBG). Microwave transmission line theory is used to accurately analyze EBG structures. The results of this analysis are compared with previously published results, including more time-intensive full-wave analysis result, and this analysis is validated. The analysis applies to any type of EBG structure, including conventional periodic EBGs and progressive-dimensions EBG with broadband response.**

## I. Introduction

Electromagnetic band gap (EBG) structures are usually periodic structures [1], [2], [3], [4]. EBG structures are equivalent to a magnetic surface at the frequency of resonance and thus have very high surface impedance; this makes a tangential current element close to the EBG structure equivalent to two current elements oriented in the same direction without the EBG structure. This helps enhance the forward radiation instead of completely cancelling it, as indicated by the image theory. This makes the EBG structures useful when mounting an antenna close to ground, provided the antenna's currents are parallel to the EBG surface.

EBG structures are equivalent to a tank circuit. Equations (1) through (3) give the impedance, frequency of resonance and bandwidth respectively of an EBG structure [1], [2], [3], [4]. The bandwidth of the EBG structure is defined as the band of frequencies where the reflection phase is between $+90^0$ to $-90^0$.

$$Z_s = \frac{j\omega L}{1-\left(\frac{\omega}{\omega_0}\right)^2} \tag{1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{2}$$

$$BW = \frac{1}{120\pi}\sqrt{\frac{L}{C}} \tag{3}$$

## II. Analysis

Assuming a plane wave incidence and using transmission line theory, the reflection phase of the EBG structure can be found. Transmission line theory is used to find the complex reflection coefficient from the EBG surface and then the reflection phase is found. The reflection coefficient from a load $Z_L$ in a transmission line with a characteristic impedance of $Z_0$ is given by:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{4}$$

The reflection coefficient from a perfect electric conductor (PEC) is -1. The reflection coefficient from a perfect magnetic conductor (PMC) is +1. In both cases the magnitude is the same, but the phase is different. So, to accurately analyse the reflection phase, we need to come up with an approach that does not change the magnitude of the reflection with frequency, but changes the reflection phase with frequency. How can this be done when the reflection coefficient has difference of impedances in the numerator and sum of the impedances in the denominator? We know that a complex number and its complex conjugate have the same magnitude. Free space has real impedance, so we need to model the EBG structure as pure imaginary surface impedance. This is a good approximation as EBG structures have low loss, which implies the real part of the surface impedance is very small compared to imaginary part. This analysis assumes that the EBG structure is lossless. Let $Z_s$ be the surface impedance of the EBG structure. The reflection coefficient and the reflection phase can be calculated as:

$$Z_s = jX \tag{5}$$

$$\Gamma = \frac{Z_s - \eta_0}{Z_s + \eta_0} \tag{6}$$

$$|\Gamma| = 1 \tag{7}$$

$$\angle\Gamma = \pi - 2\tan^{-1}\left(\frac{x}{\eta_0}\right) \tag{8}$$

The bandwidth of EBG structure is defined as the band where the reflection phase is between $+90^0$ to $-90^0$. Using this with equation (8), we can find the required conditions on the surface impedance.

$$-90^0 \le \angle\Gamma \le +90^0$$

$$-90^0 \le 180^0 - 2\tan^{-1}\left(\frac{x}{\eta_0}\right) \le 90^0$$

$$-270^0 \le -2\tan^{-1}\left(\frac{x}{\eta_0}\right) \le -90^0$$

$$+135^0 \ge \tan^{-1}\left(\frac{x}{\eta_0}\right) \ge 45^0$$

$$-1 \ge \frac{X}{\eta_0} \ge +1$$

$$-\eta_0 \ge X \ge \eta_0 \tag{9}$$

Equation (9) shows the required conditions to operate in the bandwidth of the EBG. Figure 1 shows the relationship between surface impedance and the reflection phase.

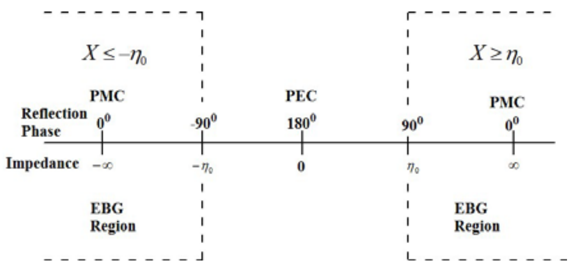


Fig. 1: Relationship between surface impedance and reflection phase of EBG structure.

The band edges of the EBG structure, as shown in Figure 1, are where the reflection phase is $+90^0$ (where the surface impedance is equal to $\eta_0$) and $-90^0$ (where the surface impedance is equal to $-\eta_0$). Let $\omega_1$ be the radian frequency where $X = \eta_0$

$$Z_s = \frac{j\omega L}{1 - \left(\frac{\omega}{\omega_0}\right)^2}$$

$$Z_s = jX$$

$$\eta_0 = \frac{\omega_1 L}{1 - \left(\frac{\omega_1}{\omega_0}\right)^2}$$

$$1 - \left(\frac{\omega_1}{\omega_0}\right)^2 = \frac{\omega_1 L}{\eta_0}$$

$$\omega_0^2 - \omega_1^2 = \frac{\omega_0^2 L \omega_1}{\eta_0} \tag{10}$$

Let $\omega 2$ be the radian frequency where $X = -\eta 0$

$$-\eta_0 = \frac{\omega_2 L}{1 - \left(\frac{\omega_2}{\omega_0}\right)^2}$$

$$1 - \left(\frac{\omega_2}{\omega_0}\right)^2 = -\frac{\omega_2 L}{\eta_0}$$

$$\omega_0^2 - \omega_2^2 = -\frac{\omega_0^2 L \omega_2}{\eta_0} \tag{11}$$

Subtracting equations (10) and (11) we get:

$$\omega_2^2 - \omega_1^2 = \frac{\omega_0^2 L}{\eta_0}(\omega_1 + \omega_2)$$

$$(\omega_1 + \omega_2)(\omega_2 - \omega_1) - \frac{\omega_0^2 L}{\eta_0}(\omega_1 + \omega_2) = 0$$

$$(\omega_1 + \omega_2)\left(\omega_2 - \omega_1 - \frac{\omega_0^2 L}{\eta_0}\right) = 0$$

But, $(\omega_1 + \omega_2)$ cannot be zero, so the only possible solution is:

$$\omega_2 - \omega_1 = \frac{\omega_0^2 L}{\eta_0}$$

Where $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\text{Bandwidth} = \frac{\omega_2 - \omega_1}{\omega_0} = \frac{\omega_0 L}{\eta_0} = \frac{1}{\eta_0}\sqrt{\frac{L}{C}} \tag{12}$$

This analysis can be performed for any desired reflection phase (using equation 8) to get the required conditions on the surface impedance of the EBG structure. This paper uses the reflection phase condition to be within $+90^0$ to $-90^0$ as this condition is widely used in defining the bandwidth of EBG structures, and these results are used in comparing the previously published results. This analysis represents any EBG structure as surface impedance and gives the required conditions on the surface impedance to get the desired range of reflection phase. Different types of EBG structures differ in the way they are implemented to get the desired surface impedance. Thus this analysis applies to any EBG structure.

## III. Validation

We see that equation (12) agrees with the published bandwidth given in equation (3), hence validating this analysis. Using this analysis the reflection phase of a uniform EBG structure is found using Matlab [6] and compared with the published results in [5], which were obtained using FDTD/PBC for the case where patch width is 3 mm, gap between the patches is 0.5 mm, via height is 1 mm and the relative permittivity of the board is 2.20. All the dimensions in figures 2 through 5 are given as a fraction of wavelength at 12 GHz. To find the surface impedance, closed form equations for the capacitance and inductance of the EBG structure are used [4]. Figure 2 shows the comparison of the effect of the patch width on the reflection phase, while Figure 3 shows the comparison of the effect of the gap width on the reflection phase. Figure 4 shows the comparison of the effect of the substrate height on the reflection phase, while Figure 5 shows the comparison of the effect of the substrate permittivity on the reflection phase. Figures 2 through 5 show good agreement and validates this analysis.

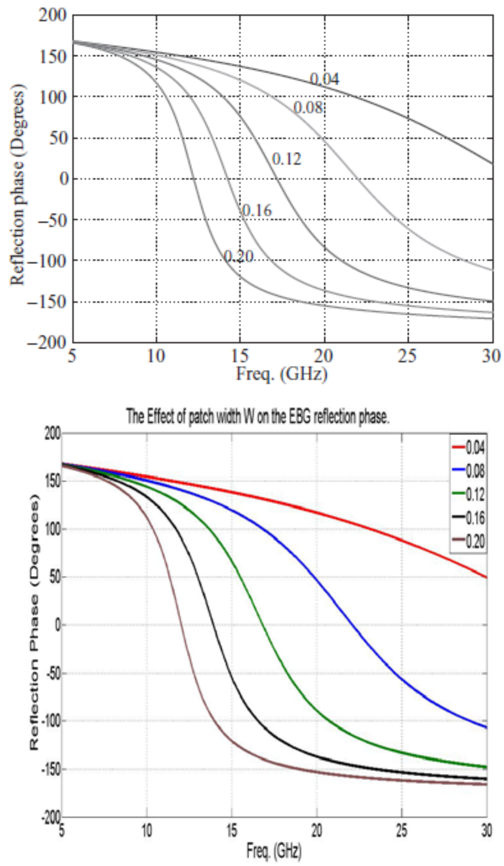


Fig. 2: Comparison of the effect of the patch width on the reflection phase: results published in [5] using FDTD/PBC, top, versus present analysis, bottom

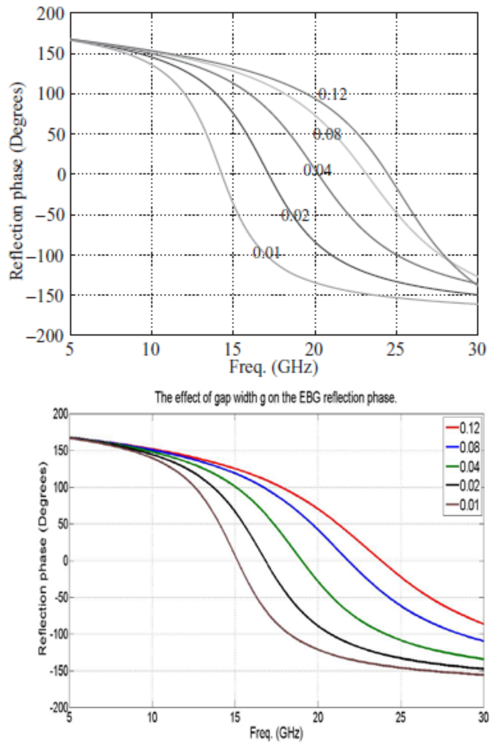


Fig. 3: Comparison of the effect of the gap width on the reflection phase: results published in [5] using FDTD/PBC, top, versus present analysis, bottom

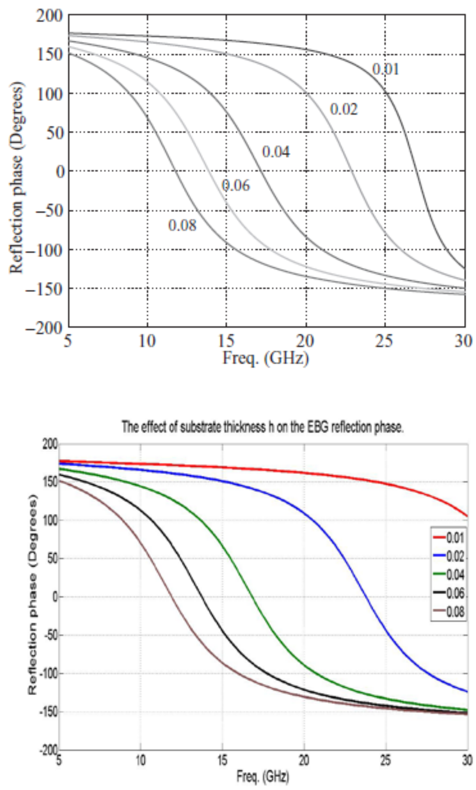


Fig. 4: Comparison of the effect of the substrate height on the reflection phase: results published in [5] using FDTD/PBC, top, versus present analysis, bottom

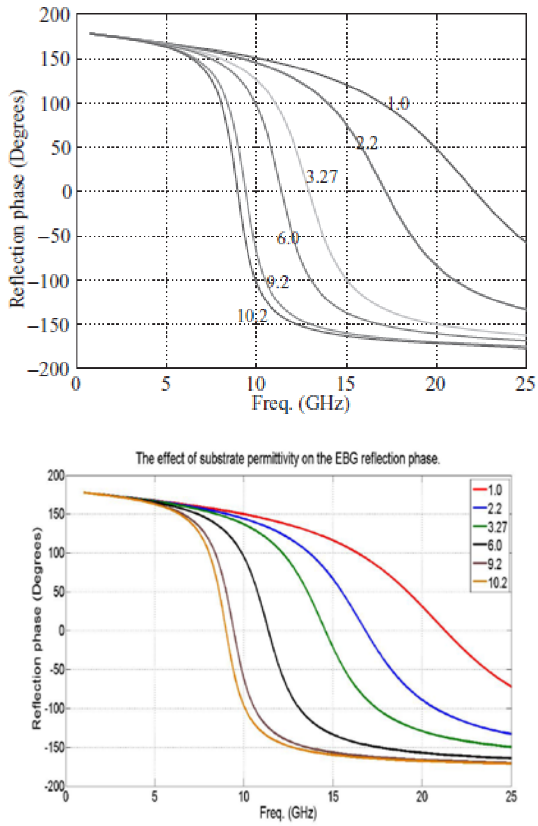


Fig. 5: Comparison of the effect of the substrate permittivity on the reflection phase: results published in [5] using FDTD/PBC, top, versus present analysis, bottom

## IV. CONCLUSION

EBG structures are helpful when mounting an antenna near a ground plane; they are compact in size and have low loss. This paper showed the analysis and characterization of EBG structures using transmission line theory, and the required conditions for a desired range of reflection phase are calculated using a closed-form equation approach and the results are compared with the published results and they agree very well.

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