Analysis of a Polarization Converter Using a Full-Vectorial Fundamental Alternating-Direction Implicit Beam-Propagation Method

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***Abstract*—A simplified algorithm for the full-vectorial alternating-direction implicit beam-propagation method (FV-ADI-BPM) is developed using a fundamental scheme. The resultant equations are reduced to those with only two spatial derivatives for a polarization coupling effect in the right-hand sides, while the conventional FV-ADI-BPM includes four derivatives for a specific polarization and four for a polarization coupling effect. We assess the performance, when analyzing a z-variant structure with a polarization coupling effect. The present method is found to yield the result identical to that from the conventional method, reducing the computation time by about 35%.**

I. Introduction

Polarization converters have many applications, including polarization controllers, switches, and polarization diversity heterodyne receivers [1], [2]. To analyze the polarization converters, we have to use a full-vectorial (FV) method that can take polarization coupling effects into account. One of the most frequently used numerical methods for the FV analysis is the beam-propagation method (BPM) [3]-[6] based on the alternating-direction implicit (ADI) method [7], [8]. The ADI-BPM has become the de facto standard technique in commercial software packages due to its high efficiency.

Recently, a fundamental scheme has been developed for efficient implementations of implicit finite-difference time-domain (FDTD) methods [9], [10], such as the ADI-FDTD [11], [12] and the locally one-dimensional (LOD) FDTD. With the fundamental scheme, we obtain matrix-operator-free forms in the right-hand sides (RHSs) of the basic equations for the ADI- and LOD-FDTDs. We have applied the fundamental scheme to the frequency-dependent LOD-FDTD [13] and the body-of-revolution LOD-FDTD [14]. Note that the formulation of the ADI-FDTD parallels that of the ADI-BPM. We were therefore motivated to apply the fundamental scheme to efficient implementation of the semi-vectorial (SV) ADI-BPM [16]. Unfortunately, the SV-ADI-BPM cannot treat polarization coupling effects.

The purpose of this article is to develop a simple FV-ADI-BPM based on the fundamental scheme and investigate its effectiveness for the analysis of devices with a polarization coupling effect. First, we present the resultant FV equations based on the fundamental scheme, in comparison with those of the conventional equations. Next, to investigate the ability to treat z-variant structures with polarization coupling effects, we analyze a mode-evolution-based polarization converter [1]. Finally, we mention the efficiency improvement of the proposed FV-ADI-BPM.

II. FV Equations Based on the Fundamental Scheme

The coupled Fresnel equations of the conventional FV-ADI-BPM [5] can be arranged to

$$\left(1-\frac{\Delta z}{2}A_y\right)E_x^{l+1/2}=\left(1+\frac{\Delta z}{2}A_x\right)E_x^l+\frac{\Delta z}{2}CE_y^l \tag{1a}$$

$$\left(1-\frac{\Delta z}{2}B_y\right)E_y^{l+1/2}=\left(1+\frac{\Delta z}{2}B_x\right)E_y^l+\frac{\Delta z}{2}DE_x^{l+1/2} \tag{1b}$$

for the first step and

$$\left(1-\frac{\Delta z}{2}B_x\right)E_y^{l+1}=\left(1+\frac{\Delta z}{2}B_y\right)E_y^{l+1/2}+\frac{\Delta z}{2}DE_x^{l+1/2} \tag{2a}$$

$$\left(1-\frac{\Delta z}{2}A_x\right)E_x^{l+1}=\left(1+\frac{\Delta z}{2}A_y\right)E_x^{l+1/2}+\frac{\Delta z}{2}CE_y^{l+1} \tag{2b}$$

for the second step, in which A_x, A_y, B_x, and B_y represent spatial second derivatives for a specific polarization, including phase variation terms, and C and D represent spatial mixed derivatives for a polarization coupling effect. The detailed definitions for these notations are given in [5].

We here resort to the fundamental scheme for efficient implementation of (1) and (2). Introducing the auxiliary variables v_x, v_y, ϕ_x and ϕ_y, we finally obtain the simplified equations

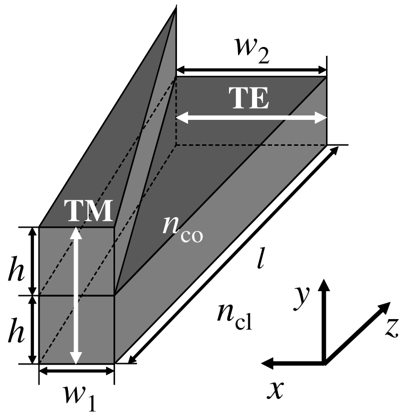

Fig. 1. Schematic view of a mode-evolution-based polarization converter.

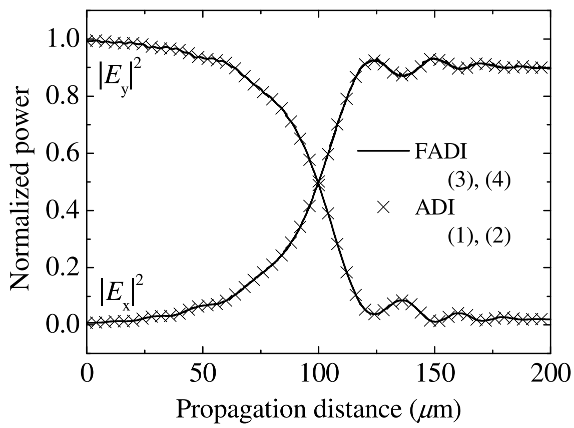

Fig. 2. Normalized powers of E_y and E_x fields.

of the fundamental ADI-BPM (FADI-BPM) as follows:

$$v_x^l = 2E_x^l - v_x^{l-1/2} + \phi_y^l \tag{3a}$$

$$\left(1 - \frac{\Delta z}{2}A_y\right)E_x^{l+1/2} = v_x^l \tag{3b}$$

$$\phi_x^{l+1/2} = \frac{\Delta z}{2}DE_x^{l+1/2} \tag{3c}$$

$$v_y^l = 2E_y^l - v_y^{l-1/2} + \phi_x^{l+1/2} \tag{3d}$$

$$\left(1 - \frac{\Delta z}{2}B_y\right)E_y^{l+1/2} = v_y^l \tag{3e}$$

for the first step and

$$v_y^{l+1/2} = 2E_y^{l+1/2} - v_y^l + \phi_x^{l+1/2} \tag{4a}$$

$$\left(1 - \frac{\Delta z}{2}B_x\right)E_y^{l+1} = v_y^{l+1/2} \tag{4b}$$

$$\phi_y^{l+1} = \frac{\Delta z}{2}CE_y^{l+1} \tag{4c}$$

$$v_x^{l+1/2} = 2E_x^{l+1/2} - v_x^l + \phi_y^{l+1} \tag{4d}$$

$$\left(1 - \frac{\Delta z}{2}A_x\right)E_x^{l+1} = v_x^{l+1/2} \tag{4e}$$

for the second step. For the initial fields E_x^0 and E_y^0, the algorithm requires the input initialization as

$$v_x^{-1/2} = \left(1 - \frac{\Delta z}{2}A_x\right)E_x^0 \tag{5}$$

$$v_y^{-1/2} = \left(1 - \frac{\Delta z}{2}B_x\right)E_y^0. \tag{6}$$

It is worth mentioning that (3) and (4) include only two spatial derivatives D and C for the polarization coupling effect in the RHSs, while the conventional method of (1) and (2) has four derivatives for the specific polarization and four for the polarization coupling effect in the RHSs. This fact leads to quite efficient implementation of the FV-BPM with fewer calculations of the spatial derivatives.

III. Numerical Results

To investigate the effectiveness of the FV-FADI-BPM, we analyze the mode-evolution-based polarization converter [1] shown in Fig. 1. The material and configuration parameters are as follows: $n_{\mathrm{co}} = 2.2, n_{\mathrm{cl}} = 1.445, h = w_1 = 0.4$ μm, $w_2 = 0.8$ μm and $l = 200$ μm. The numerical parameters are $\Delta x = \Delta y = \Delta z = 0.02$ μm.

Fig. 2 shows the normalized powers of E_y and E_x fields at $\lambda = 1.55$ μm as a function of propagation distance. The fundamental TM mode is excited at the input port. As can be seen in Fig. 2, the TM mode (the E_y field) is converted into the TE mode (the E_x field). The calculated FADI results from (3) and (4) are found to show no appreciable difference at the scale of Fig. 2, relative to the conventional ADI results from (1) and (2). In fact, the difference of the two results is within the order of 10^{-13}, demonstrating they are practically equivalent even for the z-variant structure with the polarization coupling effect. It is worth mentioning that the computation time of the FADI is reduced by 35% of that of the conventional method.

IV. Conclusion

We have developed an efficient FV-ADI-BPM based on the fundamental scheme. The resultant equations are shown to be much simpler than those of the conventional FV-ADI-BPM, i.e., only two derivatives for a polarization coupling effect are calculated in the RHSs. The effectiveness of the developed method is investigated through the analysis of a polarization converter. Even for a z-variant structure, the developed method yields results identical to those of the conventional method, with reduced computation time.

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