Scattering by Buried PEC Cylinders from an Arbitrary 2D Illumination

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***Abstract*—Cylindrical Wave Approach is used to solve scattering by buried objects from an arbitrary illumination field. The incident field is expressed by means of a spectral integral which may represent, e.g., a cylindrical wave emitted by a line source, or a Gaussian beam, used to model a finite-width illumination field.**

I. Introduction

Detection of buried objects is a topic of wide interest in the frame of environmental sensing through radar techniques [1]-[2]. Possible applications are the mapping of buried pipes and utilities, detection of voids and cavities within the ground, surveying of roads and pavements, and analysis of geological structures. Analytical and numerical techniques solving direct scattering problems may offer a useful tool for the analysis of the measured data.

Cylindrical Wave Approach (CWA) is an analytical-numerical method developed to solve two-dimensional scattering from buried objects. The method has been developed in [3], for cylinders buried under a flat interface in a semi-infinite medium. It has been extended to a layered geometry in [4]-[5]. In [6], scatterers placed below a rough interface were considered. In the abovementioned works, the excitation field was a plane wave; its interaction with one or more interfaces was dealt with, in absence of the cylinders, by means of Fresnel coefficients. Scattered fields radiated by the cylinders were expanded into cylindrical waves, and the concept of plane-wave spectrum of a cylindrical wave was used to describe the interaction with flat or rough boundaries.

The spectral analysis may be extended to successfully solve scattering from an arbitrary illumination field. In [7], the incident field radiated from a line-source, modelled by a cylindrical wave, has been expressed by means of a plane wave spectrum. Its interaction with a planar interface has been solved evaluating a Reflected and Transmitted Cylindrical Wave originated from the excitation one. A more general and practical illumination field may also be dealt with, through a suitable definition of the plane-wave spectrum. In this paper, the scattering from a two-dimensional and finite-width illumination given by a Gaussian Beam is proposed. The theory developed to solve line-source scattering is recalled, and its generalization to the Gaussian-beam excitation is shown. The theoretical analysis based on CWA is described in Section II. Numerical results, obtained from an accurate evaluation of spectral integrals, are presented in Section IV.

II. Theoretical Analysis

A two-dimensional scattering problem is solved, with a set of N perfectly conducting cylinders buried below a flat interface of separation between two semi-infinite media, which are homogeneous, isotropic, and lossless. Normalized coordinates (O, ξ, ζ) are used, being $\xi = k_0 x$ and $\zeta = k_0 z$, with k_0 the vacuum wavenumber. Cylinders have radii α_p, centres in (χ_p, η_p), with p=1, …, N, and their axes are parallel to the y-axis.

The incident field may be either a line-source with centre in (χ_L, η_L), as in Fig. 1, or a Gaussian beam, with centre in (χ_0, η_0) and width w_0, as in Fig. 2. The total field in each medium is described by a scalar function $V(\xi, \zeta)$, which stands for the electric field E_y, in the TM or E polarization state, or for the magnetic field H_y, in the TE or H one. Such a function is decomposed into several field contributions which are due to the interaction of the incident field, i.e. the scalar function V_i, with the interface and the cylinders. In particular, the reflected and transmitted fields V_r and V_t, respectively, are defined: they describe the interaction between the incident field and the interface, in the absence of the cylinders. Due to the interaction of the transmitted field with the buried objects, scattered fields are also defined (Fig. 3), i.e. the field V_s scattered by the cylinders in Medium 1, and the fields V_{sr} and V_{st}, i.e. the scattered-reflected and scattered-transmitted field, respectively, dealing with reflection and transmission of V_s by the interface.

A. Incident Field

The line source depicted in Fig. 1 radiates a *y*-directed field expressed through a Hankel function of first kind and zero order, with argument proportional to the distance from the source to the observation point:

$$V_{\mathrm{i}}(\xi,\zeta)=-V_0 H_0^{(1)}\left[\sqrt{(\xi-\chi_L)^2+(\zeta-\eta_L)^2}\right] \quad (1)$$

Reflection and transmission of the field $V_{\mathrm{i}}(\xi,\ \zeta)$ through the planar interface in ξ=0 can be evaluated expressing the cylindrical function $CW_0 = H_0^{(1)}$ as a plane-wave spectrum [8]

$$CW_0(\xi_L,\zeta_L)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F_0(\xi_L,\mathrm{n}_{\|})e^{i\mathrm{n}_{\|}\zeta_L}\mathrm{dn}_{\|} \quad (2)$$

with $n_{\|}$ the parallel component with respect to the z-axis of the generic plane wave of the spectrum. The explicit expression of the spectrum F_0, evaluated in $\xi_L > 0$, is

$$F_0(\xi_L,\mathrm{n}_{\|})=\frac{2e^{i\xi_L\sqrt{1-\mathrm{n}_{\|}^2}}}{\sqrt{1-\mathrm{n}_{\|}^2}} \quad (3)$$

To obtain the reflected field $V_{\mathrm{r}}(\xi,\ \zeta)$, the generical plane wave $F_0(\xi_L,\mathrm{n}_{\|})e^{i\mathrm{n}_{\|}\zeta_L}$ of the spectrum (3) is evaluated on the plane $\xi_{\mathrm{L}}=-\chi_{\mathrm{L}}$ and multiplied by the Fresnel reflection coefficient $\Gamma_{01}(\mathrm{n}_{\|})$. The Reflected Cylindrical Function of zero-th order $RW_0\ (\xi,\zeta)$ is derived

$$RW_0\ (\xi,\zeta)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\Gamma_{01}(\mathrm{n}_{\|})F_0(-\xi-\chi_L,\mathrm{n}_{\|})e^{i\mathrm{n}_{\|}(\zeta-\eta_L)}\mathrm{dn}_{\|} \quad (4)$$

which allows to define the Reflected Field as follows

$$V_{\mathrm{r}}(\xi,\zeta)=-V_0 RW_0\ (\xi,\zeta) \quad (5)$$

In a similar way, the transmitted field $V_{\mathrm{t}}(\xi,\ \zeta)$ is expressed through a Transmitted Cylindrical Function of zero-th order

$$TW_0\ (\xi,\zeta;-\chi_L)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{T}_{01}(\mathrm{n}_{\|})F_0(-\chi_L,\mathrm{n}_{\|})e^{i\mathrm{n}_1\sqrt{1-(\mathrm{n}_{\|}/\mathrm{n}_1)^2}\xi}\,e^{i\mathrm{n}_{\|}(\zeta-\eta_L)}\mathrm{dn}_{\|} \quad (6)$$

thus obtaining

$$V_{\mathrm{t}}(\xi,\zeta)=-V_0 TW_0\ (\xi,\zeta) \quad (7)$$

As the final solution of the scattering problem is obtained imposing boundary conditions on the cylinders' surface, an expression of the transmitted field in (7) as a function of polar coordinates centered on the p-th cylinder's axis has to be employed. For this purpose, equation (7) can be written in the following form

$$V_{\mathrm{t}}\ (\xi_{\mathrm{p}},\zeta_{\mathrm{p}})=-V_0\sum_{\ell=-\infty}^{+\infty} i^{\ell}J_{\ell}(\mathrm{n}_1\rho_{\mathrm{P}})e^{i\ell\theta_{\mathrm{P}}}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{T}_{01}(\mathrm{n}_{\|})F_0(-\chi_L,\mathrm{n}_{\|}) \times e^{i\mathrm{n}_1\sqrt{1-(\mathrm{n}_{\|}/\mathrm{n}_1)^2}\chi_{\mathrm{P}}}\,e^{i\mathrm{n}_{\|}(\eta_{\mathrm{P}}-\eta_L)}\,e^{-i\ell\arctan\left[\mathrm{n}_{\|}/\sqrt{(\mathrm{n}_1)^2-(\mathrm{n}_{\|})^2}\right]}\mathrm{dn}_{\|} \quad (8)$$

where the expansion of a plane-wave into Bessel functions has been used. The field in (7) can also be written in a more compact form

$$V_{\mathrm{t}}\ (\xi_{\mathrm{p}},\zeta_{\mathrm{p}})=-V_0\sum_{\ell=-\infty}^{+\infty} i^{\ell}J_{\ell}(\mathrm{n}_1\rho_{\mathrm{P}})e^{i\ell\theta_{\mathrm{P}}}TW_{0,\ell}(\chi_{\mathrm{P}},\eta_{\mathrm{P}}-\eta_L,-\chi_L) \quad (9)$$

The incident field reported in 1 may be expressed in a more general way, dealing with an arbitrary two-dimensional incident field, forming an angle φ with the positive ξ-axis:

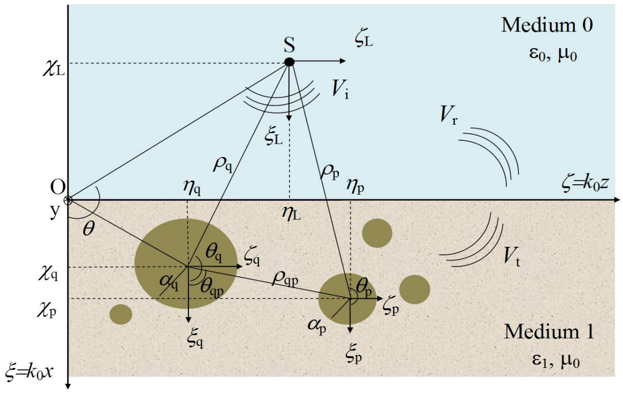

Fig. 1. Geometry of line-source scattering.

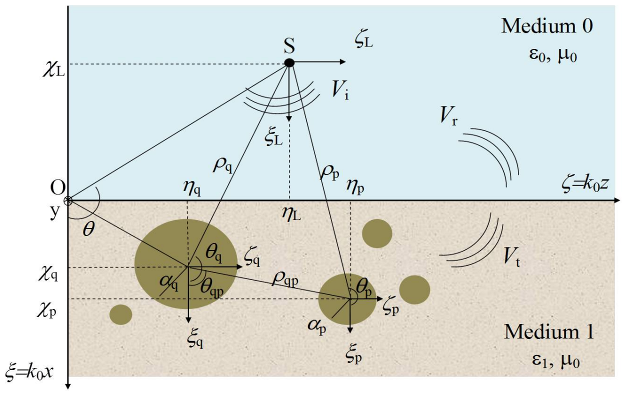

Fig. 2. Geometry of Gaussian-beam scattering.

$$V_{\mathrm{i}}(\xi,\zeta)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{V}_0^{\mathrm{i}}(\mathrm{n}_{\|})e^{i[\mathrm{n}_{\perp}(\xi-\chi_0)+\mathrm{n}_{\|}(\zeta-\eta_0)]}\left(\cos\varphi+\frac{\mathrm{n}_{\|}}{\mathrm{n}_{\perp}}\sin\varphi\right)\mathrm{dn}_{\|} \quad (10)$$

In (10), $\hat{V}_0^{\mathrm{i}}(\mathrm{n}_{\|})$ is the angular spectrum of the incident field. When the illumination field is a Gaussian Beam of width w_0, the explicit expression of the spectrum is

$$\hat{V}_0^{\mathrm{i}}(\mathrm{n}_{\|})=V_0\sqrt{\pi}\mathrm{kw}_0 e^{-\left[\mathrm{w}_0(\mathrm{n}_{\|}\cos\varphi-\mathrm{n}_{\perp}\sin\varphi)/2\right]^2} \quad (11)$$

With an approach analogous to the one applied to the line-source field, the Reflected and Transmitted Field can be obtained, respectively as

$$V_{\mathrm{r}}(\xi,\zeta)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\Gamma_{01}(\mathrm{n}_{\|})\hat{V}_0^{\mathrm{i}}(\mathrm{n}_{\|}) \times e^{i[-\mathrm{n}_{\perp}(\xi-\chi_0)+\mathrm{n}_{\|}(\zeta-\eta_0)]}\left(\cos\varphi+\frac{\mathrm{n}_{\|}}{\mathrm{n}_{\perp}}\sin\varphi\right)\mathrm{dn}_{\|} \quad (12)$$

$$V_{\mathrm{t}}\ (\xi_{\mathrm{p}},\zeta_{\mathrm{p}};-\chi_0,-\eta_0)= -V_0\sum_{\ell=-\infty}^{+\infty} i^{\ell}J_{\ell}(\mathrm{n}_1\rho_{\mathrm{P}})e^{i\ell\theta_{\mathrm{P}}}TW_{\ell}^{GB}(\chi_{\mathrm{P}},\eta_{\mathrm{P}}-\eta_0,-\chi_0) \quad (13)$$

In (13), TW_{ℓ}^{GB} is a transmitted wave of order ℓ, relevant to a Gaussian incident beam, and it is given by

$$TW_{\ell}^{GB}(\chi_{\mathrm{p}},\eta_{\mathrm{p}}-\eta_0;-\chi_0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}T_{01}(\mathrm{n}_{\parallel})\hat{V}_0^{\mathrm{i}}(\mathrm{n}_{\parallel})e^{i[-\chi_0\sqrt{1-(\mathrm{n}_{\parallel})^2}+(\eta_p-\eta_0)\mathrm{n}_{\parallel}]}$$
$$\times e^{-i\ell\arctan\left[\mathrm{n}_{\parallel}/\sqrt{(\mathrm{n}_1)^2-(\mathrm{n}_{\parallel})^2}\right]}e^{i\mathrm{n}_1\chi_{\mathrm{p}}\sqrt{1-(\mathrm{n}_{\parallel}/\mathrm{n}_1)^2}}\left[\cos\varphi+\frac{\mathrm{n}_{\parallel}}{\sqrt{1\text{-}(\mathrm{n}_{\parallel})^2}}\sin\varphi\right]\mathrm{dn}_{\parallel} \tag{14}$$

B. Scattered Fields

The transmitted field V_{t} impinging on the cylinders' surface in Medium 1 excites a scattered field $V_{\mathrm{s}}(\xi,\zeta)$. It is expressed as an expansion into cylindrical functions $CW_{\mathrm{m}}(\mathrm{n}_1\xi_{\mathrm{q}},\mathrm{n}_1\zeta_{\mathrm{q}})=H_{\mathrm{m}}^{(1)}(\mathrm{n}_1\rho_{\mathrm{q}})e^{i\ell\theta_{\mathrm{q}}}$, where $H_{\mathrm{m}}^{(1)}(\mathrm{n}_1\rho_{\mathrm{q}})$ is the first-kind Hankel function of order m, with unknown coefficients c_{qm}

$$V_{\mathrm{s}}(\xi,\zeta)=V_0\sum_{\mathrm{q=1}}^{\mathrm{N}}\sum_{\mathrm{m}=-\infty}^{+\infty}c_{\mathrm{qm}}CW_{\mathrm{m}}(\mathrm{n}_1\xi_{\mathrm{q}},\mathrm{n}_1\zeta_{\mathrm{q}}) \tag{14}$$

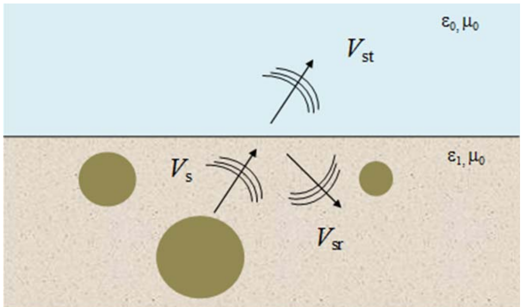

Fig. 3. Scattered fields.

Making use of the Addition Theorem of Hankel functions, the field in (14) can be written in a reference frame centred on the q-th cylinder (q=1, …, N), as follows

$$V_{\mathrm{s}}(\xi_{\mathrm{q}},\zeta_{\mathrm{q}})=V_0\sum_{\ell=-\infty}^{\infty}J_{\ell}(\mathrm{n}_1\rho_{\mathrm{p}})e^{i\ell\theta_{\mathrm{p}}}$$
$$\times\sum_{\mathrm{q=1}}^{\mathrm{N}}\sum_{m=-\infty}^{+\infty}i^{\mathrm{m}}c_{\mathrm{qm}}\left[CW_{\mathrm{m}-\ell}(\mathrm{n}_1\xi_{\mathrm{qp}},\mathrm{n}_1\zeta_{\mathrm{qp}})(1-\delta_{\mathrm{qp}})+\frac{H_{\ell}^{(1)}(\mathrm{n}_1\rho_{\mathrm{p}})}{J_{\ell}(\mathrm{n}_1\,\rho_{\mathrm{p}})}\delta_{\mathrm{qp}}\delta_{\ell\mathrm{m}}\right] \tag{15}$$

The scattered-reflected field $V_{\mathrm{sr}}(\xi,\zeta)$ is derived expanding the cylindrical function CW_{m} into a Fourier spectrum F_{m}

$$CW_{\mathrm{m}}(\xi,\zeta)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}F_{\mathrm{m}}(\xi,\mathrm{n}_{\parallel})e^{i\mathrm{n}_{\parallel}\zeta}\mathrm{dn}_{\parallel} \tag{16}$$

The spectrum F_{m} is given by [8]

$$F_{\mathrm{m}}(\xi,\mathrm{n}_{\parallel})=\frac{2e^{i|\xi|\sqrt{1-\mathrm{n}_{\parallel}^2}}}{\sqrt{1-\mathrm{n}_{\parallel}^2}}\begin{cases}e^{-i\mathrm{m}\arccos\mathrm{n}_{\parallel}}, & \xi\geq 0\\ e^{i\mathrm{m}\arccos\mathrm{n}_{\parallel}}, & \xi\leq 0\end{cases} \tag{17}$$

Reflected Cylindrical Waves, which are the basis functions of the scattered-reflected field, are obtained evaluating the reflection on each plane wave of the spectrum (17):

$$RW_{\mathrm{m}}(n_1\xi,n_1\zeta)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\Gamma_{10}(n_{\parallel})F_{\mathrm{m}}(n_1\xi,n_{\parallel})e^{in_1n_{\parallel}\zeta}dn_{\parallel} \tag{18}$$

An expression of the Cylindrical Wave (18), which is a function of polar coordinates centered on the p-th cylinder, is used when boundary conditions are imposed. The resulting expression of the scattered-reflected field is

$$V_{\mathrm{sr}}(\xi,\zeta)=V_0\sum_{\ell=-\infty}^{+\infty}J_{\ell}(\mathrm{n}_1\rho_{\mathrm{p}})e^{i\ell\theta_{\mathrm{p}}}$$
$$\times\sum_{\mathrm{q=1}}^{\mathrm{N}}\sum_{\mathrm{m}=-\infty}^{+\infty}c_{\mathrm{qm}}RW_{\mathrm{m}+\ell}\left[-\mathrm{n}_1(\chi_{\mathrm{q}}+\chi_{\mathrm{p}}),\mathrm{n}_1(\eta_{\mathrm{q}}-\eta_{\mathrm{p}})\right] \tag{19}$$

In (19), the expansion of a plane wave into Bessel functions has been used.

In a similar way, Transmitted Cylindrical Waves are defined

$$TW_{\mathrm{m}}(\xi,\zeta,\chi)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}T_{10}(n_{\parallel})F_{\mathrm{m}}(-n_1\chi,n_{\parallel})$$
$$\times e^{-i\sqrt{1-(n_1n_{\parallel})^2}(\xi+\chi)}e^{in_1n_{\parallel}\zeta}dn_{\parallel} \tag{20}$$

as basis functions of the scattered-transmitted field, which is defined as follows

$$V_{\mathrm{st}}(\xi,\zeta)=V_0\sum_{\mathrm{q=1}}^{\mathrm{N}}\sum_{\mathrm{m}=-\infty}^{+\infty}c_{\mathrm{qm}}TW_{\mathrm{m}}(\xi_{\mathrm{q}},\zeta_{\mathrm{q}},\chi_{\mathrm{q}}) \tag{21}$$

C. Solving System

Boundary conditions of zero electric field on the cylinders' surface are imposed, which are

$$V_{\mathrm{t}}+V_{\mathrm{s}}+V_{\mathrm{sr}}\big|_{\rho_{\mathrm{p}}=\alpha_{\mathrm{p}}}=0 \tag{22}$$

in TM polarization, and

$$\frac{\partial}{\partial\rho_{\mathrm{p}}}(V_{\mathrm{t}}+V_{\mathrm{s}}+V_{\mathrm{sr}})\big|_{\rho_{\mathrm{p}}=\alpha_{\mathrm{p}}}=0 \tag{23}$$

in TE polarization.

From boundary conditions (22) and (23), after some algebra, a linear system in the unknown expansion coefficients c_{qm} is derived

$$\sum_{\mathrm{q=1}}^{\mathrm{N}}\sum_{\mathrm{m}=-\infty}^{+\infty}\mathrm{A}_{\mathrm{qp}}^{\ell\mathrm{m(TM,TE)}}\mathrm{c}_{\mathrm{qm}}=\mathrm{B}_{\mathrm{p}}^{\ell\mathrm{(TM,TE)}}$$
$$\mathrm{p}=1,...,\mathrm{N} \tag{24}$$
$$\ell=0,\pm1,...,\pm\infty$$

being

$$\mathrm{A}_{\mathrm{qp}}^{\ell\mathrm{m(TM,TE)}}=i^{-\ell}\mathrm{G}_{\ell}^{\mathrm{(TM,TE)}}(\mathrm{n}_1\rho_{\mathrm{p}})\Big\{CW_{\mathrm{m}-\ell}(\mathrm{n}_1\xi_{\mathrm{qp}},\mathrm{n}_1\zeta_{\mathrm{qp}})(1-\delta_{\mathrm{qp}})+$$
$$+RW_{\mathrm{m}+\ell}\left[-\mathrm{n}_1(\chi_{\mathrm{q}}+\chi_{\mathrm{p}}),\mathrm{n}_1(\eta_{\mathrm{q}}-\eta_{\mathrm{p}})\right]+\frac{\delta_{\mathrm{qp}}\delta_{\ell\mathrm{m}}}{\mathrm{G}_{\ell}^{\mathrm{(TM,TE)}}(\mathrm{n}_1\rho_{\mathrm{p}})}\Big\} \tag{25}$$

and

$$\mathrm{B}_{\mathrm{p}}^{\ell\mathrm{(TM,TE)}}=-\mathrm{G}_{\ell}^{\mathrm{(TM,TE)}}(\mathrm{n}_1\rho_{\mathrm{p}})TW_{0,\ell}(\chi_{\mathrm{p}},\eta_{\mathrm{p}}-\eta_{\mathrm{L}},-\chi_{\mathrm{L}}) \tag{26}$$

when the incident field V_{i} is the line-source excitation (1), whereas it is

$$\mathrm{B}_{\mathrm{p}}^{\ell(\mathrm{TM,TE})} = -\mathrm{G}_{\ell}^{(\mathrm{TM,TE})}(\mathrm{n}_1\rho_{\mathrm{p}}) TW_{\ell}^{GB}(\chi_{\mathrm{p}}, \eta_{\mathrm{p}} - \eta_0; -\chi_0) \quad (27)$$

when V_{i} is the two-dimensional one given in (10).

In (25), (26), and (27), it is $\mathrm{G}_{\ell}^{(\mathrm{TM})}(\cdot) = J_{\ell}(\cdot)/\mathrm{H}_{\ell}^{(1)}(\cdot)$ and $\mathrm{G}_{\ell}^{(\mathrm{TE})}(\cdot) = J'_{\ell}(\cdot)/\mathrm{H}_{\ell}^{(1)}{}'(\cdot)$.

III. Numerical Analysis and Results

A. Numerical Analysis

Spectral integrals relevant to Cylindrical Waves require an accurate numerical evaluation. In particular, a general algorithm which takes into account the infinite integration domain, and deals with highly oscillating functions has been developed in [9] to solve equations (18) and (20), and it can be directly applied to solve equation (4). As to spectral integrals (8), (10), (12) and (13), a generalization of the algorithm developed in [9] has been performed, on the basis of the different terms defined in the integrand. Three main subintervals of integration have been detected, corresponding to totally-evanescent plane waves, homogeneous plane waves, and homogeneous-evanescent plane waves. The homogeneous part of the spectrum, which is the most oscillating, has been solved by means of an adaptive integration technique. Suitable decompositions have been introduced on evanescent and homogeneous-evanescent spectra, and low-order Gauss-Legendre formulas have been used. Solution of system (24) has been performed truncating the order of cylindrical waves to a finite number, according to the rule $\mathrm{M_t}=3\alpha_{\max}n_1$ [10], where $\alpha_{\max}$ is the radius of the largest cylinder. Such a rule turned out to be a good compromise between accuracy and computational heaviness.

B. Results

In Fig. 4 an example of results is reported. Near field scattering by a perfectly-conducting circular cross-section cylinder with normalized radius $\alpha = 1$, and center in (2.57, 0) is solved. The illumination is given by a line source centered in χ_{L}=1.29, η_{L}=0. The refraction index of Medium 1 is 2. Results are found for normal incidence and in TM polarization, along a line in ξ = -1. In the same plot, the possibility to characterize a buried scatterer with arbitrary cross-section is also shown. The cylinder has been simulated by N small cylinders of radius α_{i} (i=1, …, N) placed along its border, with dimensions satisfying the Same-Area Rule [8]. A fast convergence to the exact solution can be obtained, as the number N is increased.

IV. Conclusions

The fundamentals of CWA and its application to two-dimensional sources have been introduced. The method is based on a general approach capable to deal with general two dimensional scattering problems, which can be solved in the spectral domain by means of expansion into suitable Cylindrical Waves. The accurate evaluation of the involved spectral integrals allows to get results both in near and far-field region.

A future extension of the method will be the time-domain solution of scattering from buried objects by an arbitrary pulsed source.

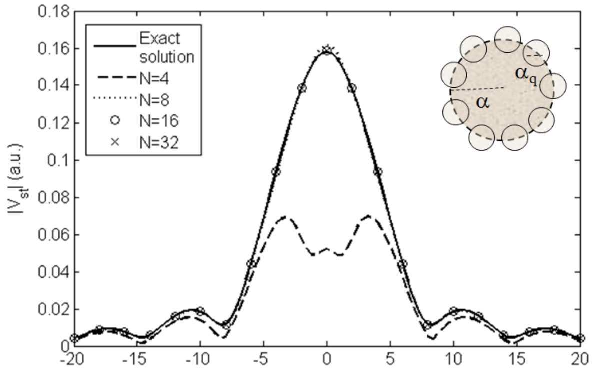

Fig. 4. Near-field scattering of a line source from a perfectly conducting cylinder.

References

[1] D. J. Daniels, *Surface Penetrating Radar*, 2nd ed., London: IEE, 2004.

[2] H.M. Jol, *Ground Penetrating Radar Theory and Applications*, Elsevier, UK, 2009.

[3] M. Di Vico, F. Frezza, L. Pajewski, and G. Schettini, "Scattering by a finite set of perfectly conducting cylinders buried in a dielectric half-space: A spectral-domain solution," *IEEE Trans. Antennas Propag.*, vol. 53, no. 2, February 2005.

[4] F. Frezza, L. Pajewski, C. Ponti, and G. Schettini, "Electromagnetic scattering by perfectly-conducting cylinders buried in a dielectric slab through the cylindrical wave approach," *IEEE Trans. Antennas Propag.*, vol. 57, pp. 1208–1217, April 2009.

[5] F. Frezza, L. Pajewski, C. Ponti, and G. Schettini, "Scattering by dielectric circular cylinders buried in a dielectric slab," *J. Opt. Soc Am. A*, vol. 27, no. 4, pp. 687–695, April 2010.

[6] M.A. Fiaz, F. Frezza, L. Pajewski, C. Ponti, and G. Schettini, "Electromagnetic scattering by perfectly-conducting cylinders buried in a dielectric slab through the cylindrical wave approach," *IEEE Trans. Antennas Propag.*, vol. 57, pp. 1208–1217, April 2009.

[7] F. Frezza, L. Pajewski, C. Ponti, and G. Schettini, "Line source scattering by buried perfectly conducting circular cylinders," *Int. Journal of Antennas and Propag.*, vol. 2012, 7 pp., doi:10.1155/2012/261818.

[8] G. Cincotti, F. Gori, M. Santarsiero, F. Frezza, F. Furnò, and G. Schettini, "Plane wave expansion of cylindrical functions," *Optics Communications*, vol. 95, no. 4–6, pp. 192–198, 1993.

[9] M. Di Vico, F. Frezza, L. Pajewski, and G. Schettini, "Scattering by buried dielectric cylindrical structures," *Radio Science*, vol. 40, no. 6, Article ID RS6S18, 2005.

[10] A.Z. Elsherbeni, "A comparative study of two-dimensional multiple scattering techniques," *Radio Science*, vol. 29, no. 4, pp. 1023–1033, 1994.