Transformation Optical Design for 2D Flattened Maxwell Fish-Eye Lens

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***Abstract*-In this paper, a 2D Flattened Maxwell fish-eye (MFE) lens is proposed using transformation optics, which transforms a traditional MFE lens into a flattened MFE lens. The coordinate transformation is presented from a circular to a rectangular domain. We also discussed the rotated ray trajectories in MFE lens. The required permittivity and permeability tensors are derived for the flattened MFE lens and full-wave simulations are obtained. The simulated results show that the flatted lens can also provide the image focus opposite to the object focus on the boundary of rectangle.**

I. INTRODUCTION

Transformation optics (TO) as a new and powerful technique for microwave and optical device design has drawn great attention since it was reported by Leonhardt and Pendry et al. in 2006 [1,2]. Because Maxwell's equations possess the form-invariance regardless of the coordinate systems used, TO can provide the permittivity and permeability tensors which are functions of spatial coordinates. Then metamaterials with the rebuilt constructive parameters can guide and manipulate light. There have been various transformation devices which are usually fantastic, such as invisibility cloak [3], illusion generators and so on.

In 1968, the electromagnetic and optical behavior of two classes of dielectric lenses including Luneburg lens and Maxwell fish-eye lens was studied through optical ray path by Uslenghi et al. [4]. The refractive index distribution of these dielectric lenses are all functions of radius and the constructive parameters are all isotropic. These lenses have found success in commercial applications, but heavily restricted from broad use in imaging tools for two reasons. One of those is the spherical locus of focal points represents an inherent mismatch to conventional detector/receiver arrays, which are generally planar. To deal with the problem, Kundtz et al. suggested to flatten the some part of spherical surface into a planar one using the quasi-conformal mapping technique [5] for Luneburg lens.

In this paper, TO is employed to transform a 2D conventional MFE lens into a flatten MFE which has a planar surface. A circular domain is transformed into a rectangular domain and the ray trajectories are also presented when the object point is moved on the original circumference of the circle. The required permittivity and permeability are derived and evaluated when the original space is free space. Combining the refractive index distribution of the original MFE and the constructive parameters derived above, the rebuilt parameters are obtained for flattened MFE. The full-wave simulated results show the proposed MFE lens can give the image focus opposite to the object point on the boundary square or rectangle.

II. EQUATIONS UNDER TRANSFORMATION OPTICS

In TO, geometry and coordinate transformations play the dominant role in the design process. The coordinate transformation is interpreted as anisotropic expansion and compression of the original space. In Figure 1, the circular domain with the radius a in the original coordinate system (x, y, z) is transformed into a rectangular domain with the width w and the height h in the transformed system (x', y', z'). To flatten the lens, the following transformation is chosen

$$x' = \frac{wx}{a}, \quad (1)$$

$$y' = \frac{ly}{\sqrt{a^2 - x^2}}, \quad (2)$$

$$z' = z, \quad (3)$$

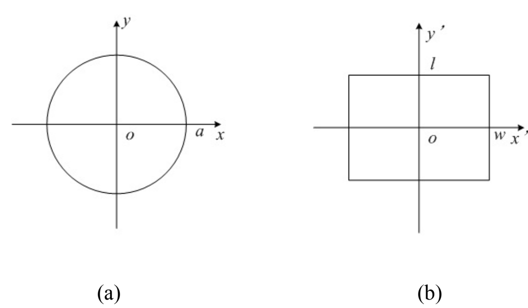

Figure 1 Spatial coordinate transformation
(a) the original coordinate system (b) the transformation coordinate system

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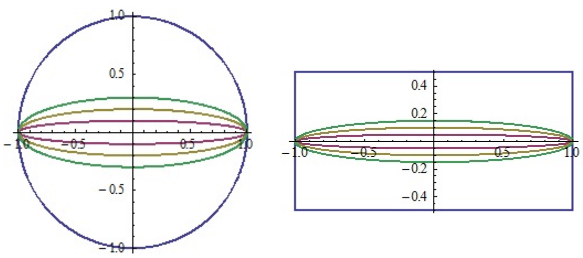
When the geometry is specified by $a = 1$, $w = 1$, $l = 0.5$, the ray trajectories in spherical MFE is transformed into those in flattened MFE, shown in Figure 2. If the object points on the circumference of the circle is rotated θ around the origin, the original coordinate system for 2D should be multiplied by a coordinate rotation matrix firstly as follows

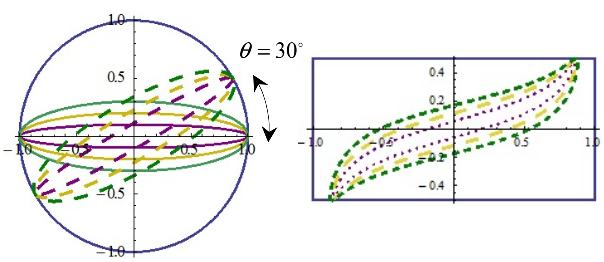
$$\begin{pmatrix} x_{rt} \\ y_{rt} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{4}$$

where x_{rt} and y_{rt} are in the rotated coordinate system. In Figure 3, the flattened region and the ray trajectories are illustrated with $\theta = 30^\circ$.



(a) (b)

Figure 2 Ray trajectories in Flattened MFE
(a) the original coordinate system (b) the transformation coordinate system



(a) (b)

Figure 3 Rotated ray trajectories in Flattened MFE
(a) the original coordinate system (b) the transformation coordinate system

Spherical MFE, as a gradient index (GRIN) lens, has a radial distribution of index that varies as

$$n(r) = \frac{n_0}{1 + (r/r_0)^2} \tag{5}$$

where n_0 is the index of refraction at the center of the lens and r_0 is the radius of the lens [6]. Then the permittivity and permeability tensors can be derived from the usual TO [7] as

$$\varepsilon' = \frac{\Lambda \varepsilon \Lambda^T}{|\Lambda|} n^2(x, y) \qquad \mu' = \frac{\Lambda \varepsilon \Lambda^T}{|\Lambda|} \tag{6}$$

Where $n(x, y)$ is the initial index distribution, and the tensor Λ is the Jacobian matrix relating differential distance between the two coordinate systems which has the explicit form as

$$\Lambda = \begin{pmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} & \frac{\partial x'}{\partial z} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} & \frac{\partial y'}{\partial z} \\ \frac{\partial z'}{\partial x} & \frac{\partial z'}{\partial y} & \frac{\partial z'}{\partial z} \end{pmatrix} \tag{7}$$

Using (1)-(3) in (6), the constructive parameters can be obtained in terms of the transformed system coordinates as

$$\varepsilon'_{xx} = \frac{\sqrt{w^2 - x'^2}}{l} \tag{8a}$$

$$\varepsilon'_{xy} = \varepsilon'_{yx} = \frac{x'y'}{l\sqrt{w^2 - x'^2}} \tag{8b}$$

$$\varepsilon'_{yy} = \frac{x'^2 y'^2}{l(w^2 - x'^2)^{3/2}} + \frac{l}{\sqrt{w^2 - x'^2}} \tag{8c}$$

$$\varepsilon'_{zz} = \frac{a^2\sqrt{w^2 - x'^2}}{w^2 l} \tag{8d}$$

and $\varepsilon'_{xz} = \varepsilon'_{zx} = \varepsilon'_{yz} = \varepsilon'_{zy} = 0$. For TE wave, the permittivity tensors in (8a)-(8d) should be replaced by permeability tensors.

III. SMILATION AND ANALYSIS

Considering the original circular domain is free space, the values of tensors are compared in Figure 4 with a=1, w=1, l=0.125.

When the line source is embedded in the center of the rectangular region with $a = w = 60$mm and $l = 7.5$mm, the total electric field distribution is simulated for TE wave at 10GHz by full-wave software Comsol Multiphysics for both free space and transformation material in Figure 5(a) and 5(b), which indicates that when the rectangular region is full of transformed permeability tensors, it can be used as a wave collimator.

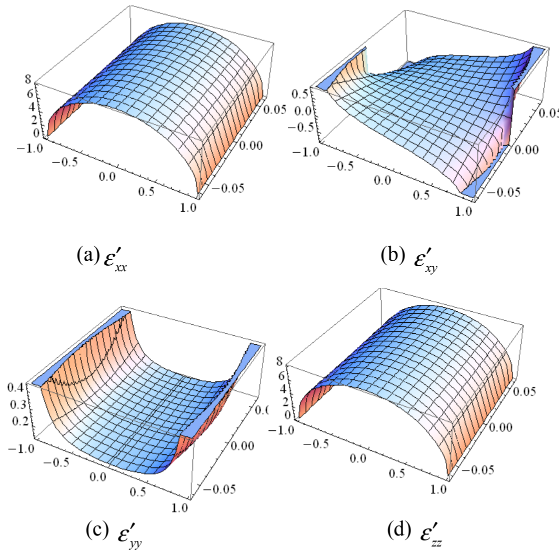


Figure 4 Material permittivity in the transformation coordinate system

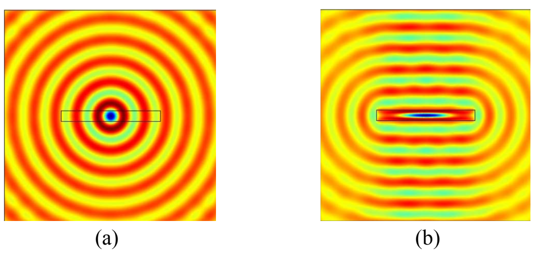


Figure 5 Electric field distribution with a line source located at the coordinate origin: (a) for free space and (b) for transformation material

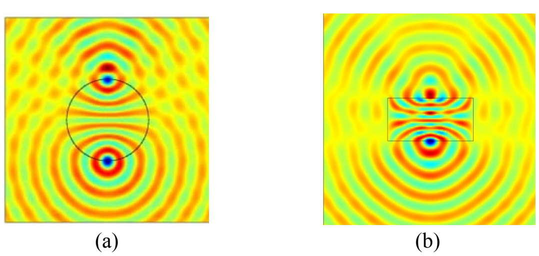


Figure 6 Electric field distribution (a) for sphere MFE (b) for flattened MFE

Combining (5) and (6), the required permittivity and permeability for flattened MFE can be derived. The electric field distributions for sphere MFE and flattened MEF are simulated respectively shown in Figure 6(a) and 6(b) at 10GHz, where a = w = 60mm and l =30mm. It is obvious that the flattened MFE can also provide the image point on the planar surface opposite to the object point.

IV. CONCLUSIONS

A flattened Maxwell fish-eye lens is proposed in this paper, which has a full and compressed flattened structure. The required constructive parameters are derived for the coordinate transformation from sphere to flattened structure. The components of tensors are compared quantitatively. Furthermore, the ray trajectories in flattened MFE lens is also presented using coordinate rotation transformation. The flattened MFE can be believed to have a potential application in imaging system with detector/receiver arrays.

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