Analysis of Coupled Nonuniform Transmission Lines Through Analysis of Uncoupled Ones

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Abstract

A general method is proposed to simplify the analysis of coupled Nonuniform Transmission Lines (NTLs). The modal decomposition approach is used to decouple the sets of differential equations, first. Then, after analyzing the modal lines, the *S* parameters of the coupled NTLs are obtained.

1. Introduction

Coupled Nonuniform Transmission Lines (NTLs) are widely used in microwave circuits as analog signal processors [1], interconnectors [2] and etc.. The differential equations describing these structures have non-diagonal matrices because of coupling between the lines. So, except for a few special cases, no analytical solution exists for NTLs. Coupled NTLs with exponential variation is an example for these special cases [3]. Although, the method of using power series expansion directly [3] or indirectly [4-7] has been utilized to solve many types of coupled NTLs. Of course, the conventional and most straightforward method to analyze arbitrary coupled NTLs is subdividing them into many short uniform sections [8]-[9]. The subject of this paper is using analysis of single NTLs to simplify the analysis of coupled NTLs. In this method, the modal decomposition approach is used to decouple the sets of differential equations, first. Then, the modal lines are analyzed using a suitable method and finally the *ABCD* and then the *S* parameters of the coupled NTL are obtained. Some closed relations are obtained for this purpose. This method is applicable to many arbitrary coupled NTLs.

2. The Equations of Coupled NTLs

In this section, the equations related to the lossless coupled NTLs in the frequency domain are reviewed. It is assumed that the principal propagation mode of the lines is TEM or quasi-TEM. This assumption is valid when the lengths in the cross section are being small enough compared to the wavelength. Figure 1 shows a typical coupled NTL consisting of *N* lines with length of *d*.

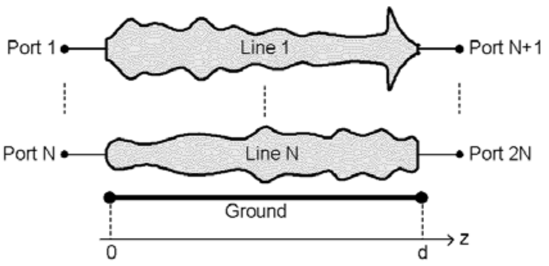

Figure 1. A typical coupled nonuniform transmission line consisting of *M* lines with length of *d*

It is assumed the inductance and capacitance matrices of the structure can be written, respectively, as follows

$$\mathbf{L}(z) = \mathbf{L}_0 f(z) \quad (1)$$

$$\mathbf{C}(z) = \mathbf{C}_0 g(z) \quad (2)$$

The partial differential equations describing the assumed structure are given by

$$\frac{d\mathbf{V}(z)}{dz} = -j\omega f(z)\mathbf{L}_0\mathbf{I}(z) \quad (3)$$

$$\frac{d\mathbf{I}(z)}{dz} = -j\omega g(z)\mathbf{C}_0\mathbf{V}(z) \quad (4)$$

in which **V** and **I** are $N\times1$ voltage and current vectors, respectively. Combining (3) and (4), gives the following differential equations for the voltage and current vectors

$$\frac{d^2\mathbf{V}(z)}{dz^2} - \frac{1}{f(z)}\frac{df(z)}{dz}\frac{d\mathbf{V}(z)}{dz} + \omega^2 f(z)g(z)\mathbf{L}_0\mathbf{C}_0\mathbf{V}(z) = \mathbf{0} \quad (5)$$

$$\frac{d^2\mathbf{I}(z)}{dz^2} - \frac{1}{g(z)}\frac{dg(z)}{dz}\frac{d\mathbf{I}(z)}{dz} + \omega^2 f(z)g(z)\mathbf{L}_0\mathbf{C}_0\mathbf{I}(z) = \mathbf{0} \quad (6)$$

3. Decoupling The Equations of Coupled NTLs

One sees from (3)-(6) that, solving analytically the equations of coupled NTLs is a quite difficult problem generally. The main difficulty at first step is the existing of non-diagonal matrices $\mathbf{L}_0$ and $\mathbf{C}_0$ in them. To overcome this problem, the following modal voltage and current vectors can be defined

$$\mathbf{V}^m(z) = \mathbf{T}_V^{-1}\mathbf{V}(z) \quad (7)$$

$$\mathbf{I}^m(z) = \mathbf{T}_I^{-1}\mathbf{I}(z) \quad (8)$$

where $\mathbf{T}_V$ and $\mathbf{T}_I$ are transfer matrices, which must be satisfy two following relations [13]

$$\mathbf{T}_V^{-1}\mathbf{L}_0\mathbf{T}_I = \mathbf{L}_0^m \quad (9)$$

$$\mathbf{T}_I^{-1}\mathbf{C}_0\mathbf{T}_V = \mathbf{C}_0^m \quad (10)$$

in which $\mathbf{L}_0^m$ and $\mathbf{C}_0^m$ are diagonal inductance and capacitance matrices. Setting (7)-(10) in (3)-(6), the following decoupled equations are obtained

$$\frac{d\mathbf{V}^m(z)}{dz} = -j\omega f(z)\mathbf{L}_0^m\mathbf{I}^m(z) \quad (11)$$

$$\frac{d\mathbf{I}^m(z)}{dz} = -j\omega g(z)\mathbf{C}_0^m\mathbf{V}^m(z) \quad (12)$$

and

$$\frac{d^2\mathbf{V}^m(z)}{dz^2} - \frac{1}{f(z)}\frac{df(z)}{dz}\frac{d\mathbf{V}^m(z)}{dz} + f(z)g(z)(\boldsymbol{\beta}_0^m)^2\mathbf{V}^m(z) = \mathbf{0} \quad (13)$$

$$\frac{d^2\mathbf{I}^m(z)}{dz^2} - \frac{1}{g(z)}\frac{dg(z)}{dz}\frac{d\mathbf{I}^m(z)}{dz} + f(z)g(z)(\boldsymbol{\beta}_0^m)^2\mathbf{I}^m(z) = \mathbf{0} \quad (14)$$

in which

$$\boldsymbol{\beta}_0^m = \omega(\mathbf{L}_0^m\mathbf{C}_0^m)^{1/2} \quad (15)$$

is a diagonal matrix, consisting of propagation coefficients of decoupled modes at z = 0. One sees that each of the resulted equations consists of *N* decoupled equations. In fact, we have to analyze *N* nonuniform decoupled single lines (Modal Lines) instead of *N* nonuniform coupled lines, now. The *ABCD* parameters of the *n*-th modal line, defined by

$$\begin{bmatrix} V_n^m(0) \\ I_n^m(0) \end{bmatrix} = \begin{bmatrix} A_n^m & B_n^m \\ C_n^m & D_n^m \end{bmatrix}\begin{bmatrix} V_n^m(d) \\ I_n^m(d) \end{bmatrix} \quad (16)$$

can be obtained after analyzing it, using a suitable method.

4. Analysis of Coupled Lines

After finding the *ABCD* parameters of all modal lines, the following relation is obtained

$$\begin{bmatrix} \mathbf{V}^m(0) \\ \mathbf{I}^m(0) \end{bmatrix} = \mathbf{T}^m\begin{bmatrix} \mathbf{V}^m(d) \\ \mathbf{I}^m(d) \end{bmatrix} \quad (17)$$

where

$$\mathbf{T}^m = \begin{bmatrix} \mathbf{A}^m & \mathbf{B}^m \\ \mathbf{C}^m & \mathbf{D}^m \end{bmatrix} \quad (18)$$

in which $\mathbf{A}^m$, $\mathbf{B}^m$, $\mathbf{C}^m$ and $\mathbf{D}^m$ are diagonal matrices, whose main diagonals consist of A_n^m, B_n^m, C_n^m and D_n^m, respectively, for *n* = 1, 2, …, *N*. Finally, the relation between the voltages and currents of all physical ports is obtained using (17) and (7)-(8), as follows

$$\begin{bmatrix} \mathbf{V}(d) \\ \mathbf{I}(d) \end{bmatrix} = \boldsymbol{\Phi}\begin{bmatrix} \mathbf{V}(0) \\ \mathbf{I}(0) \end{bmatrix} \quad (19)$$

where $\boldsymbol{\Phi}$ is the chain parameter matrix and is given by

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\varphi}_{11} & \boldsymbol{\varphi}_{12} \\ \boldsymbol{\varphi}_{21} & \boldsymbol{\varphi}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_V^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_I^{-1} \end{bmatrix}^{-1}(\mathbf{T}^m)^{-1}\begin{bmatrix} \mathbf{T}_V^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_I^{-1} \end{bmatrix} \quad (20)$$

Of course, one can determine the *S* parameter matrix from the chain parameter matrix after some manipulations, as follows

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \quad (21)$$

in which

$$\mathbf{S}_{11} = -(\boldsymbol{\varphi}_{11} - \boldsymbol{\varphi}_{12}/Z_0 - Z_0\boldsymbol{\varphi}_{21} + \boldsymbol{\varphi}_{22})^{-1} \times(\boldsymbol{\varphi}_{11} + \boldsymbol{\varphi}_{12}/Z_0 - Z_0\boldsymbol{\varphi}_{21} - \boldsymbol{\varphi}_{22}) \quad (22)$$

$$\mathbf{S}_{12} = 2(\boldsymbol{\varphi}_{11} - \boldsymbol{\varphi}_{12}/Z_0 - Z_0\boldsymbol{\varphi}_{21} + \boldsymbol{\varphi}_{22})^{-1} \quad (23)$$

$$\mathbf{S}_{21} = [(\boldsymbol{\varphi}_{11} - \boldsymbol{\varphi}_{12}/Z_0)^{-1} + (\boldsymbol{\varphi}_{22} - Z_0\boldsymbol{\varphi}_{21})^{-1}]^{-1} \times[(\boldsymbol{\varphi}_{11} - \boldsymbol{\varphi}_{12}/Z_0)^{-1}(\boldsymbol{\varphi}_{11} + \boldsymbol{\varphi}_{12}/Z_0) + (\boldsymbol{\varphi}_{22} - Z_0\boldsymbol{\varphi}_{21})^{-1}(\boldsymbol{\varphi}_{22} + Z_0\boldsymbol{\varphi}_{21})] \quad (24)$$

$$\mathbf{S}_{22} = -[(\boldsymbol{\varphi}_{11} - \boldsymbol{\varphi}_{12}/Z_0)^{-1} + (\boldsymbol{\varphi}_{22} - Z_0\boldsymbol{\varphi}_{21})^{-1}]^{-1} \times[(\boldsymbol{\varphi}_{11} - \boldsymbol{\varphi}_{12}/Z_0)^{-1} - (\boldsymbol{\varphi}_{22} - Z_0\boldsymbol{\varphi}_{21})^{-1}] \quad (25)$$

5. Conclusions

A general method was proposed to simplify the analysis of coupled Nonuniform Transmission Lines (NTLs). The modal decomposition approach is used to decouple the sets of differential equations, first. Then, the *ABCD* and so the *S* parameters of the coupled NTLs are obtained, after analyzing the modal lines using a suitable method. Some closed relations were obtained for this purpose. This method is applicable to many arbitrary coupled NTLs.

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