Analytic Solution for the Problem of Excited Coupled Exponential Transmission Lines

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Abstract

An exact and explicit frequency domain solution is derived for the problem of coupled exponential transmission lines (CETL) excited by both external electromagnetic plane wave and the terminal sources. The solutions obtained are exact and are applicable to all arbitrarily loaded, lossless, inhomogeneous and dispersive coupled exponential transmission lines. The problem is formulated in some matrix equations and the voltages and currents of the lines are obtained along their length.

1. Introduction

Current advances in technology enables the use of increased speed in devices, which require much higher frequency bandwidth, in the present microwave integrated circuits (MICs). However, with increased frequency, more emphasis is now placed upon greater concentration of active devises in the ICs to even smaller volumes, which results in shorter interconnects and hence increased density. An important aspect of electromagnetic interference (EMI) is the determination of electromagnetic coupling in transmission lines (TL's) and the response of the lines to an incident electromagnetic plane wave. The response of several kinds of single or coupled TL's to external electromagnetic fields has been studied by some authors [1]-[7]. The method of these studies is based on subdividing total electric and magnetic fields into a primary (in the absence of metal conductors) and a secondary part (TEM or quasi-TEM mode). The primary part is modeled by distributed voltage and current forcing sources and the secondary part is modeled by transmission lines equations. On the other hand, there are many occasions where the transmission lines are nonuniform, e.g. connection of indoor wires with power lines, wiring of electronic equipments, impedance matching [8], analog signal processing [9], VLSI interconnect [10], etc. The differential equations describing these structures have non-constant coefficients because the per-unit-length matrices vary along the lines. The most straightforward method to analyze arbitrary coupled nonuniform TL's is subdividing them into many short uniform sections [6], [11]. There are exact and analytic methods only for a few special cases such as binomial [12], power law [13], exponential power law [14] and hermite [15] variations.

Most of the works discuss about excited coupled uniform TL's [1]-[7] or non-excited nonuniform TL's [8]-[15]. In this paper, we derive an analytic solution for the excited coupled exponential TL's (CETL's) by both the incident wave and the terminal sources. The solutions obtained are exact and are applicable to all arbitrarily loaded, lossless, inhomogeneous and dispersive CETL's. The effect of distributed voltage and current forcing sources and also the source voltages to the voltages and currents of the lines are studied using an example.

2. Analytic Solution of CETLs

Solving the equations of externally excited coupled nonuniform transmission lines analytically is a very hard problem. However, these equations will be solved analytically without approximation for the special case of externally excited CETLs, in this section. Fig. 1 shows a typical CETL consisting of N lines with length of d and with arbitrary terminal loads of $Z_{S,n}(\omega)$ and $Z_{L,n}(\omega)$, in which n=1, 2, …, N. The structure is excited by both source voltages $V_{S,n}(\omega)$, where n=1, 2, …, N in the terminals, and an external electromagnetic plane wave.

In general, the partial differential equations describing lossless and dispersive externally excited coupled nonuniform transmission lines in the angular frequency of ω are given by [1]-[7]

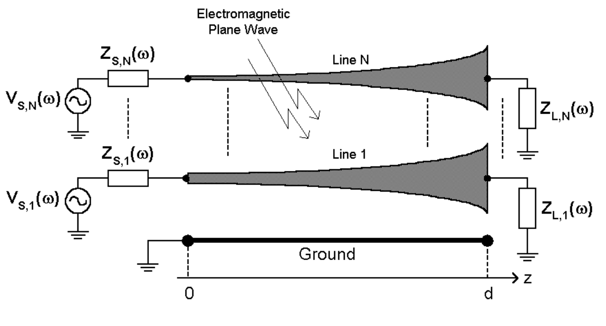

Fig. 1. A typical coupled exponential transmission lines with length of d and excited by source voltages and an electromagnetic plane wave

$$\frac{d\mathbf{V}(z,\omega)}{dz}+j\omega\mathbf{L}(z,\omega)\mathbf{I}(z,\omega)=\mathbf{V}_F(\omega)\exp(-jk_z z) \quad (1)$$

$$\frac{d\mathbf{I}(z,\omega)}{dz}+j\omega\mathbf{C}(z,\omega)\mathbf{V}(z,\omega)=\mathbf{I}_F(\omega)\exp(-jk_z z) \quad (2)$$

in which $\mathbf{V}$ and $\mathbf{I}$ are $N\times 1$ voltage and current vectors, respectively. The parameter k_z is the phase constant of the incident wave along the lines. Also, $\mathbf{V}_F$ and $\mathbf{I}_F$ are $N\times 1$ distributed forcing voltage and current vectors, respectively, which are related to the specifications of the incident wave (the frequency, the angle of incidence and the polarization) and the exposed structure [1]-[7]. Moreover, $\mathbf{L}$ and $\mathbf{C}$ are frequency dependent per-unit-length inductance and capacitance matrices of the coupled transmission lines, written as follows

$$\mathbf{L}(z,\omega)=\mathbf{L}_0(\omega)f(z) \quad (3)$$

$$\mathbf{C}(z,\omega)=\mathbf{C}_0(\omega)g(z) \quad (4)$$

In addition to (1)-(2), there are two following terminal conditions for coupled nonuniform transmission lines

$$\mathbf{V}(0,\omega)+\mathbf{Z}_S(\omega)\mathbf{I}(0,\omega)=\mathbf{V}_S(\omega) \quad (5)$$

$$\mathbf{V}(d,\omega)-\mathbf{Z}_L(\omega)\mathbf{I}(d,\omega)=\mathbf{0} \quad (6)$$

where $\mathbf{Z}_S$ and $\mathbf{Z}_L$ are diagonal source and load matrices, respectively. The differential equations in (1) can be decoupled using the definition of modal voltage and current vectors as

$$\mathbf{V}^m(z,\omega)=\mathbf{T}_V^{-1}\mathbf{V}(z,\omega) \quad (7)$$

$$\mathbf{I}^m(z,\omega)=\mathbf{T}_I^{-1}\mathbf{I}(z,\omega) \quad (8)$$

where $\mathbf{T}_V$ and $\mathbf{T}_I$ are transfer matrices determined from inductance and capacitance matrices [6]. Setting (7)-(8) in (1)-(2), the decoupled equations are obtained as

$$\frac{d\mathbf{V}^m(z,\omega)}{dz}+j\omega f(z)\mathbf{L}_0^m\mathbf{I}^m(z,\omega) =\mathbf{T}_V^{-1}\mathbf{V}_F(\omega)\exp(-jk_z z) \quad (9)$$

$$\frac{d\mathbf{I}^m(z,\omega)}{dz}+j\omega g(z)\mathbf{C}_0^m\mathbf{V}^m(z,\omega) =\mathbf{T}_I^{-1}\mathbf{I}_F(\omega)\exp(-jk_z z) \quad (10)$$

in which $\mathbf{L}_0^m$ and $\mathbf{C}_0^m$ are diagonal inductance and capacitance matrices derived from the following matrix equations [6].

$$\mathbf{T}_V^{-1}\mathbf{L}_0\mathbf{T}_I=\mathbf{L}_0^m \quad (11)$$

$$\mathbf{T}_I^{-1}\mathbf{C}_0\mathbf{T}_V=\mathbf{C}_0^m \quad (12)$$

Combining (9) and (10), gives the following differential equation for modal voltage vector

$$\begin{aligned}&\frac{d^2\mathbf{V}^m(z,\omega)}{dz^2}-\frac{f'(z)}{f(z)}\frac{d\mathbf{V}^m(z,\omega)}{dz}\\&+\omega^2 f(z)g(z)\mathbf{L}_0^m\mathbf{C}_0^m\mathbf{V}^m(z,\omega)=\\&-\left[\left(jk_z+\frac{f'(z)}{f(z)}\right)\mathbf{T}_V^{-1}\mathbf{V}_F(\omega)+j\omega f(z)\mathbf{L}_0^m\mathbf{T}_I^{-1}\mathbf{I}_F(\omega)\right]\\&\times\exp(-jk_z z)\end{aligned} \quad (13)$$

Now, we consider the following functions for exponentially varied coupled lines

$$f(z)=\exp(qz) \quad (14)$$

$$g(z)=\exp(-qz) \quad (15)$$

Using (14)-(15) in (13), gives us

$$\begin{aligned}&\frac{d^2\mathbf{V}^m(z,\omega)}{dz^2}-q\frac{d\mathbf{V}^m(z,\omega)}{dz}+\mathbf{B}_0^m\mathbf{V}^m(z,\omega)=\\&-(jk_z+q)\mathbf{T}_V^{-1}\mathbf{V}_F(\omega)\exp(-jk_z z)\\&-j\omega\mathbf{L}_0^m\mathbf{T}_I^{-1}\mathbf{I}_F(\omega)\exp\big((q-jk_z)z\big)\end{aligned} \quad (16)$$

in which

$$\mathbf{B}_0^m=\omega^2\mathbf{L}_0^m\mathbf{C}_0^m \quad (17)$$

is a diagonal matrix. It is simple to show that the voltage vector satisfying (16) is as follows

$$\begin{aligned}\mathbf{V}(z,\omega)&=\mathbf{T}_V\mathbf{exp}(\boldsymbol{\lambda}_1 z)\mathbf{K}_1+\mathbf{T}_V\mathbf{exp}(\boldsymbol{\lambda}_2 z)\mathbf{K}_2\\&-\mathbf{K}_V\mathbf{V}_F(\omega)\exp(-jk_z z)-j\omega\mathbf{K}_I\mathbf{I}_F(\omega)\exp\big((q-jk_z)z\big)\end{aligned} \quad (18)$$

in which $\mathbf{K}_1$ and $\mathbf{K}_2$ are unknown column vectors of coefficients and also

$$\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2=\frac{q}{2}\mathbf{I}\mp\left(\frac{q^2}{4}\mathbf{I}-\mathbf{B}_0^m\right)^{1/2} \quad (19)$$

$$\mathbf{K}_V=(q+jk_z)\mathbf{T}_V\left(jk_z(q+jk_z)\mathbf{I}+\mathbf{B}_0^m\right)^{-1}\mathbf{T}_V^{-1} \quad (20)$$

$$\mathbf{K}_I=\mathbf{T}_V\left(-jk_z(q-jk_z)\mathbf{I}+\mathbf{B}_0^m\right)^{-1}\mathbf{L}_0^m\mathbf{T}_I^{-1} \quad (21)$$

In (19)-(21), $\mathbf{I}$ is an N by N identity matrix. Also, the current vector is determined using (1), (3), (14) and (18), as follows

$$\begin{aligned}\mathbf{I}(z,\omega)&=\frac{1}{j\omega f(z)}\mathbf{L}_0^{-1}\left(\mathbf{V}_F(\omega)\exp(-jk_z z)-\frac{d\mathbf{V}(z,\omega)}{dz}\right)\\&=\frac{1}{j\omega}\mathbf{L}_0^{-1}\big[-\mathbf{T}_V\boldsymbol{\lambda}_1\mathbf{exp}\big((\boldsymbol{\lambda}_1-q\mathbf{I})z\big)\mathbf{K}_1\\&-\mathbf{T}_V\boldsymbol{\lambda}_2\mathbf{exp}\big((\boldsymbol{\lambda}_2-q\mathbf{I})z\big)\mathbf{K}_2\\&+(\mathbf{I}-jk_z\mathbf{K}_V)\mathbf{V}_F(\omega)\exp\big(-(q+jk_z)z\big)\\&+j\omega(q-jk_z)\mathbf{K}_I\mathbf{I}_F(\omega)\exp(-jk_z z)\big]\end{aligned} \quad (22)$$

Setting (18) and (22) in the terminal conditions (5)-(6), one gets to the following set of equations to find the unknown vectors $\mathbf{K}_1$ and $\mathbf{K}_2$

$$\mathbf{a}_1\mathbf{K}_1+\mathbf{a}_2\mathbf{K}_2=\mathbf{V}_S(\omega)+\mathbf{b}_1\mathbf{V}_F(\omega)+\mathbf{b}_2\mathbf{I}_F(\omega) \quad (23)$$

$$\mathbf{a}_3\mathbf{K}_1 + \mathbf{a}_4\mathbf{K}_2 = \mathbf{b}_3\mathbf{V}_F(\omega) + \mathbf{b}_4\mathbf{I}_F(\omega) \tag{24}$$

where the following eight coefficients have been defined

$$\mathbf{a}_1 = \mathbf{T}_V - \frac{1}{j\omega}\mathbf{Z}_S\mathbf{L}_0^{-1}\mathbf{T}_V\boldsymbol{\lambda}_1 \tag{25}$$

$$\mathbf{a}_2 = \mathbf{T}_V - \frac{1}{j\omega}\mathbf{Z}_S\mathbf{L}_0^{-1}\mathbf{T}_V\boldsymbol{\lambda}_2 \tag{26}$$

$$\mathbf{a}_3 = \mathbf{T}_V\mathbf{exp}(\boldsymbol{\lambda}_1 d) + \frac{1}{j\omega}\mathbf{Z}_L\mathbf{L}_0^{-1}\mathbf{T}_V\boldsymbol{\lambda}_1\mathbf{exp}\big((\boldsymbol{\lambda}_1 - q\mathbf{I})d\big) \tag{27}$$

$$\mathbf{a}_4 = \mathbf{T}_V\mathbf{exp}(\boldsymbol{\lambda}_2 d) + \frac{1}{j\omega}\mathbf{Z}_L\mathbf{L}_0^{-1}\mathbf{T}_V\boldsymbol{\lambda}_2\mathbf{exp}\big((\boldsymbol{\lambda}_2 - q\mathbf{I})d\big) \tag{28}$$

and

$$\mathbf{b}_1 = \mathbf{K}_V - \frac{1}{j\omega}\mathbf{Z}_S\mathbf{L}_0^{-1}(\mathbf{I} - jk_z\mathbf{K}_V) \tag{29}$$

$$\mathbf{b}_2 = j\omega\mathbf{K}_I - (q - jk_z)\mathbf{Z}_S\mathbf{L}_0^{-1}\mathbf{K}_I \tag{30}$$

$$\begin{aligned}\mathbf{b}_3 &= \mathbf{K}_V\exp(-jk_z d)\\ &+ \frac{1}{j\omega}\mathbf{Z}_L\mathbf{L}_0^{-1}(\mathbf{I} - jk_z\mathbf{K}_V)\exp\big(-(q + jk_z)d\big)\end{aligned} \tag{31}$$

$$\begin{aligned}\mathbf{b}_4 &= j\omega\mathbf{K}_I\exp\big((q - jk_z)d\big)\\ &+ (q - jk_z)\mathbf{Z}_L\mathbf{L}_0^{-1}\mathbf{K}_I\exp(-jk_z d)\end{aligned} \tag{32}$$

Finally, the unknown coefficients $\mathbf{K}_1$ and $\mathbf{K}_2$ are determined using (23)-(24) as follows

$$\begin{aligned}\mathbf{K}_1 &= (\mathbf{a}_4\mathbf{a}_2^{-1}\mathbf{a}_1 - \mathbf{a}_3)^{-1}\\ &\times\big[\mathbf{a}_4\mathbf{a}_2^{-1}\mathbf{V}_S(\omega) - (\mathbf{b}_3 - \mathbf{a}_4\mathbf{a}_2^{-1}\mathbf{b}_1)\mathbf{V}_F(\omega)\\ &- (\mathbf{b}_4 - \mathbf{a}_4\mathbf{a}_2^{-1}\mathbf{b}_2)\mathbf{I}_F(\omega)\big]\end{aligned} \tag{33}$$

$$\begin{aligned}\mathbf{K}_2 &= (\mathbf{a}_3\mathbf{a}_1^{-1}\mathbf{a}_2 - \mathbf{a}_4)^{-1}\\ &\times\big[\mathbf{a}_3\mathbf{a}_1^{-1}\mathbf{V}_S(\omega) - (\mathbf{b}_3 - \mathbf{a}_3\mathbf{a}_1^{-1}\mathbf{b}_1)\mathbf{V}_F(\omega)\\ &- (\mathbf{b}_4 - \mathbf{a}_3\mathbf{a}_1^{-1}\mathbf{b}_2)\mathbf{I}_F(\omega)\big]\end{aligned} \tag{34}$$

It is necessary to mention that, the relations (23)-(24) have to be solved for each excitation frequency of the incident wave and the terminal sources, separately.

3. EXAMPLES AND RESULTS

In this section, a CETL is analyzed using the obtained analytical formulas. Consider a lossless coupled microstrip structure with $N = 2$ strips. The substrate permittivity is $\varepsilon_r = 10$, the width of the strips and the gap between them are equal to the thickness of the substrate. This inhomogeneous structure has the following exponential the per-unit-length matrices.

$$\mathbf{L}(z) = \mathbf{L}_0\exp(qz) = \begin{bmatrix}425.6 & 74.83\\ 74.83 & 425.6\end{bmatrix}\exp(qz) \quad nH/m$$

$$\mathbf{C}(z) = \mathbf{C}_0\exp(-qz) = \begin{bmatrix}174.9 & -14.25\\ -14.25 & 174.9\end{bmatrix}\exp(-qz) \quad pF/m$$

Assume that d = 20 cm, f = 1.0 GHz, $k_z = \omega/(\sqrt{2}c)$, where c is the velocity of the light, $Z_{S,1} = Z_{S,2} = 50\ \Omega$, $Z_{L,1} = Z_{L,2} = 50\ \Omega$. Figures 2 shows the amplitude of voltages of two lines for $q = 0$ and $q = 1$, assuming $V_{S,1} = 1$ V, $V_{S,2} = V_{F,1} = V_{F,2} = I_{F,1} = I_{F,2} = 0$. Figures 3 shows the amplitude of voltages of two lines for $q = 0$ and $q = 1$, assuming $V_{F,1} = 1$ V, $V_{S,1} = V_{S,2} = V_{F,2} = I_{F,1} = I_{F,2} = 0$. Also, figures 4 shows the amplitude of voltages of two lines for $q = 0$ and $q = 1$, assuming $I_{F,1} = 1$ A, $V_{S,1} = V_{S,2} = V_{F,1} = V_{F,2} = I_{F,2} = 0$. In fact, Figs. 2-4 show the effects of source voltage and distributed forcing voltage and current sources, separately. Of course, all of these sources are simultaneously existed, in a real problem. The time consumed for the above example was only a few seconds using a Pentium-4 PC and MATLAB program.

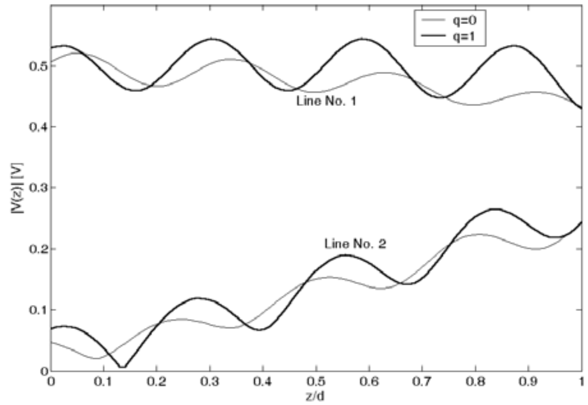

Fig. 2. The amplitude of the voltage of CETL for two cases of q=0 and q=1, assuming $V_{S,1}$=1 V, $V_{S,2}$=$V_{F,1}$=$V_{F,2}$=$I_{F,1}$=$I_{F,2}$=0

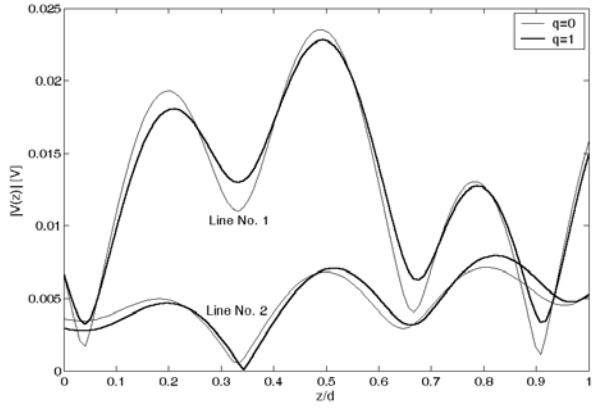

Fig. 3. The amplitude of the voltage of CETL for two cases of q=0 and q=1, assuming $V_{F,1}$=1 V, $V_{S,1}$=$V_{S,2}$=$V_{F,2}$=$I_{F,1}$=$I_{F,2}$=0

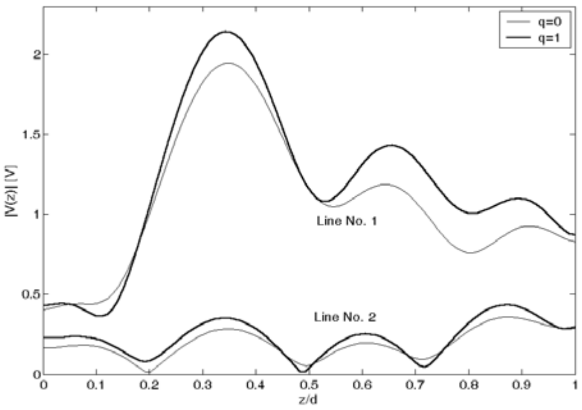

Fig. 4. The amplitude of the voltage of CETL for two cases of q=0 and q=1, assuming $I_{F,1}$=1 A, $V_{S,1}=V_{S,2}=V_{F,1}=V_{F,2}=I_{F,2}$=0

4. CONCLUSIONS

An analytic solution is derived for externally and internally excited coupled exponential transmission lines problem in the frequency domain. The solutions obtained are exact and are applicable to all arbitrarily loaded, lossless, inhomogeneous and dispersive coupled exponential transmission lines. The problem is formulated in some matrix equations and the voltages and currents of the lines are obtained along their length. The effect of distributed voltage and current forcing sources and also the source voltages to the voltages and currents of the lines are studied using an example.

REFERENCES

1. C. R. Paul, "Efficient numerical computation of the frequency response of cables illuminated by an electromagnetic field," *IEEE Trans. Mic. Theory and Tech.*, pp. 454-457, Apr. 1974.
2. C. R. Paul, "Frequency response of multiconductor transmission lines illuminated by an electromagnetic field," *IEEE Trans. Electromagn. Compat.*, pp. 183-190, Nov. 1976.
3. Y. Kami and R. Sato, "Circuit concept approach to externally excited transmission lines," *IEEE Trans. Electromag. Compat.*, pp. 177-183, Nov. 1985.
4. A. T. Adams, J. Perini, M. Miyabayashi, D. H. Shau, and K. Heidary, "Electromagnetic field-to-wire coupling in the SHF frequency range and beyond," *IEEE Trans. Electrormagn. Compat.*, pp. 126-131, May 1987.
5. P. Bernardi and R. Cichetti, "Response of Planar Microstrip Line Excited by an External Electromagnetic Field", *IEEE Trans. Electromag. Compat.*, pp. 98-105, May 1990.
6. C. R. Paul, *"Analysis of multiconductor transmission lines"*, John Wiley and Sons Inc., 1994.
7. A. Cheldavi, D. Ansari and M. Khalaj Amirhosseini, "Electromagnetic coupling to circulant symmetric multi-conductor microstrip line", Progress in Electromagnetics Research (PIER 49), Vol. 49, pp. 189-201, 2004.
8. R. E. Collin, "*Foundations for Microwave Engineering*", New York: McGraw-Hill, 1996.
9. L. A. Hayden and V. K. Tripathi, "Nonuniform coupled microstrip transversal filters for analog signal processing", *IEEE Trans. Mic. Theory and Tech.*, pp. 47-53, Jan. 1991.
10. T. Dhaene, L. Martens and D. D. Zutter, "Transient simulation of arbitrary nonuniform interconnection structures characterized by scattering parameters", *IEEE Trans. Circuits and Sys.-I*, pp. 928-937, Nov. 1992.
11. M. Khalaj Amirhosseini and A. Cheldavi, "The use of grounded lines in non-uniform interconnects", *J. Electromagn. Waves and Appl.*, pp. 637-647, May 2004.
12. K. Kobayashi, Y. Nemoto, R. Sato, "Equivalent circuits of binomial form nonuniform coupled transmission lines", *IEEE Trans. Mic. Theory and Tech.*, pp. 817-824, Aug. 1981.
13. A. Cheldavi, M. Kamarei, S. Safavi Naeini, "Analysis of coupled transmission lines with power-law characteristic impedance", *IEEE Trans. Electromagn. Compat.*, pp. 308-312, Aug. 2000.
14. A. Cheldavi, "Exact analysis of coupled nonuniform transmission lines with exponential power law characteristic impedance", *IEEE Trans. Mic. Theory and Tech.*, pp. 197-199, Jan. 2001.
15. A. Cheldavi, "Analysis of coupled Hermite transmission lines", *IEE Proc. Mic., Ant. and Prop.*, pp. 279-284, Aug. 2003.