A DAUBECHIES'-WAVELET-BASED TIME DOMAIN ELECTROMAGNETIC FIELD MODELING TECHNIQUE

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I. Introduction

Numerical analysis has become an important technique for the modeling of electromagnetic field. However, there still exist many restrictive factors that make numerical analysis difficult to apply to practical problems. The finite-difference time-domain (FDTD) technique [1] and the transmission line matrix method [2] have been applied to many problems and proven to be promising techniques by virtue of their versatility. However for modeling electrically large structures, they are often restricted by memory shortage like other space-discretization methods. The multi-resolution time-domain (MRTD) method based on the Battle-Lemarié wavelets [3] has a highly linear numerical dispersion property. It has been demonstrated that, with this technique, space discretization with only a few cells per wavelength gives accurate results, leading to a reduction of both memory requirement and computation time.

Recently, the wavelet-Galerkin scheme based on Daubechies' compactly supported wavelet with two vanishing moments (D_2) was proposed by Cheong et.al. [4]. Although the numerical dispersion of this technique is larger than that of the MRTD method, it has advantages over MRTD in that the Daubechies' scaling function has compact support, and the stencil size or the number of coefficients in the time-evolution equations is kept to a minimum. Cheong's method also adopts the so-called "shifted interpolation property", which enables local field sampling in spite of the asymmetry of the Daubechies' scaling function and a support larger than unity; with this property, the evaluation of the constitutive equations can be omitted even for inhomogeneous media.

The authors have already extended the method to the use of Daubechies' scaling functions with three and four vanishing moments (denoted as D_3 and D_4, respectively) [5]. By using basis functions of higher regularity and minimum support, better accuracy and minimum stencil sizes can be expected, resulting in an optimally efficient algorithm.

In this paper, a three-dimensional formulation of the wavelet-Galerkin scheme is presented with an implementation of uniaxial perfectly matched layer (UPML) absorbing boundary conditions (ABC) [6]. The accuracy of the present method is first verified, and then it is applied to the analysis of an electrically-large optical waveguide, which is too expensive to solve with the conventional FDTD method, and the advantage of the present method is demonstrated. This is, to the best of authors' knowledge, the first attempt to apply the wavelet based approach to the full-wave time-domain analysis of electrically large inhomogeneous structures such as optical waveguides.

II. Theory

Maxwell's equations for the three-dimensional lossless case

$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} \quad (1)$$

$$\nabla \times \boldsymbol{H} = \epsilon \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J} \quad (2)$$

are discretized on the standard Yee grid [1]. The field values are first expanded in Daubechies' compactly supported scaling functions ϕ [7] as

$$E_x(x,y,z,t) = \sum_{i,j,k,n=-\infty}^{+\infty} E^{\phi,x}_{i+1/2,j,k,n-1/2} \phi_{i+1/2}(x)\phi_j(y)\phi_k(z)h_{n-1/2}(t) \quad (3)$$

and the other field components E_y, E_z and $\boldsymbol{H}$ are expanded analogously, where

$$h_n(t) = h(\frac{t}{\Delta t} - n + \frac{1}{2}) \tag{4}$$

is the well known Haar scaling function and

$$\phi_i(x) = \phi(\frac{x}{\Delta x} - i + M_1) \tag{5}$$

is the translated scaling function with $M_1 = \int_{-\infty}^{+\infty} x\phi(x)dx$ being the first-order moment of the scaling function; $M_1 = 0.6339743121$ for D$_2$, and $M_1 = 0.1005392384$ for D$_4$. Daubechies' compactly supported scaling functions approximately satisfy the so-called "shifted interpolation property"

$$\phi(k + M_1) = \delta_{k,0} \tag{6}$$

for k integer, where δ is the Kronecker delta function. This property yields a simple algorithm for the inhomogeneous problems through the local sampling of the field values [4].

The standard Galerkin's procedure leads to a system of updating equations similar to the S-MRTD method [3]. The UPML formulation in [6] yields straightforwardly the standard two-step updating procedure for the UPML medium in the wavelet-Galerkin scheme. The first step is to update the electric flux density $\boldsymbol{D}$ from $\boldsymbol{H}$

$$\begin{aligned} D^{\phi,x}_{i+1/2,j,k,n+1/2} = {} & \frac{2\epsilon_0\kappa_y - \sigma_y\Delta t}{2\epsilon_0\kappa_y + \sigma_y\Delta t} D^{\phi,x}_{i+1/2,j,k,n-1/2} + \frac{2\epsilon_0\Delta t}{2\epsilon_0\kappa_y + \sigma_y\Delta t} \\ & \cdot \left[\sum_{l=-L_s}^{L_s-1} a(l) \left(\frac{H^{\phi,z}_{i+1/2,j+l+1/2,k,n}}{\Delta y} - \frac{H^{\phi,y}_{i+1/2,j,k+l+1/2,n}}{\Delta z} \right) - J^{\phi,x}_{i+1/2,j,k,n+1/2} \right] \end{aligned} \tag{7}$$

where L_s denotes the effective support size of the basis function, that is, the stencil size or the number of connection coefficients per side included in the update equations. The second step is to update $\boldsymbol{E}$ from $\boldsymbol{D}$ locally without including the neighboring $\boldsymbol{E}$ or $\boldsymbol{D}$ due to the shifted interpolation property of the scaling functions

$$\begin{aligned} E^{\phi,x}_{i+1/2,j,k,n+1/2} = {} & \frac{2\epsilon_0\kappa_z - \sigma_z\Delta t}{2\epsilon_0\kappa_z + \sigma_z\Delta t} E^{\phi,x}_{i+1/2,j,k,n-1/2} + \frac{2\epsilon_0\Delta t}{\epsilon_0\epsilon_r\,(2\epsilon_0\kappa_z + \sigma_z\Delta t)} \\ & \cdot \left[\left(\frac{\kappa_x}{\Delta t} + \frac{\sigma_x}{2\epsilon_0} \right) D^{\phi,x}_{i+1/2,j,k,n+1/2} - \left(\frac{\kappa_x}{\Delta t} - \frac{\sigma_x}{2\epsilon_0} \right) D^{\phi,x}_{i+1/2,j,k,n-1/2} \right] . \end{aligned} \tag{8}$$

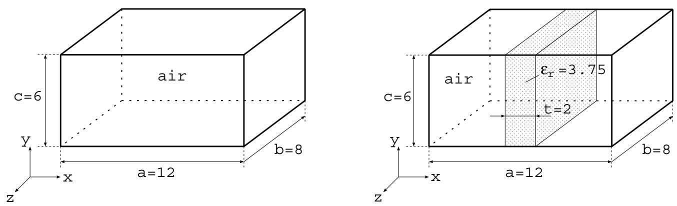
The update equations for the other field components E_y, E_z, D_y and D_z are obtained by permutating x, y and z, and the equations for the magnetic flux density $\boldsymbol{B}$ and $\boldsymbol{H}$ is obtained similarly. The coefficients $a(l)$ are obtained by numerically evaluating the inner products between the scaling functions and their derivatives in Fourier domain [5]; for D$_2$, $a(l, l = 0, 1, 2) = \{1.2291666667,\ -0.0937500000,\ 0.0104166667\}$ and for D$_4$, $a(l, l = 0, \cdots, 6) = \{1.3110340773,\ -0.1560100710,\ 0.0419957460,\ -0.0086543236,\ 0.0008308695,\ 0.0000108999,\ -0.0000000041\}$.

The UPML material parameters are chosen to be $\sigma_{\xi(\xi=x,y,z)} = 0$ for the inner computation region, and $\sigma_{\xi(\xi=x,y,z)} > 0$ with fourth-order polynomial scaling $(m = 4)$ for the UPML region. The maximum value of σ at the end of the UPML region is chosen to be $\sigma_{\max} = (m+1)/(150\pi\Delta\sqrt{\epsilon_r})$ [6], where Δ is the cell dimension perpendicular to the UPML interface to the regular region. The other parameter is $\kappa_{\xi(\xi=x,y,z)} = 1$ over the whole region in this paper.

The UPML region is backed by a perfect electric conductor wall implemented using the mirror principle. One might wonder about the accuracy of the mirror image because of the asymmetry of the basis functions. However, by virtue of the shifted interpolation property, the basis functions have unity value at the origin of the function and zero values at other integer points, thus the mirror image is accurate at integer points. Of course, the basis functions take non-zero asymmetric values at non-integer points; nevertheless, this does not affect the numerical procedure.

III. Validation

The resonator structures shown in Fig. 1 were analyzed with the wavelet-Galerkin method based on D_2 and D_4 scaling functions as well as the standard FDTD method. The stability factor for D_2 and D_4 was chosen to be 0.2 of the corresponding FDTD Courant limit, while for FDTD, it was the maximum Courant limit to obtain the best accuracy. The final calculation time is 2000 in the normalized unit. The computations were conducted on a Sun Ultra-Spark workstation with 300MHz clock rate and 256MB memory. The code is not optimized, however, the results give a good estimate of the required computer resources. The results are summarized in Table I.



(a) (b)

Fig. 1. Cavity resonators. Numbers denote normalized dimensions of the structures.

TABLE I

Dominant resonant frequencies F_r of the resonators. The theoretical values of the normalized resonant frequencies of the resonators are 0.075116 for (a) and 0.05221 for (b). N_{max} denotes the number of time steps.

Resonator	No. of Yee cells	Conditions	FDTD	$D_2\,(L_s=3)$	$D_4\,(L_s=5)$
a	12 x 6 x 8	N_{max}	3464	17320	17320
		Δt	0.57735	0.11547	0.11547
		F_r	0.07495	0.07533	0.07535
		error (%)	-0.23	+0.28	+0.30
		CPU time (s)	7	45	68
	3 x 2 x 3	N_{max}	—	5605	5605
		Δt	—	0.35678	0.35678
		F_r	—	0.07556	0.07526
		error (%)	—	+0.59	+0.19
		CPU time (s)	—	3	3
b	12 x 6 x 8	N_{max}	6708	33541	33541
		Δt	0.29814	0.059628	0.059628
		F_r	0.05230	0.05282	0.05282
		error (%)	+0.17	+1.17	+1.17
		CPU time (s)	12	85	130
	12 x 2 x 3	N_{max}	—	21665	21665
		Δt	—	0.092312	0.092312
		F_r	—	0.05418	0.05440
		error (%)	—	+3.77	4.19
		CPU time (s)	—	8	10

For the empty cavity (a), the present scheme gives results comparable to FDTD. It was also found that D_4 gives better accuracy than D_2. However, for the inhomogeneous structure (b), the present scheme is less accurate than FDTD. When the inhomogeneity of the structure is smaller than the wavelength, the accuracy of the present scheme is not sufficient. We will show in the following example that the present scheme gives better accuracy than FDTD when the structure is inhomogeneous but electrically larger than the wavelength.

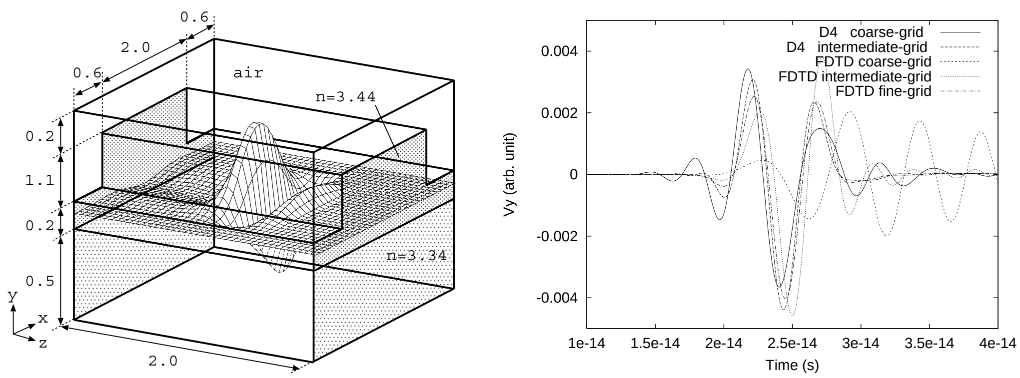
IV. Analysis of an Optical Waveguide Structure

Finally the present method was applied to an optical rib waveguide shown in Fig. 2 (a), which was previously analyzed by many authors with various numerical techniques, mostly with beam propagation methods [8] to keep the numerical analysis effort within reasonable limits.

The waveguide was analyzed with the D_4-based scheme as well as the standard FDTD using three different discretization levels: a coarse-grid ($\Delta x \times \Delta y \times \Delta z = 0.1 \times 0.1 \times 0.1 \mu m^3$); an intermediate-grid ($0.1 \times 0.1 \times 0.05 \mu m^3$); and a fine-grid ($0.05 \times 0.05 \times 0.025 \mu m^3$). The analysis region was completely surrounded by five-layer UPML-ABCs.

Figure 2 (a) also demonstrates the E_y field propagating in the waveguide analyzed with the D_4 scheme using the intermediate-gird, and Fig. 2 (b) shows the comparison of the resulting time series data picked up at 1.6 μm from the source point.

Note that results of D_4 with the intermediate-grid and of FDTD with the fine-grid agree well compared to the other results for the coarser girds that show highly dispersive waveforms. Although the CPU time was about 70 minutes for both D_4-intermediate-grid and FDTD-fine-gird, the executable sizes were 15 MB for D_4-intermediate-grid and 55 MB for FDTD-fine-gird. The executable size will be further reduced once the code is optimized. The details of the analysis will be presented at the conference.



(a) (b)

Fig. 2. (a) A straight optical rib waveguide and a snapshot of the E_y field distribution. The dimensions are in μm. n denotes the refractive indices. The wave travels from left to right. (b) Comparison of the time series data.

V. Conclusions

The three-dimensional time-domain wavelet-Galerkin method based on Daubechies' compactly supported scaling functions with three and four vanishing wavelet moments has been presented together with the implementation of the uniaxial-medium PML ABCs.

The proposed method was verified by analyzing resonant cavities, and then applied to an electrically large optical waveguide problem. The minimum support of Daubechies' scaling functions yields an effective algorithm, and the highly linear numerical dispersion property reduces the number of cells required in the analysis, thus reducing the full-wave analysis of electrically large inhomogeneous structures to a size that can be handled by workstations or personal computers.

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