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An Efficient Algorithm of 2-D FDTD Method Based on Finite-Difference Approximation of Wave Equation

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1 Introduction

Finite-Difference Time-Domain (FDTD) method [1], one of the most flexible tools to analyze electromagnetic fields, is the application result of the mid-point method to Maxwell's equations. Since Maxwell's equations are first-order simultaneous differential equations, the application of the mid-point method causes a half cell and a half time-step distances between electric and magnetic filed components. The distance does not matter by Yee's spatial arrangement of field components [1], and this considerable arrangement causes the widespread use of his algorithm. However, it would be more adequate if any approximation does not derive such distances. Actually it is natural to evaluate all the components at the same discrete points.

Besides, Yee's FDTD method requires large number of computational resources (computer memories) and long computation time. One of the reasons may be the number of computed components. Since Yee's method is a leapfrog algorithm which computes electric and magnetic field components alternatively, each component requires the other components to update itself. For example, we have to evaluate the components E_z, H_x and H_y to analyze two-dimensional (2-D) field of TM-mode even if we want to know only E_z. It would also be helpful if we can develop an algorithm to update each component irrespectively to other components. It enables shorter computation time.

This paper investigates an efficient algorithm of 2-D FDTD method by applying the mid-point method to wave equations. The proposed algorithm is called "WE-FDTD method" abbreviated from "Wave Equation-based FDTD method" to be distinguished with the conventional Yee's FDTD method. Throughout this paper, the 2-D TM-mode case is studied in detail. However, similar approach can easily be applied to 2-D TE-mode case.

2 Preliminaries

Maxwell's equations can be written as the following three simultaneous differential equations in 2-D TM-mode case:

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_0}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right), \quad \frac{\partial H_x}{\partial t} = \frac{1}{\mu_0}\frac{\partial E_z}{\partial y}, \quad \frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_z}{\partial x}, \tag{1}$$

where E_z, H_x and H_y denote the z-component of the electric field, the x- and y-component of the magnetic field, respectively. Besides, ε_0 and μ_0 respectively denote the permittivity and the permeability of a vacuum. The direct application of the 1st-order midpoint method to the above equations yields the Yee's method [1].

A wave equation which corresponds to the component E_z in 2-D TM-mode case is given as

$$\frac{\partial^2 E_z}{\partial t^2} = c_0^2 \nabla^2 E_z = c_0^2\left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2}\right), \tag{2}$$

where $c_0 := 1/\sqrt{\varepsilon_0 \mu_0}$ denotes the speed of light in a vacuum. The same equation holds for the magnetic components H_x and H_y. Also, the 2-D TE-mode case can be derived for H_z, E_x and E_y.

3 Proposed Algorithm

Applying the 2nd-order midpoint method

$$\frac{\partial^2 f}{\partial x^2}(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{(\Delta x)^2}, \quad \Delta x = x_{i+1} - x_i,$$

to (2), we have the following finite-difference equation:

$$\begin{aligned} E_z^{n+1}(i,j) &= 2E_z^n(i,j) - E_z^{n-1}(i,j) + \left(\frac{c_0 \Delta t}{\Delta x}\right)^2 \{E_z^n(i+1,j) - 2E_z^n(i,j) + E_z^n(i-1,j)\} \\ &\quad + \left(\frac{c_0 \Delta t}{\Delta y}\right)^2 \{E_z^n(i,j+1) - 2E_z^n(i,j) + E_z^n(i,j-1)\}. \end{aligned} \tag{3}$$

where $E_z^n(i,j)$ denotes the component E_z at the point $(i\Delta x, j\Delta y)$ when $t = n\Delta t$. Besides, the stability condition of (3) can be written as

$$\Delta t \leq \frac{\Delta}{c_0\sqrt{2}}, \quad \Delta = \min\{\Delta x, \Delta y\}.$$

Introducing the parameter a which satisfy

$$\Delta t = a\frac{\Delta}{c_0}, \quad 0 < a \leq \frac{1}{\sqrt{2}},$$

the proposed algorithm (3) can be rewritten as

$$\begin{aligned} E_z^{n+1}(i,j) = (2-4a^2)E_z^n(i,j) - E_z^{n-1}(i,j) + a^2\{&E_z^n(i+1,j) + E_z^n(i-1,j) \\ &+ E_z^n(i,j+1) + E_z^n(i,j-1)\}. \end{aligned} \tag{4}$$

From (3) and (4), we see that the component $E_z^{n+1}(i,j)$ can be computed from the previous values at the same point: $E_z^n(i,j)$, $E_z^{n-1}(i,j)$, and the previous values at the surrounding points: $E_z^n(i+1,j)$, $E_z^n(i-1,j)$, $E_z^n(i,j+1)$ and $E_z^n(i,j-1)$. The equations (3) and (4) are of only E_z, thus we can recursively update E_z without computing the magnetic components H_x and H_y. It leads the smaller memories to be required in computer programs.

The equations (3) and (4) cannot correspond to conditions for magnetic components, however a slight modification enables the equations to correspond to such conditions. It is studied with an actual model in Section 4.

4 Simulation

4.1 Analysis of an Example 2-D Field

In this section, we analyze an example 2-D field to see if the proposed algorithm really works. The analyzed field is illustrated in Fig. 1, and the specifications of this simulation is summarized as Table 1. It is mentioned here that the proposed algorithm can employ the Mur's 2nd-order boundary condition [2], similarly as the Yee's method could. The incident voltage is given by

$$E_z^n = e^{-\alpha(n-\beta)^2}, \quad (\beta = 32.0,\ \alpha = 4\beta^{-2}),$$

also it is illustrated in Fig. 2.

In this simulation, an electric and a magnetic walls are placed in the field as drawn in Fig. 1. The magnetic condition $H_y = 0$ should be replaced into a condition of E_z to correspond to the proposed algorithm. The condition $H_y = 0$ is first reduced into the partial difference: $\partial H_y/\partial t = 0$, and then it is replaced by $\partial E_z/\partial x = 0$ from the last equation of (1). Based on the above discussion and the parabolic approximation, the components E_z inside the magnetic wall are updated by

$$E_z^{n+1}(i,j) = \frac{4}{3}E_z^{n+1}(i-1,j) - \frac{1}{3}E_z^{n+1}(i-2,j).$$

On the other hand, the components E_z at the right side of the magnetic wall are updated by

$$E_z^{n+1}(i,j) = \left(2 - \frac{8}{3}a^2\right) E_z^n(i,j) - E_z^{n-1}(i,j) \ +a^2 \left\{\frac{2}{3}E_z^n(i+1,j) + E_z^n(i,j+1) + E_z^n(i,j-1)\right\},$$

without using the components inside the magnetic wall. This equation is also based on the parabolic approximation.

Figure 3 and 4 illustrate the behavior of E_z in the cases $t = 30\Delta t$ and $50\Delta t$, respectively. These figures show that the proposed method really works. The results by the proposed method are quite similar but slightly different to those by Yee's method. This is because that Yee's method cannot follow the original magnetic condition. The magnetic wall has to be located with a half cell distance from the original place in Yee's method.

4.2 Comparison of the Computing Process

Comparing the Yee's and the proposed methods with respect to the computer memories in the 2-D case, the Yee's method requires $7N$: corresponding to E_z^{n+1}, E_z^n, E_z^{n-1}, $H_x^{n+\frac{1}{2}}$, $H_x^{n-\frac{1}{2}}$, $H_y^{n+\frac{1}{2}}$ and $H_y^{n-\frac{1}{2}}$, where N denotes the number of cells in the analyzed field (for example, $N = 40 \times 40 = 1600$ in the case of Fig. 1). Note that the E_z^{n-1} components are preserved to compute boundary elements. On the other hand, the proposed method requires $3N$: corresponding to E_z^{n+1}, E_z^n and E_z^{n-1}. The required memories are reduced into three-seventh.

Table 2 shows the required time to compute one thousand time-steps for the assigned fields. The simulation has been done by Gateway G6-300. The proposed algorithm spends 25–50% of the time required to Yee's method.

5 Concluding Remarks

This paper proposed a new algorithm of 2-D FDTD method by applying the mid-point method to wave equations. A simple example analysis in 2-D TM-mode shows that the algorithm really works effectively. It was also presented that the proposed algorithm reduces both required memories and computing time into three-seventh and 25–50% respectively, in comparison with the Yee's method. Besides, the 2-D TE-mode case can be considered similarly to TM-mode case.

It should be noted that the length of one cell does not make any sense in Yee's method, since the magnetic conditions are given with a half cell distance against the electric ones. The cell length must be shortened to dissolve such problem, and it requires more memories and computation time. On the other hand, the one cell length works as it is in the proposed method. Magnetic conditions are replaced by electric conditions without any distance as shown in Section 4, thus we do not have to shorten the cell length.

The 3-D algorithm would be considered in the similar manner. It should be studied as a future study.

References

[1] K. S. Yee: "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media," *IEEE Trans. Antennas and Propagation*, vol. 14, no. 4, pp. 302–307, 1966.

[2] G. Mur: "Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic-Field Equations," *IEEE Trans. Electromagnetic Compatibility*, vol. 23, no. 4, pp. 377–382, 1981.

Table 1: Specifications of the simulation

Analyzed field	45×45 [mm^2]
Cell size $\Delta x, \Delta y$	1mm
Time step Δt	$\Delta / c_0\sqrt{2}$ $(a = 1/\sqrt{2})$
Boundary condition	Mur's 2nd-order [2]

Table 2: Comparison of the required time

Number of Cells	Yee's	Proposed
40×40	1.04sec	0.49sec
100×100	13.41sec	3.29sec
200×200	61.03sec	25.92sec
400×400	246.94sec	106.50sec

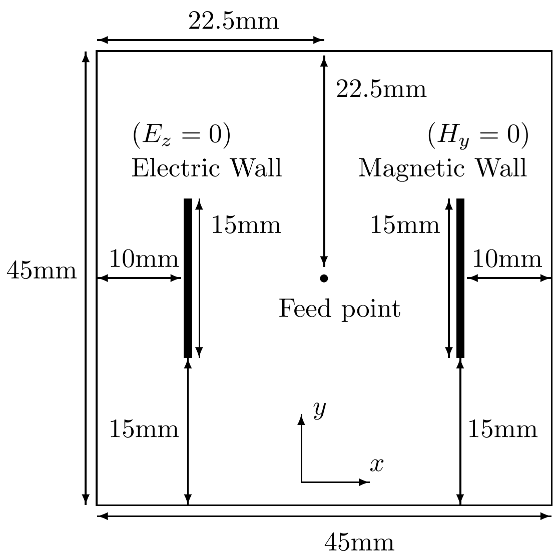

Figure 1: The field to be analyzed

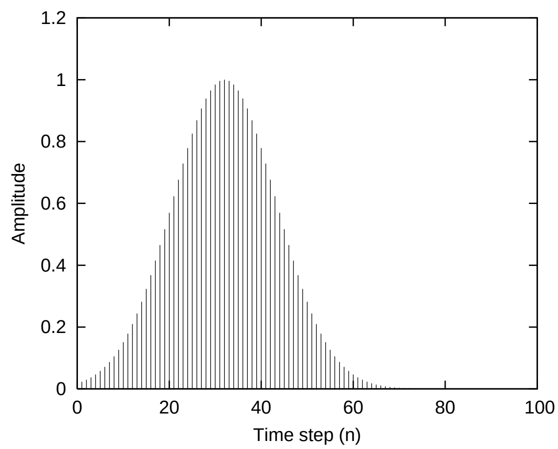

Figure 2: The incident voltage

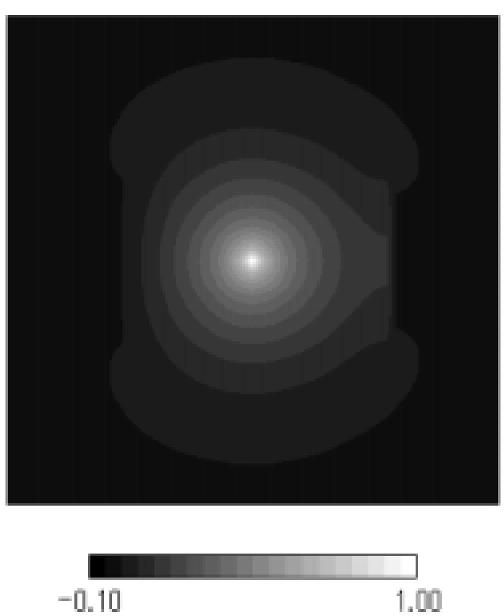

Figure 3: Behavior of E_z in the case $t = 30\Delta t$

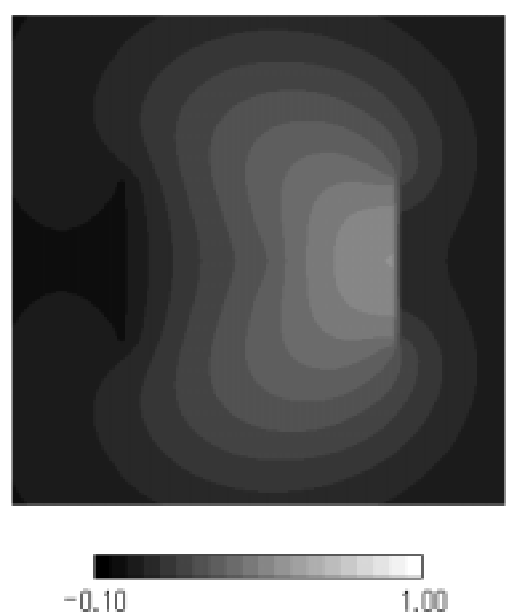

Figure 4: Behavior of E_z in the case $t = 50\Delta t$