# TREATMENT OF RADIATION FIELDS FROM CHARGES IN THE VECTOR POTENTIAL SPATIAL NETWORK BY USING THE NETWORK FOR SCALAR POTENTIAL BASED ON THE LORENTZ GAUGE CONDITION

Norinobu YOSHIDA, Hiroaki SATOH, and Yoshikazu MIYANAGA
Division of Electronics & Information Engineering
Graduate School of Engineering, Hokkaido University
North-13, West-8, Sapporo-shi, Hokkaido 060-8628, Japan
e-mail:yoshidan@rose.hucc.hokudai.ac.jp

## 1. Introduction

"Electromagnetic Interference (EMI)" has become serious problems as development of the digital technology, such as the downsizing in devices and performing high-speed operation. Since these problems are so complicated that the numerical analyses on the three-dimenional space and time domain become very effective. But the conventional time-dependent methods such as FD-TD or TLM essentially simulate only the solenoidal propagation field due to the assumption in the formulation that the divergence of both electric and magnetic field is assumed to be zero. Therefore, some radiation mechanisms in the EMI problems such as "Electro Static Discharge (ESD)", in which the radiation from not only current source but also space charges is included, cannot be analyzed by only above methods. The scalar field must be analyzed separately.

I have already proposed the adaptation of the vector potential field to the time-dependent numerical methods, especially, to the Condensed Node "Spatial Network Method (SNM)"[1]-[4]. In the formulation, both the vector and scalar potential can be connected generally by using the equivalent current or voltage sources for the Lorentz gauge condition. This gauge corresponds physically to the conservation law between charges and discharge currents. Therefore, the proposed method may be useful in the analysis of the field radiated from both current and charge sources. In this paper, basic validity of the treatment of the radiation fields from charges by using such the equivalent circuits for the vector and scalar potential is presented.

## 2. Formulation

The characteristic equations for the vector potential are given as follows by using the magnetic vector potential "A" and the electric vector potential "S".

$$\nabla\times A = \sigma^* S + \mu_0\frac{\partial S}{\partial t} \quad (\equiv B) \qquad (1a), \qquad \nabla\times S = -\sigma A - \varepsilon_0\frac{\partial A}{\partial t} \quad (\equiv D) \qquad (1b)$$

Here, $\sigma^*$ and $\sigma$ are the hypothetical magnetic conductivity and the electric conductivity, respectively. In Figure 1, the condensed node expression for the vector potential is shown [3]. At each port, each of the propagation quantities " $A_1 \sim A_{12}$;($V \pm z_0 I$)" is assignedto give the connection between adjacent nodes. The wave equation for the magnetic vector po-tential is given as follows.

$$\nabla^2 A - (\varepsilon_0\sigma^* + \mu_0\sigma)\frac{\partial A}{\partial t} - \varepsilon_0\mu_0\frac{\partial^2 A}{\partial t^2} = \nabla\nabla\cdot A \qquad (2)$$

The Lorentz gauge condition between the magnetic vector potential A and the electric scalar potential $\phi$ is given as follows;

$$\nabla\cdot A = -\varepsilon_0\mu_0\frac{\partial\phi}{\partial t} - \mu_0\sigma\phi \quad (\equiv -\mu_0 F) \qquad (3)$$

Here, "F" function is defined as follows ;

$$F = \varepsilon_0\frac{\partial\phi}{\partial t} + \sigma\phi \qquad (4a)$$

On the other hand, the scalar electric fieldhas the following conventional definition.

$$\nabla\cdot E_S = \frac{\rho}{\varepsilon_0} \qquad (4b)$$

From Equs.(4), the characteristic equations for the scalar field can be given as follows;

$$-\nabla F = \varepsilon_0\frac{\partial E_S}{\partial t} + \sigma E_S \qquad (5a), \qquad -\nabla\cdot E_S = \mu_0\frac{\partial F}{\partial t} - \frac{\rho}{\varepsilon_0} \qquad (5b)$$

The equivalent circuit for the electric scalar field is presented in Figure 2. By using these equations and the divergence relation of between currents and chrgaes such as $\nabla\cdot J_s = -\partial\rho/\partial t - \sigma\rho/\varepsilon_0$, the left hand side of Equ.(2) can be rewritten by the equivlent current source $J_s$ as follows;

$$\nabla\nabla\cdot A = -\mu_0\nabla F \equiv -\mu_0 J_s \qquad (6)$$

This current source is used in the spatial network for the vector potential in Figure 1.

## 3. Results and Discussion

In Figure 3, the analyzed model is shown. Figure 4 for the field characteristics from a sinusoidally changing monopole current source gives only the rotational component, that can be simulated by the conventional time-dependent analysis methods. Figure 5 gives that the divergent component is dominant for the sinusoidally changing point charge. The solenoidal component is slightly produced by the numerical errors. Figures 6 and 7 give the field characteristics for a dipole which changes as the raised cosine form. The latter shows the largerotational component from the current in the connecting conductor. In each figure, the variations in (a) and (b) seem to be nearly coincident because of the definition of "F" based on the term " $\nabla\cdot A$ ".


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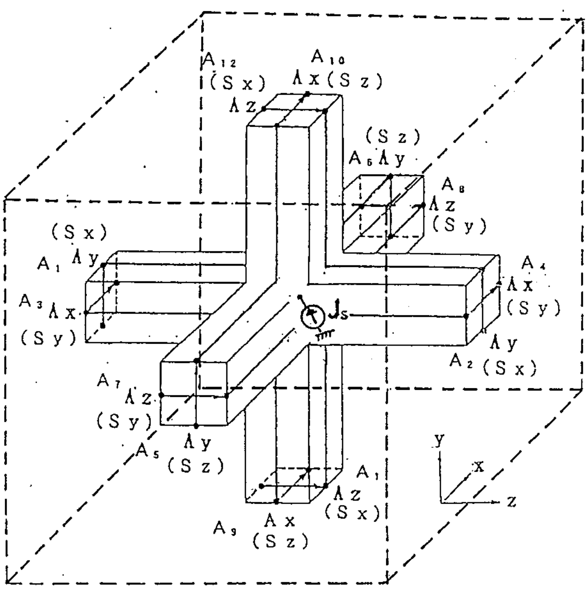



**Fig.1 Conceptual condensed node spatial network for vector potential with the current source defined in Equ.(6).**

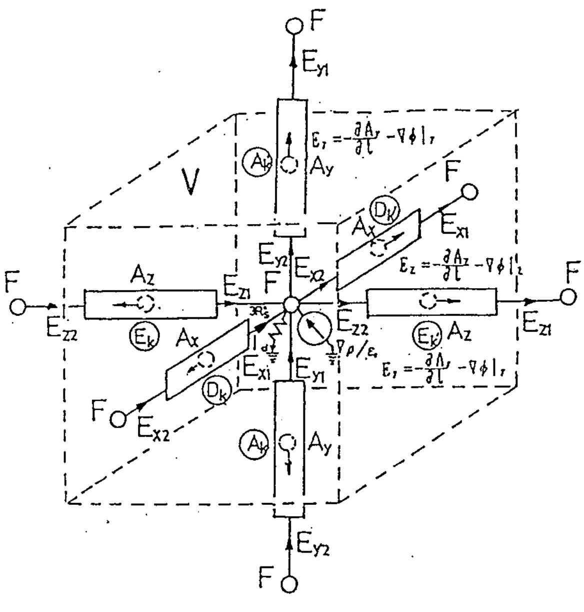



**Fig.2 Equivalent spatial network in each node for scalar electric field.**

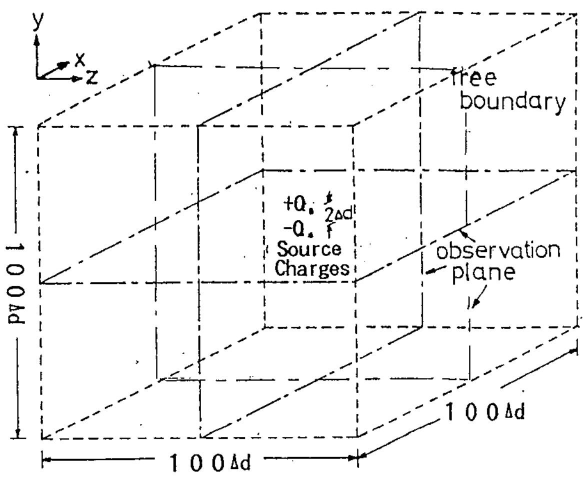
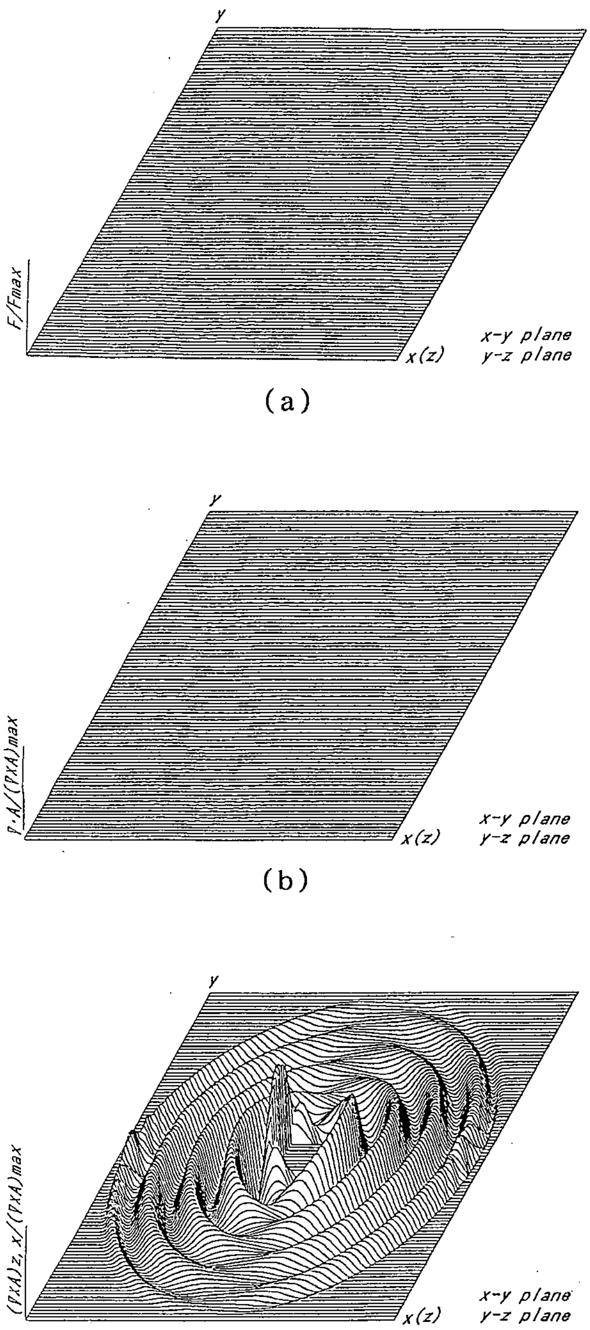



**Fig.3 Analyzed Model**



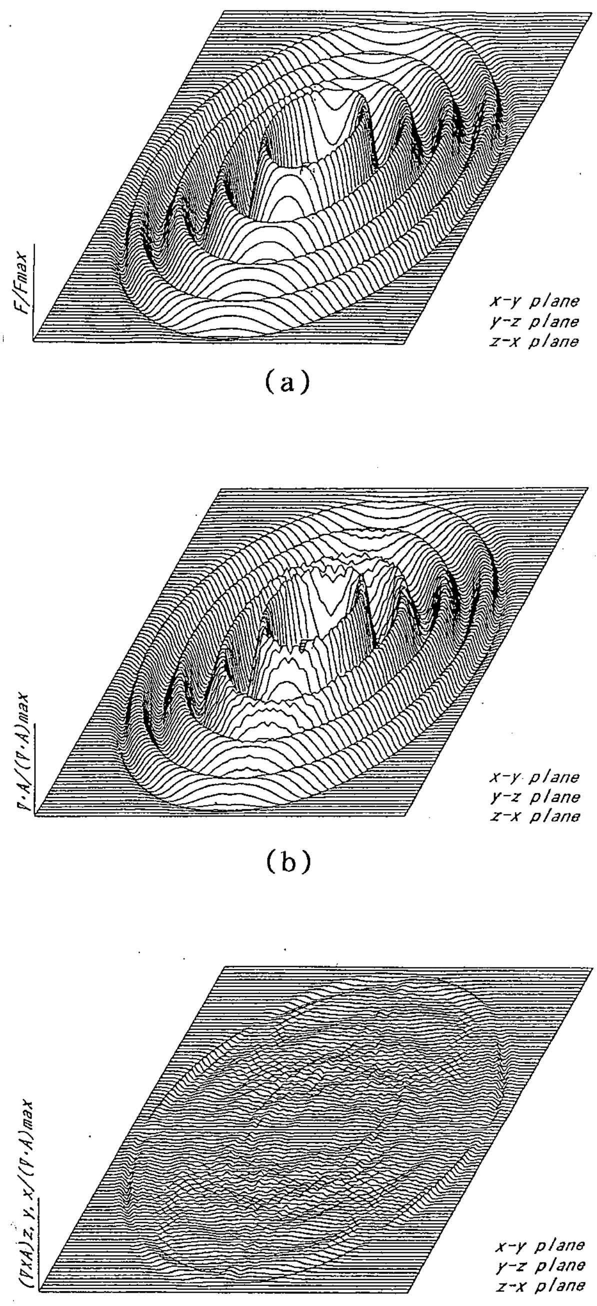
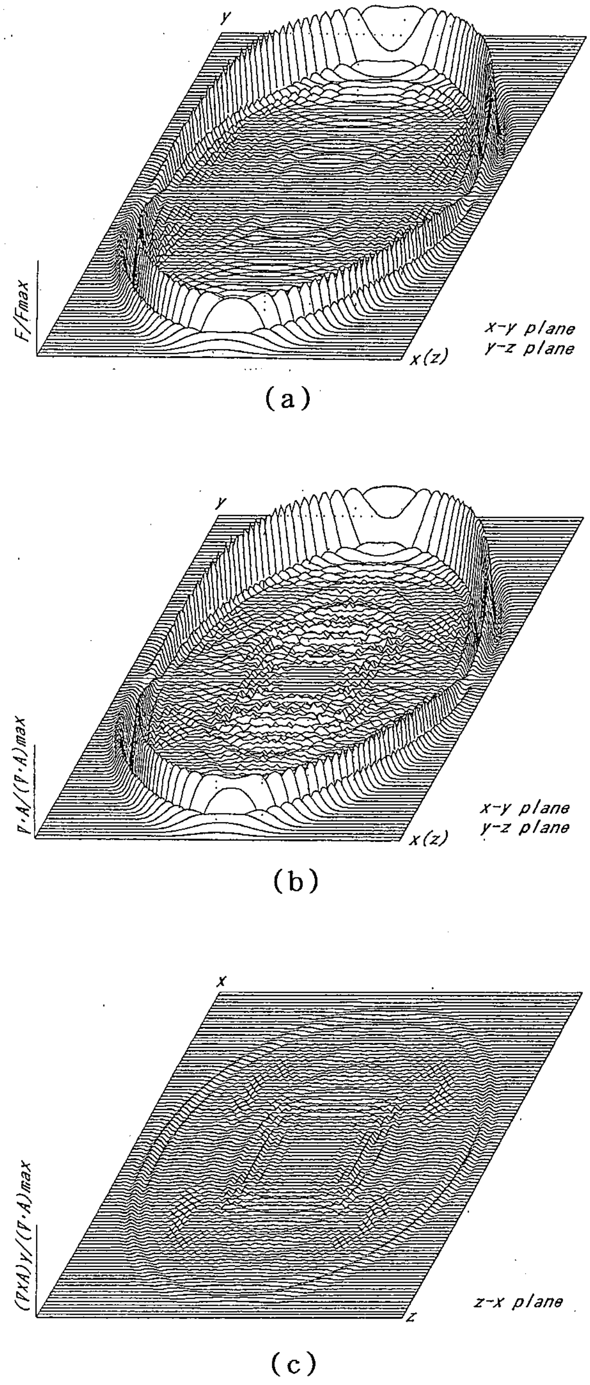




**Fig.4 Field Characteristics for the changing monopole current source.**
($J(t)=\sin(2\pi t/T)$)

(a) $F/F_{max}$
(b) $\nabla\cdot A/(\nabla\times A)_{max}$
(c) $\nabla\times A/(\nabla\times A)_{max}$

**Fig.5 Field Characteristics for the changing point charge.**
($Q(t)=\sin(2\pi t/T)$)

(a) $F/F_{max}$
(b) $\nabla\cdot A/(\nabla\cdot A)_{max}$
(c) $\nabla\times A/(\nabla\cdot A)_{max}$



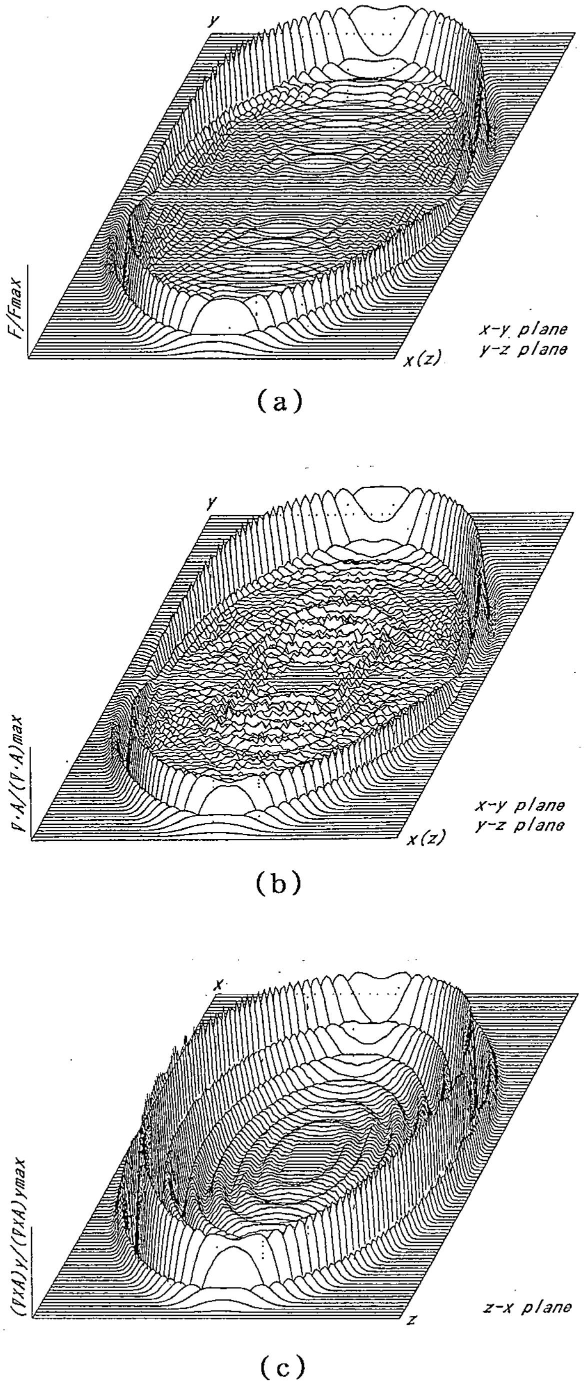




**Fig.6 Field Characteristics for the changing dipole.**
$(Q(t)=\pm(1-\cos(2\pi t/T)^2\ )$

(a) $F/F_{max}$
(b) $\nabla\cdot A/(\nabla\cdot A)_{max}$
(c) $\nabla\times A/(\nabla\cdot A)_{max}$

**Fig.7 Field Characteristics for the changing dipole with a connecting conductor.**
$(Q(t)=\pm(1-\cos(2\pi t/T)^2\ )$

(a) $F/F_{max}$
(b) $\nabla\cdot A/(\nabla\cdot A)_{max}$
(c) $\nabla\times A/(\nabla\times A)_{max}$