ANALYSIS OF COUPLED TRANSMISSION LINES IN ANISOTROPIC LAYERED MEDIA

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1. Introduction

In this paper, the effect of material anisotropy on the coupled transmission lines is studied. Dielectric anisotropy is present in many substrate materials (such as boron nitride and sapphire) used for microwave/millimeter wave integrated circuits. It could have important implication on circuit operation at high operating frequency. Accurate evaluation for the effect of the material anisotropy is crucial for the microwave circuit design. In the past, striplines or microstrip lines embedded in anisotropic multilayered media were analyzed by numerous researchers in this field, and results can be found in the related literature [1]-[4]. Most of the previous work treated only single stripine structures or edge-coupled multiple striplines, and less effort was placed on the broadside-coupled stripline systems. In this research, a volume integral equation formulation is proposed for analysis of the coupled striplines embedded in anisotropic lossy nonmagnetic layered media. In the formulation, an electric–type dyadic Green's function for the planar multilayered media based on the transmission-line network analog along the axis normal to the stratification is developed first [5]. The number of the background dielectric layers is assumed arbitrary, and the dielectrics can be lossy in nature. In addition, the microstrip lines can reside in different layers, and their cross sections can be of arbitrary shape. The electric field can then be expressed as a convolution integral of the dyadic Green’s function and the equivalent currents on the metal strips. Finally, the integral equation is solved using Galerkin's method of moments to obtain the propagation and attenuation constants for the coupled striplines numerically.

In this work, the multilayered structures with strip thickness on the order of the skin depth are of primary interest. The effects of the material anisotropy on edge coupling as well as broadside coupling behaviors were examined. The results were compared with other researchers' data, which showed good agreement.

2. Theoretical Analysis

The multilayered structure with conductor strips embedded in layers ℓ and m is depicted in Fig. 1. Both the dielectrics and the conductors can be lossy. To solve the problem, the spectral Green’s function for the planar multilayered medium is first derived. Assuming uniformity along the propagation direction y for the whole structure, and an $e^{j\omega t}$ time dependence, the electric field in this layer ℓ can be expressed, in terms of the dyadic Green's function, as

$$\bar{E}_\ell(\bar{r}) = \int_{V_\ell} dv' \left[\bar{\bar{G}}^{PV}_{\ell\ell}(\bar{r},\bar{r}') + \frac{j}{\omega\varepsilon_0\varepsilon'_{rz,\ell}}\delta(\bar{r}-\bar{r}')\hat{z}\hat{z} \right]\cdot \bar{J}_{eq,\ell}(\bar{r}') + \int_{V_m} dv' \bar{\bar{G}}_{\ell m}(\bar{r},\bar{r}')\cdot \bar{J}_{eq,m}(\bar{r}') \,. \quad (1)$$

Here $\bar{\bar{G}}_{\alpha\beta}(\bar{r},\bar{r}')$ is the dyadic Green’s function representing the electric field in layer α generated by a source of unit strength located in layer β, and the one with superscript PV stands for the principal value part of the corresponding Green’s function. In addition, $\frac{j}{\omega\varepsilon_0\varepsilon'_{rz}}\delta(\bar{r}-\bar{r}')\hat{z}\hat{z}$ is the source dyadic, $\vec{J}_{eq,\alpha} = \Delta\overline{\overline{\sigma}}_\alpha(\bar{r})\cdot\vec{E}(\bar{r})$ is the equivalent current (conduction and polarization) of the conductor in layer α with $\Delta\overline{\overline{\sigma}}_\alpha(\bar{r}) = \overline{\overline{\sigma}}_{s\alpha}(\bar{r}) - \overline{\overline{\sigma}}_\alpha + j\omega\left[\overline{\overline{\varepsilon}}^{\,r}_{s\alpha}(\bar{r}) - \overline{\overline{\varepsilon}}^{\,r}_\alpha\right]$, and $\alpha = \ell, m$. Here, $\overline{\overline{\sigma}}_{s\alpha}$ and $\overline{\overline{\varepsilon}}^{\,r}_{s\alpha}$ represent, respectively, the conductivity and dielectric constant of the conductor strip in layer α; $\overline{\overline{\sigma}}_\alpha$ and $\overline{\overline{\varepsilon}}^{\,r}_\alpha$ denote the corresponding quantities of the background dielectric. Similarly, the electric field in layer m can be expressed as

$$\vec{E}_m(\bar{r}) = \int_{V_\ell} dv' \vec{G}_{m\ell}(\bar{r},\bar{r}')\cdot\vec{J}_{eq,\ell}(\bar{r}') + \int_{V_m} dv' \left[\vec{G}^{PV}_{mm}(\bar{r},\bar{r}') + \frac{j}{\omega\varepsilon_0\varepsilon'_{rz,m}}\delta(\vec{r}-\vec{r}')\hat{z}\hat{z}\right]\cdot\vec{J}_{eq,m}(\bar{r}'). \quad (2)$$

The coupled integral equations (1) and (2) can be solved using Galerkin's method of moments, where eigenmode electric field on the conductor cross-section S is represented by a set of pulse basis functions. The Galerkin's procedure will result in a matrix equation, from which both the phase constant and the attenuation constant are solved. Computed results of some sample structures of Fig. 1 using the approach addressed above are presented in the next section.

3. Results and Discussions

In this section we will present the numerical results computed for some sample structures of Fig. 1. For the study of coupled striplines in an anisotropic medium, we first compute an edge-coupled stripline structure with background dielectric constant fixed. The results plotted in Fig. 2 show a similar behavior with that of the isotropic case studied by Kiang *et. al* [6], and good agreement is observed as compared with the data presented in [7]. As is expected, the propagation constant is smaller for the odd modes than for the even modes.

Next, to check the effect of the material anisotropy on the coupling behavior, the edge-coupled structures with ε_z varied, ε_t fixed, and with ε_t fixed, ε_z varied are calculated. For the even modes, the field lines emerged from inner edges of both striplines will turn the path toward the upper and lower ground planes due to repulsion, and hence varying ε_z will dominates the variation of dispersion characteristics. On the other hand, the field lines are going in the transverse direction from one edge to the other in the region between the striplines for the odd modes. So in this case, ε_t will have stronger effect on the dispersion relation. Based on the above statements, if ε_z of the background medium is increased, the propagation constant and the conductor loss of the even modes will increase more than as they do for the odd modes, as shown in Fig. 3. However, if ε_t is increased, the propagation constant and the conductor loss of the odd modes are seen to increase more as it shows in Fig. 4. The broadside-coupled structures, which show the contrast behavior to those edge-coupled striplines, are also calculated, and the results are shown in Fig. 4 and Fig. 5.

4. Conclusions

A volume integral equation formulation was proposed to study the dispersion relation of the coupled striplines in anisotropic layered media. Results show that for the edge-coupled striplines, the variation of transverse (longitudinal) dielectric constant has stronger effect on the odd mode (even mode) dispersion relations due to different field line distributions. The dispersion relations of the broadside-coupled structures, which show contrast behavior to those of the edge-coupled striplines due to material anisotropy, are also examined.

Acknowledgement

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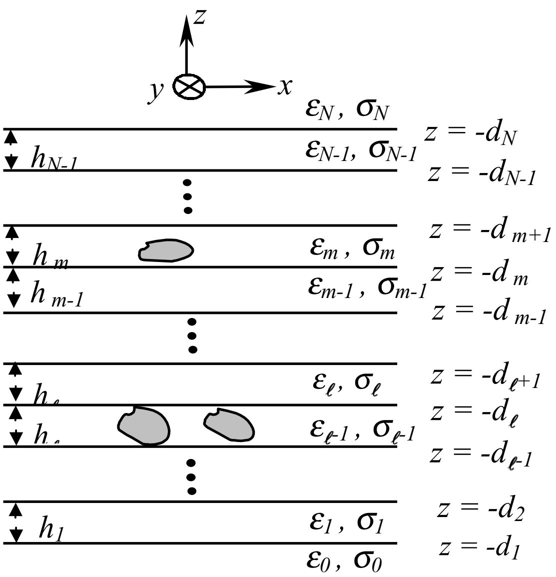


Fig. 1 Conductor strips of arbitrary cross section embedded in a layered medium.

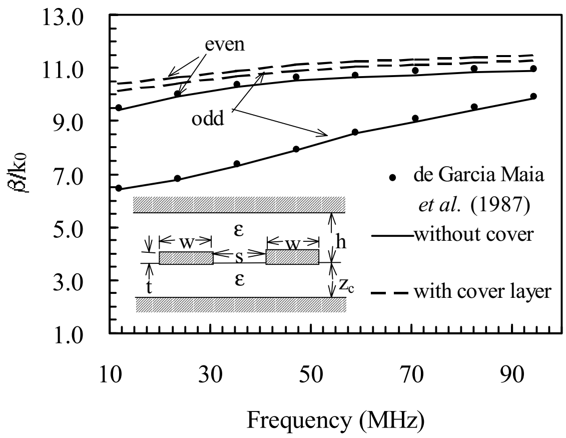
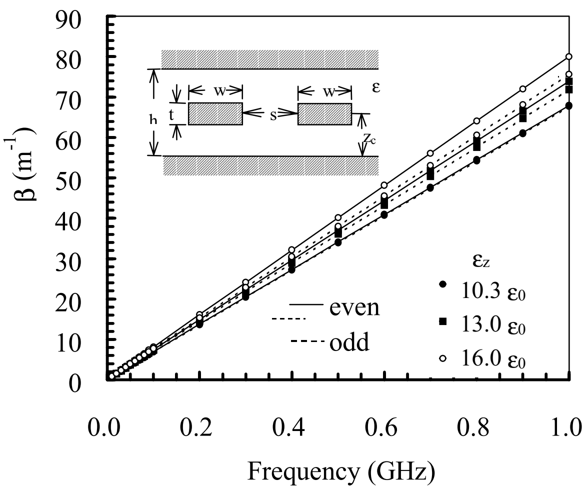
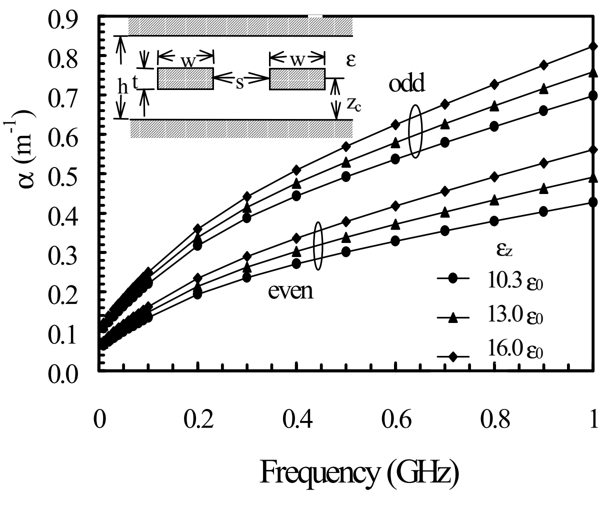


Fig. 2 Propagation constant of edge-coupled striplines in an anisotropic meduium : w = 507 μm, t = 0.1 μm, h = 507 μm, z_c = 507 μm, s = 507 μm, $\varepsilon_{xx} = \varepsilon_{yy} = 9.4\varepsilon_0$, $\varepsilon_{zz} = 11.6\varepsilon_0$, $\sigma = 5.92 \times 10^7$ S/m.

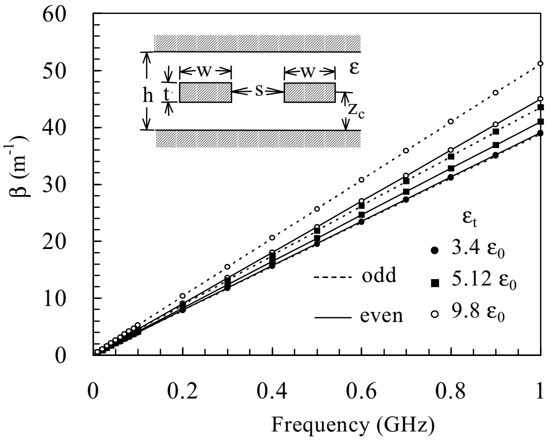


(a) Propagation constant

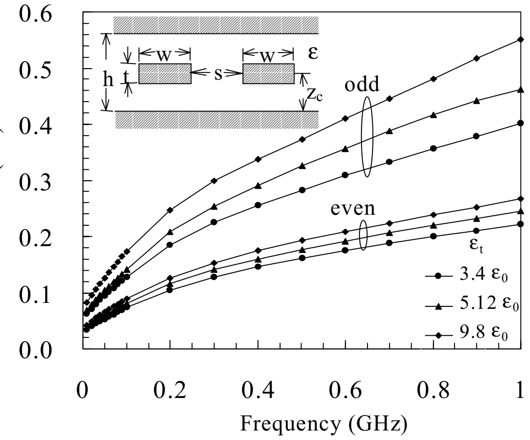


(b) Attenuation constant

Fig. 3 Dispersion relation of edge-coupled striplines for different ε_z: w = 100 μm, t = 25 μm, h = 600 μm, s=100 μm, z_c = 300 μm, $\varepsilon_t = 10.3\varepsilon_0$, $\sigma = 5.92 \times 10^7$ S/m.

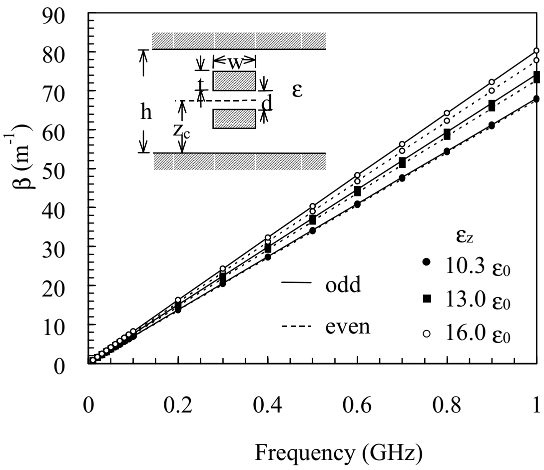


(a) Propagation constant

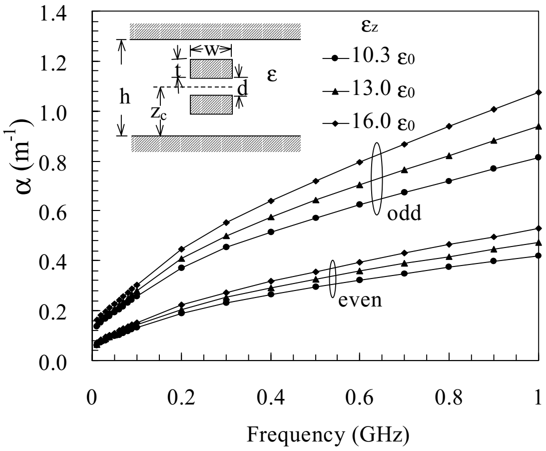


(b) Attenuation constant

Fig. 4 Dispersion relation of edge-coupled striplines for different ε_t: w = 100 μm, t = 25 μm, h = 600 μm, s=100 μm, z_c = 300 μm, $\varepsilon_z = 3.4\varepsilon_0$, $\sigma = 5.92 \times 10^7$ S/m.

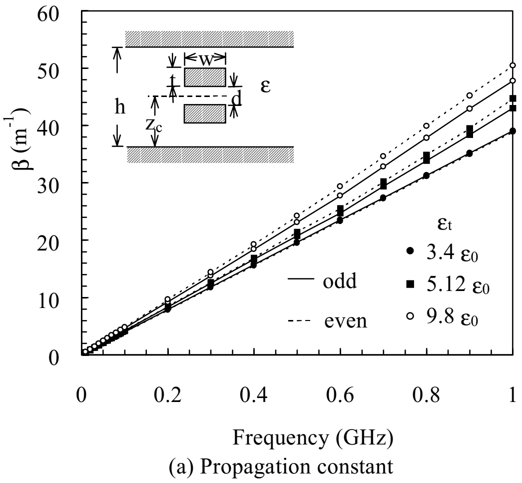


(a) Propagation constant

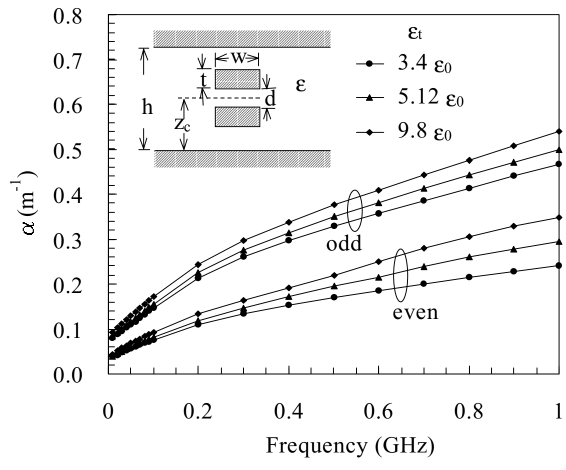


(b) Attenuation

Fig. 5 Dispersion relation of broadside-coupled striplines for different ε_z: w = 100 μm, t = 25 μm, h = 600 μm, d = 100 μm, z_c = 300 μm, $\varepsilon_t = 10.3\varepsilon_0$, $\sigma = 5.92 \times 10^7$ S/m.



(a) Propagation constant



(b) Attenuation constant

Fig. 6 Dispersion relation of broadside-coupled striplines for different ε_t: w = 100 μm, t = 25 μm, h = 600 μm, d=100 μm, z_c = 300 μm, $\varepsilon_z = 3.4\varepsilon_0$, $\sigma = 5.92 \times 10^7$ S/m.