ANALYSIS OF BICONICAL ANTENNAS USING GENERALIZED SCATTERING MATRICES

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1. Introduction

Biconical antennas are widely employed as transmitting antennas in various communication systems. They mainly utilize TEM (rarely TE_{01}) coaxial mode excitation to produce omnidirectional radiation patterns in the azimuth plane. Angular dependent modes can also be used to generate multibeam radiation patterns [1].

Theoretical analysis of biconical antennas is mostly based on spherical waves, which satisfy the boundary conditions at linear conical surfaces under assumption, that both cones are excited symmetrically at the apices by a voltage source [2, 3]. Therefore a feed unit is not taken into account and has to be completed experimentally.

In this paper a new approach to analysis of biconical antennas is presented. A key feature of the approach is building of a generalized scattering matrix of a biconical antenna.

2. Mathematical Model of a Biconical Antenna

A biconical antenna is divided by cylindrical surfaces into sections of regular radial waveguides with length much less than wavelength, as schematically shown in Fig. 1. An open end of the outer radial waveguide is an antenna aperture. Fields in the waveguides are represented using series of eigenfunctions. By cascading the generalized scattering matrix of the junction between coaxial and radial waveguides and the generalized scattering matrix of the radially irregular structure formed by the conical surfaces an overall generalized scattering matrix of the biconical antenna is determined. This matrix completely defines the antenna including the reflection coefficient and field amplitudes at the aperture under any chosen excitation mode. The Kirchhoff-Huygens method is used to obtain the far-field radiation pattern of the antenna.

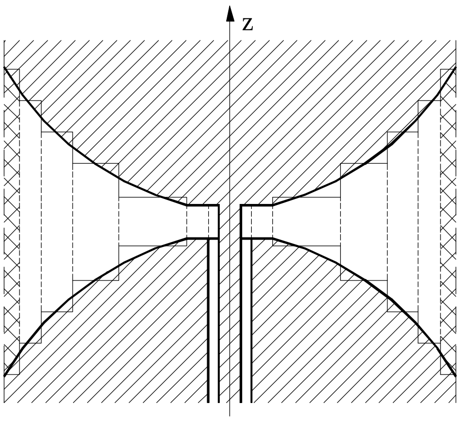

Fig. 1

The proposed method allows to analyze biconical antennas with non-linear elements of cones and to take into account a feed geometry. Furthermore, partial or full cylindrical dielectric filling can also be included into analysis (Fig. 1).

The solution for a general junction between two radial waveguides (for instance, r_1 and r_2 in Fig. 2) and determining of the generalized scattering matrix for a radially irregular structure have been considered in [4]. Closed form expressions for the components of the radiated far fields for an open ended radial waveguide also can be found in [4]. In this paper we will consider the analysis of a coaxial waveguide to radial waveguide junction.

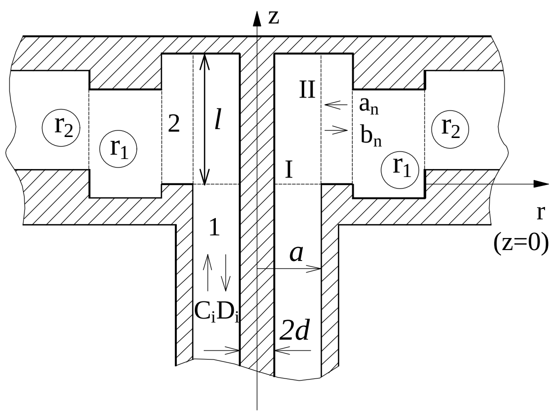

Fig. 2

3. Multiwave Analysis of a Coaxial Waveguide to Radial Waveguide Junction

The diffraction problem of a coaxial waveguide to radial waveguide junction has not been solved in a generalized form so far. Known mathematical models are limited to the TEM mode in a coaxial line and formulated in terms of input admittance normalized to the coaxial aperture [5]. In this paper the solution of the problem is obtained in terms of generalized scattering matrix, which describes all possible operation modes of the junction.

Transversal (to the *r*-direction) components of the electric and magnetic fields in a radial waveguide are given by (azimuthal index m is omitted for brevity)

$$\begin{aligned}\vec{E}_t &= \sum_{n=0}^{\infty}\left(a_n^E H_m^{(1)}(k_n r)+b_n^E H_m^{(2)}(k_n r)\right)\vec{\Phi}_n^E+\sum_{n=1}^{\infty}\left(a_n^H H_m'^{(1)}(k_n r)+b_n^H H_m'^{(2)}(k_n r)\right)\left(\vec{\Phi}_n^H\vec{e}_j\right)\vec{e}_j = \\ &= \sum_{n=0}^{\infty}A_n^E\vec{\Phi}_n^E+\sum_{n=1}^{\infty}A_n^H\left(\vec{\Phi}_n^H\vec{e}_j\right)\vec{e}_j\,,\end{aligned} \tag{1}$$

$$\begin{aligned}\vec{e}_r\times\vec{H}_t &= \sum_{n=0}^{\infty}y_n^E\left(a_n^E H_m'^{(1)}(k_n r)+b_n^E H_m'^{(2)}(k_n r)\right)\left(\vec{\Phi}_n^E\vec{e}_z\right)\vec{e}_z - \\ &-\sum_{n=1}^{\infty}y_n^H\left(a_n^H H_m^{(1)}(k_n r)+b_n^H H_m^{(2)}(k_n r)\right)\vec{\Phi}_n^H=\sum_{n=0}^{\infty}B_n^E\left(\vec{\Phi}_n^E\vec{e}_z\right)\vec{e}_z-\sum_{n=1}^{\infty}B_n^H\vec{\Phi}_n^H\,,\end{aligned} \tag{2}$$

where indices *E* and *H* mean TM and TE waves, respectively; $H_m^{(1)}$ and $H_m^{(2)}$ are Hankel functions of the first and second kind; $k_n=\sqrt{k_0^2-g_n^2}$; k_0 is the wave propagation constant in free space; $g_n=np/l$; $\vec{\Phi}_n^E$ and $\vec{\Phi}_n^H$ are the radial waveguide eigenfunctions

$$\vec{\Phi}_n^E=\cos(g_n z)\cos(mj)\vec{e}_z+\frac{mg_n}{k_n^2 r}\sin(g_n z)\sin(mj)\vec{e}_j\,; \tag{3}$$

$$\vec{\Phi}_n^H=-\frac{mg_n}{k_n^2 r}\cos(g_n z)\cos(mj)\vec{e}_z+\sin(g_n z)\sin(mj)\vec{e}_j\,; \tag{4}$$

$y_n^E=-j\dfrac{wee_0}{k_n}$ and $y_n^H=-j\dfrac{k_n}{wmm_0}$ are the wave admittances. When $m=0$

$$\vec{\Phi}_n^E=\cos(g_n z)\vec{e}_z\,;\qquad \vec{\Phi}_n^H=\sin(g_n z)\vec{e}_j\,. \tag{5}$$

At first we define a boundary condition on the surface II as $E_t^{II}=0$ and an exciting electric field on the surface I by

$$\vec{E}_t^I=\sum_i\left(C_i^{H,E}+D_i^{H,E}\right)Z_i^{H,E}\vec{h}_i^{H,E}\times\vec{e}_z=\sum_i G_i^{H,E}Z_i^{H,E}\vec{h}_i^{H,E}\times\vec{e}_z\,, \tag{6}$$

where $\vec{h}_i$ is the transverse magnetic field of mode *i* ($m\neq 0$):

for TE modes : $$\vec{h}_i^H=-j\frac{g_i^H}{k_i^H}X_m'(k_i^H r)\sin(mj)\vec{e}_r-j\frac{m}{k_i^H r}\frac{g_i^H}{k_i^H}X_m(k_i^H r)\cos(mj)\vec{e}_j \tag{7}$$

for TM modes : $$\vec{h}_i^E=-j\frac{m}{k_i^E r}\frac{k_0}{k_i^E}eL_m(k_i^E r)\sin(mj)\vec{e}_r-j\frac{k_0}{k_i^E}eL_m'(k_i^E r)\cos(mj)\vec{e}_j \tag{8}$$

when $m=0$: $$\vec{h}_i^H=-j\frac{g_i^H}{k_i^H}X_0'(k_i^H r)\vec{e}_r\,;\qquad \vec{h}_i^E=-j\frac{k_0}{k_i^E}eL_0'(k_i^E r)\vec{e}_j\,; \tag{9}$$

Z_i is the wave impedance of mode *i*: $Z_i^H=\dfrac{k_0 Z_0}{g_i^H}m$, $Z_i^E=\dfrac{g_i^E Z_0}{k_0 e}$; $g_i^{H,E}=\sqrt{k_0^2 em-\left(k_i^{H,E}\right)^2}$ is the propagation constant of mode *i*; $k_i^{H,E}=l_i^{H,E}/a$ is the wave number of mode *i*; l_i^E is the *i*-th root of the function $L_m(x)=J_m(x)-Y_m(x)J_m(xd/a)/Y_m(xd/a)$;

l_i^H is the *i*-th root of the function $X_m'(x) = J_m'(x) - Y_m'(x) J_m'(xd/a) / Y_m'(xd/a)$;
J_m and Y_m are Bessel functions of the first and second kind, order *m*.

Then, the tangential magnetic fields on the surfaces II and I are found using Green's function for a coaxial resonator. At the same time, these magnetic fields can be represented through equation (2) and $\vec{H}_t^I = \sum_p F_p^{H,E} \vec{h}_p^{H,E}$. Making use of continuity of tangential magnetic fields on the surfaces I and II and by following the Galerkin procedure for the resulting equations we obtain the elements of the generalized admittance matrix $Y^{(11)}$ and $Y^{(21)}$: TM$_i$ → TM$_i$ and TE$_i$ → TE$_i$ mode coupling: $Y_{ii}^{(11)} = j\cos\left(g_i^{H,E} l\right) / \sin\left(g_i^{H,E} l\right)$;

TEM→TEM mode coupling: $Y_{ii}^{(11)} = j\cos\left(k_0\sqrt{em}\, l\right) / \sin\left(k_0 \sqrt{em}\, l\right)$;

TM$_i$→TM$_n$ mode coupling: $Y_{ni}^{(21)} = \dfrac{4}{e_n l} \dfrac{k_0 e}{k_i^E} \dfrac{g_i^E}{\left(g_i^E\right)^2 - g_n^2} L_m'\left(k_i^E a\right)$;

TE$_i$→TM$_n$ mode coupling: $Y_{ni}^{(21)} = \dfrac{4}{e_n l} \left\{ \dfrac{m g_n}{\left(k_n\right)^2 a} g_n + \dfrac{m g_i^H}{\left(k_i^H\right)^2 a} g_i^H \right\} \dfrac{X_m\left(k_i^H a\right)}{g_n^2 - \left(g_i^H\right)^2}$;

TM$_i$→TE$_n$ mode coupling: $Y_{ni}^{(21)} = 0$;

TE$_i$→TE$_n$ mode coupling: $Y_{ni}^{(21)} = -\dfrac{4}{e_n l} \dfrac{g_n}{g_n^2 - \left(g_i^H\right)^2} X_m\left(k_i^H a\right)$;

TEM→TM$_n$ mode coupling: $Y_n^{(21)} = j \dfrac{4d}{e_n a l} \dfrac{k_0 \sqrt{em}}{k_0^2 em - g_n^2}$; $\quad e_n = \begin{cases} 2, n = 0 \\ 1, n \neq 0 \end{cases}$.

Similary, through defining $E_t^I = 0$ and E_t^{II} by equation (1), we obtain the other elements of the generalized admittance matrix $Y^{(12)}$ and $Y^{(22)}$:

TM$_n$→TM$_i$ mode coupling: $Y_{in}^{(12)} = 2 j y_n^E \dfrac{k_i^E}{k_0 e} \dfrac{k_n a}{k_n^2 - \left(k_i^E\right)^2} \dfrac{L_m'\left(k_i^E a\right)}{a^2 L_m'^2\left(k_i^E a\right) - d^2 L_m'^2\left(k_i^E d\right)}$;

TE$_n$→TM$_i$ mode coupling: $Y_{in}^{(12)} = 0$;

TM$_n$→TE$_i$ mode coupling:

$$Y_{in}^{(12)} = -2 j m y_n^E \frac{1}{g_i^H k_n} \frac{X_m\left(k_i^H a\right)}{a^2 X_m^2\left(k_i^H a\right)\left[\left(m / k_i^H a\right)^2 - 1\right] - d^2 X_m^2\left(k_i^H d\right)\left[\left(m / k_i^H d\right)^2 - 1\right]};$$

TE$_n$→TE$_i$ mode coupling:

$$Y_{in}^{(12)} = -2 j y_n^H \frac{\left(k_i^H\right)^2 g_n a}{g_i^H k_n \left(k_n^2 - \left(k_i^H\right)^2\right)} \frac{X_m\left(k_i^H a\right)}{a^2 X_m^2\left(k_i^H a\right)\left[\left(m / k_i^H a\right)^2 - 1\right] - d^2 X_m^2\left(k_i^H d\right)\left[\left(m / k_i^H d\right)^2 - 1\right]};$$

TM$_n$→TEM mode coupling: $Y_n^{(12)} = y_n^E \left(k_n d \ln(a/d)\right)^{-1}$;

TM$_n$→TM$_n$ mode coupling: $Y_{nn}^{(22)} = y_n^E \Lambda_m'\left(k_n a\right) / \Lambda_m\left(k_n a\right)$;

TE$_n$→TE$_n$ mode coupling: $Y_{nn}^{(22)} = y_n^H \Psi_m\left(k_n a\right) / \Psi_m'\left(k_n a\right)$,

where $\Lambda_m(x) = J_m(x) - H_m^{(2)}(x) J_m(xd/a) / H_m^{(2)}(xd/a)$;

$\Psi_m(x) = J_m(x) - H_m^{(2)}(x) J_m'(xd/a) / H_m'^{(2)}(xd/a)$.

Transition from the admittance matrix to a scattering matrix yields the generalized scattering matrix of the junction between coaxial and radial waveguides.

Consider a cylindrical cavity formed by a short circuit in the radial waveguide (Fig. 3). Experimental results have been presented by Keam and Williamson [5] for the phase of the reflection coefficient for the partially filled with dielectric and hollow coaxial-line/cylindrical cavity junctions. The dimensions are a = 1.525 mm, d = 3.5 mm, R = 41 mm, l = 150 mm, c = 24.12 mm and e = 2.1. Both cases for TEM coaxial mode excitation have been analyzed using developed mathematical model. For the first case four generalized scattering matrices have been progressively cascaded: of the junction between the hollow and filled coaxial waveguides, of the coaxial waveguide to radial waveguide junction, of the junction between the filled and hollow radial waveguides, and of the short circuit in the radial waveguide. The computed results are plotted in Fig. 4 along with Keam and Williamson's experimental values. One can see that the theoretical results agree very well with experiment.

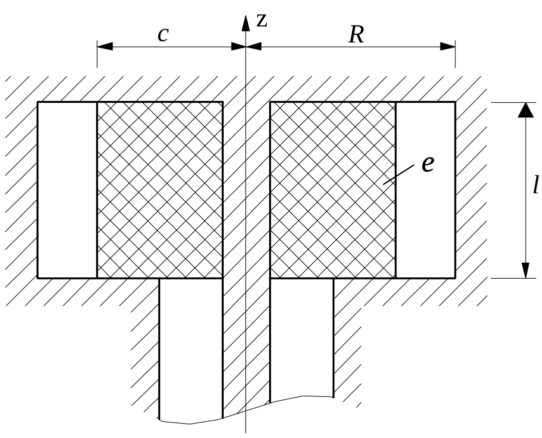

Fig. 3

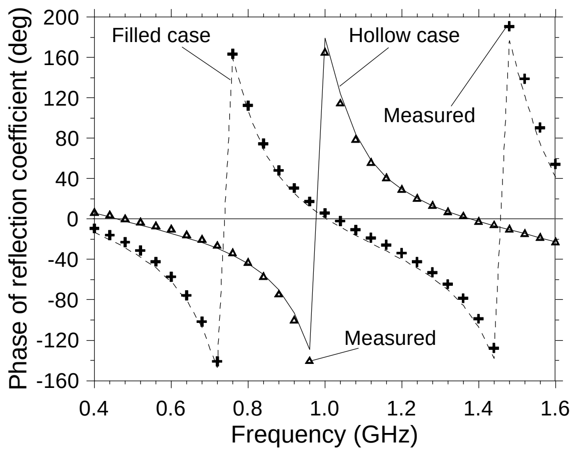

Fig. 4

4. Conclusion

A mathematical model of biconical antennas that based on generalized scattering matrices has been presented. Expressions for the elements of the generalized admittance matrix of a coaxial waveguide to radial waveguide junction have been given. Owing to this a feed geometry is incorporated into analysis. Bicionical antennas with non-linear elements of cones and with partial or full cylindrical dielectric filling can be analyzed using the developed mathematical model.

Experimental results for a biconical antenna as a whole are not yet available. But correct results that have been obtained for the coaxial waveguide to radial waveguide junction and for the radially irregular structure [6] allow to draw a conclusion about correctness of the developed mathematical model of biconical antennas.

References

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