

# Effective Transmission Area of a Resonant Rectangular Aperture

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## Abstract

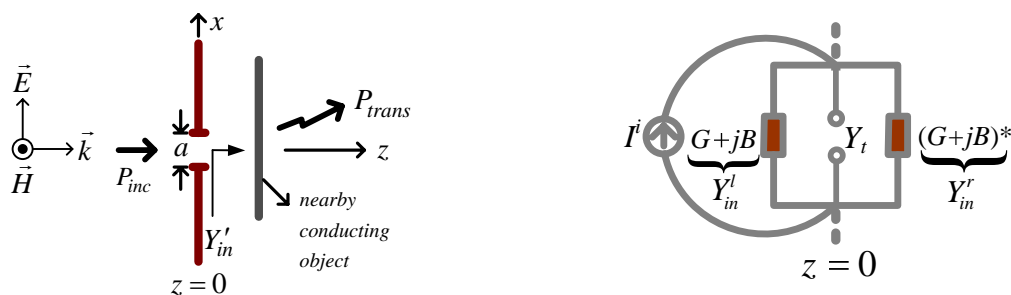
We derive a simple expression for the effective transmission area for the resonant rectangular aperture by use of the equivalent circuit approach based upon the aperture-body resonance concept. This work may be helpful to understand better the electromagnetic (EM) energy coupling mechanism via subwavelength apertures.

## 1. Introduction

Since the work by Ebbesen et al. [1] on the extraordinary optical transmission (EOT) phenomenon through two dimensional arrays perforated in optically thick silver films, the research concern for the transmission properties of subwavelength apertures has been rekindled in the area of electromagnetism. Several studies have been reported on the influence of aperture shape on the optical transmission properties of the single subwavelength [2, 3] apertures and their 2D arrays [4]. These studies showed that transmission through a rectangular aperture presents higher transmittance than square or circular holes with the same area.

Recently, some studies [5] on the transmission resonance phenomena via a small aperture were reported in connection with the EOT problem. These studies have dealt with the enhanced transmission phenomenon from the view point of the aperture-body resonance (ABR) concept. So it would be interesting to treat the resonant rectangular aperture from the ABR viewpoint. This is a main aim of this article.

## 2. Aperture-Body Resonance (ABR) Concept



(a) present structure (b) equivalent circuit representation(at ABR)

Figure 1: One-dimensional aperture-body resonance problem and its equivalent circuit representation

Let us begin by 1-dimensional ABR problem which is composed of the conducting screen with a narrow slit along with nearby scattering object as shown in Fig. 1-(a). As discussed in [6], when the nearby scatterer (metal strip) is not present, the transmitted (or coupled) power through the narrow slit is very small, if the aperture size is much smaller than the operating wavelength. But

with the nearby scatterer present near the slit, the amount of the transmitted power via the slit can be made to be remarkably increased. The maximum coupling (i.e., ABR) condition can be explained with its equivalent circuit representation, which is shown in Fig.1-(b). When the nearby scatterer is not present, the admittance looking into left and right half spaces,  $Y_{in}^l$  and  $Y_{in}^r$ , are identically given as  $Y_{in}^l = Y_{in}^r = G + jB$ , where  $G$  and  $B$  are, respectively, the radiation conductance and the susceptance of the slit. In this case the transmitted power is very small. However when the nearby scatterer is brought in the near field region of the slit such that the input admittance looking into the right half space may become the complex conjugate of the input admittance looking into the left half space, i.e.,  $Y_{in}^r = (Y_{in}^l)^* = (G + jB)^*$ , the transmitted power via the slit can be remarkably increased. When this condition is met, i.e., the total admittance  $Y_t (= Y_{in}^r + Y_{in}^l)$  becomes  $2G$ , the maximum transmission via the slit occurs. Here if we define the effective transmission width to be the ratio of  $P_{trans} / P_{inc}$  where  $P_{inc}$  means the incident power density [ $W/m^2$ ], whereas  $P_{trans}$  [ $W/m$ ] means the transmitted power through the slit per unit length along the y-direction, the maximum of the effective transmission width the slit becomes  $\lambda / \pi$  [m], for normal incidence case. This can be interpreted physically as funnelling width as shown in Fig. 2. That is, the incident wave power upon the area corresponding to  $\lambda / \pi$  [m] (even outside the slit area) is funnelled into the slit and transmitted through the slit. As this result, the remarkable transmission through the slit is thought to result.

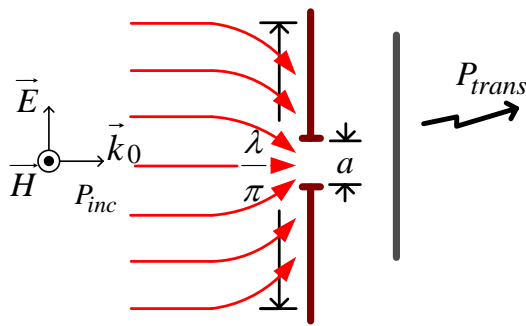


Figure 2: EM energy funnelling under the aperture-body resonance (ABR) condition

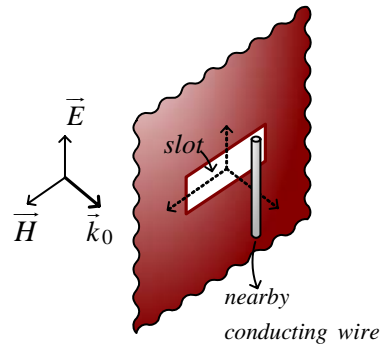


Figure 3: Practical aperture-body resonance problem

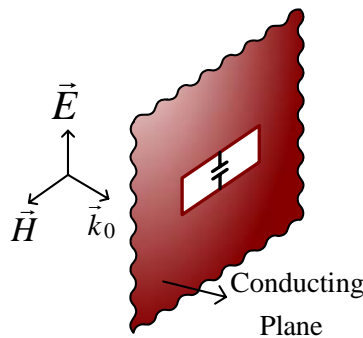


Figure 4: Alternative implementation of the ABR concept (by loading the appropriate lumped element on the aperture plane)

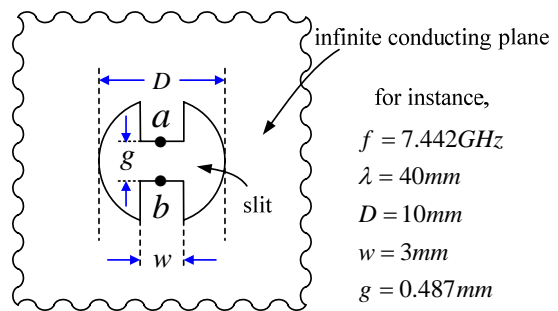


Figure 5: Alternative implementation of the ABR concept (by perturbing appropriately the aperture shape)

This ABR concept can be applied to the more general ABR problem of the two dimensional geometries as shown in Fig. 3. Alternative methods for ABR concept can be implemented, instead of putting a nearby scatterer in the near field region of the slit as above,

- i) by loading some appropriate lumped element on the aperture plane as shown in Fig. 4, or

ii) by introducing an appropriate coplanar perturbation on the aperture shape, for example by employing C-shaped aperture as shown in Fig. 5.

The narrow rectangular aperture where the length of the long side is chosen to be the resonant length close to half wavelength may belong to the category ii). Here we are going to deal with the effective transmission area of the resonant rectangular slit from the viewpoint of the ABR problem.

### 3. Effective Transmission Area of the Resonant Rectangular Aperture (Equivalent Circuit Approach)

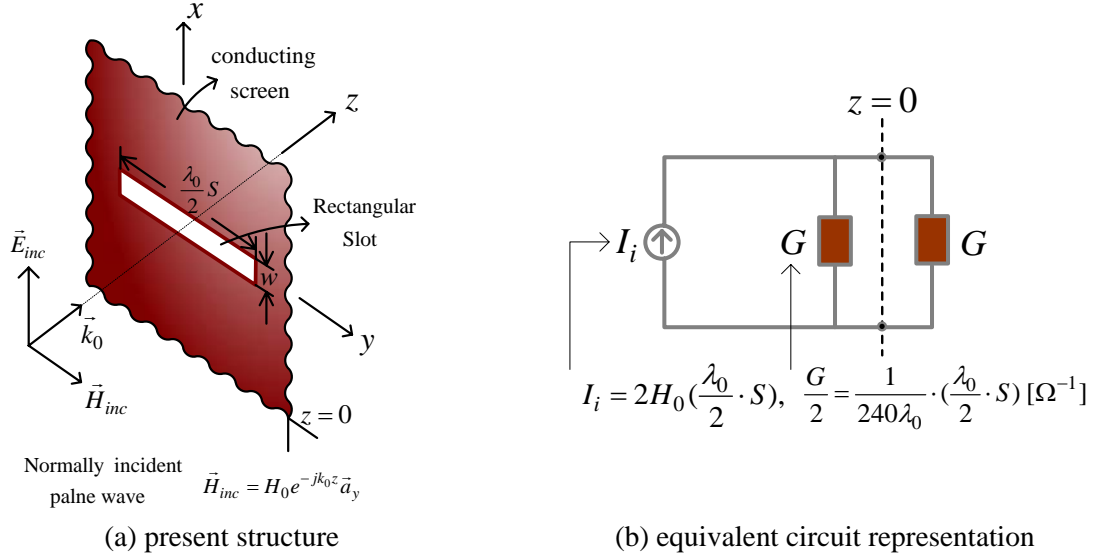


Figure 6: The present structure (a) and its equivalent circuit representation (b).

Here  $\lambda_0$ : wavelength in free space,  $S$ : Fractional shortening,

The present structure and its equivalent circuit representation are given in Fig. 6. Under the ABR condition where the length of the long side of the rectangular aperture is chosen to be resonant length ( $\cong \frac{\lambda_0}{2} S$ ) such that the imaginary part of the total admittance  $Y_t$  vanish to zero, the aperture admittance becomes purely real  $G$ . Here  $G = \bar{G} \cdot (\frac{\lambda_0}{2} S)$ , where  $\bar{G}$  is a radiation conductance  $\bar{G} = 1/(120\lambda_0)$  per unit length for the infinitely long narrow slit [5, 6] and  $S$  is a fractional shortening. From the equivalent circuit, the transmitted power  $P_{trans}$  through the slit into the region for  $z > 0$  is obtained to be

$$P_{trans} = \frac{1}{2} \cdot \left( \frac{I_i}{2} \right)^2 \cdot \frac{2}{G} = 60 H_0^2 \lambda_0^2 S [W].$$

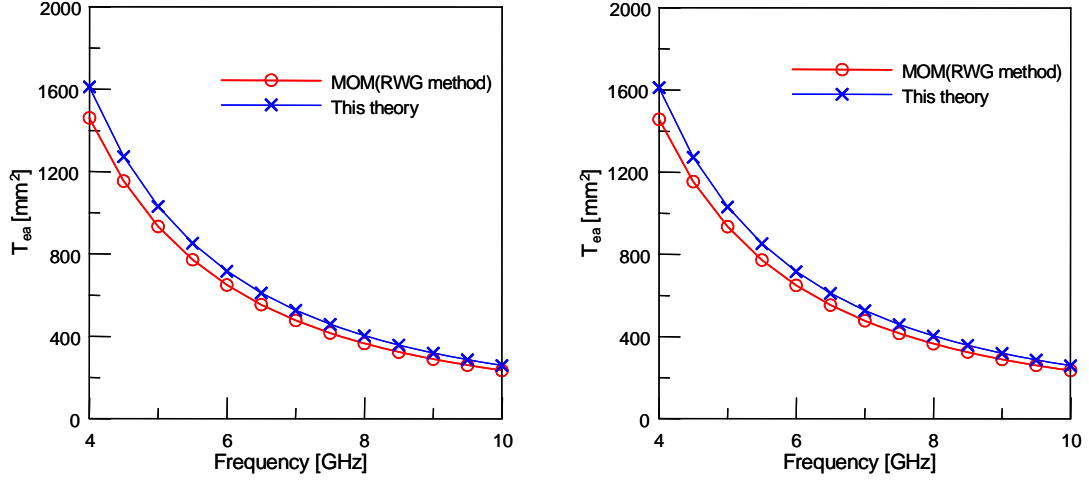
Dividing this by  $P_{inc} (= \frac{1}{2} \eta_0 H_0^2 [W/m^2])$  leads to the desired expression for the effective transmission area of the slit as

$$T_{ea} = \frac{P_{trans}}{P_{inc}} = \frac{\lambda_0^2 S}{\pi} [m^2].$$

### 4. Discussions and Conclusion

In order to validate the expression for the effective transmission area  $T_{ea}$ , we have calculated the variation of  $T_{ea}$  versus the frequency and compared these two results in Fig. 7. Comparison shows fairly good agreement. As expected from the funnelling width for the infinitely

long narrow slit under the ABR condition, the effective transmission area for the resonant (narrow) rectangular aperture can be given simply as product of the funnelling width for the long slit problem and the resonant length of the narrow slit with inclusion of the fractional shortening. This is the main result of this article.



$$(a) T_{ea} = \frac{\lambda^2 S}{\pi}, S = 1, w = 1[mm]$$

$$(b) T_{ea} = \frac{\lambda^2 S}{\pi}, S = 1, w = 2[mm]$$

Figure 7: Effective transmission area of the resonant slit versus the frequency; Comparison between this result and that obtained by use of the MOM (RWG) method.

In conclusion, we have considered the maximum power transmission problem through a resonant rectangular aperture and derived the simple expression for the effective transmission area for the resonant aperture. This work may be helpful to obtain better understanding for the EM energy transmission problem through subwavelength aperture coupling.

## Acknowledgments

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