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Multiple Plasmonic Resonances of a Coated NanoSphere in Light Scattering

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1. Introduction

Light scattering by spherical particles has been a classical subject attracting lots of interests over the past few decades and also studied thoroughly using the Mie theory [1]. As the electrical parameter k_0a (where k_0 is the wave number of the free space and a is the radius of the sphere) of the scattering objects becomes much smaller than 1, Rayleigh scattering which is derived approximately from the of the first order Mie scattering will dominate [2]. Scattering of electromagnetic waves from two concentric spheres was first worked out by Aden and Kerker [3]. Recently, with the development of nanotechnologies, it becomes desirable to investigate the microcosmic world. Nano-scaled objects are attracting considerable attentions as they have shown interesting optical properties. The modern photonic applications of nano-scaled objects are of great interest [4]. It is found that the resonances can happen when $\Re(\epsilon_r) = -(1+1/n)$ (electric resonances), where $n = 1, 2, \cdots$. The near-field energy intensity can be increased significantly.

In this paper, we will discuss the characteristics of light scattering by a coated sphere (with plasmonic coating) and investigate some interesting optical phenomenon.

2. Theoretical Formulas

A plasmon has great impact on the optical properties of metals. The plasma frequency ω_p can be expressed as:

$$\omega_p^2 = \frac{ne^2}{\epsilon_0 m_{eff}}. \tag{1}$$

It produces an effective relative dielectric permittivity of the form

$$\epsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - j\tau)} \tag{2}$$

which is nearly independent of the wave number, where τ is the damping term. So the value of the permittivity depends on certain parameters. When the real part of $\omega_p^2/[\omega(\omega + i\tau)]$ is bigger than 1, we can obtain negative real part of $\epsilon_r(\omega)$.

The geometry of the problem is shown in Fig. 1. The inner radius of the coated sphere is a and its outer radius is b. We take $x = k_0a$ and $y = k_0b$ to be the electrical parameters for the inner and outer radii of the coated sphere. we assume m_1 and m_2 to be the complex refractive indices of the core and the coating regions, respectively. m is the refractive index of the free space and here $m = 1$. It is clear that $m_1^2 = \epsilon_1$ and $m_2^2 = \epsilon_2$, where ϵ_1 and ϵ_2 are the relative permittivities of the core and the coating materials.

For a nonabsorbing host medium, the scattering coefficients for a coated sphere are given in Eqn. (3) and Eqn. (4). $\psi_n(\rho) = \rho j_n(\rho)$ and $\xi_n(\rho) = \rho h_n^{(2)}(\rho)$, where j_n and $h_n^{(2)}$ denote the spherical Bessel function of the first kind and the spherical Hankel function of the second kind. $\chi_n(\rho) = -\rho y_n(\rho)$, where y_n is the spherical Bessel function of the second kind. A_n and B_n are given in Eqn. (5) and Eqn. (6).

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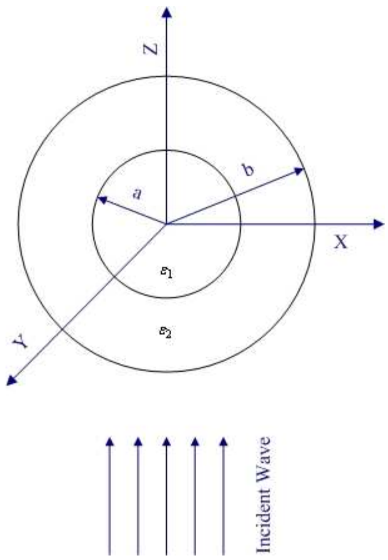

Figure 1: Geometry of light scattering by a coated sphere in free space.

$$a_n = \frac{m\psi_n(my)[\psi_n'(m_2y) - A_n\chi_n'(m_2y)] - m_2\psi_n'(my)[\psi_n(m_2y) - A_n\chi_n(m_2y)]}{m\xi_n(my)[\psi_n'(m_2y) - A_n\chi_n'(m_2y)] - m_2\xi_n'(my)[\psi_n(m_2y) - A_n\chi_n(m_2y)]}, \tag{3}$$

$$b_n = \frac{m_2\psi_n(my)[\psi_n'(m_2y) - B_n\chi_n'(m_2y)] - m\psi_n'(my)[\psi_n(m_2y) - B_n\chi_n(m_2y)]}{m_2\xi_n(my)[\psi_n'(m_2y) - B_n\chi_n'(m_2y)] - m\xi_n'(my)[\psi_n(m_2y) - B_n\chi_n(m_2y)]}. \tag{4}$$

$$A_n = \frac{m_2\psi_n(m_2x)\psi_n'(m_1x) - m_1\psi_n'(m_2x)\psi_n(m_1x)}{m_2\chi_n(m_2x)\psi_n'(m_1x) - m_1\chi_n'(m_2x)\psi_n(m_1x)}, \tag{5}$$

$$B_n = \frac{m_2\psi_n(m_1x)\psi_n'(m_2x) - m_1\psi_n'(m_1x)\psi_n(m_2x)}{m_2\chi_n'(m_2x)\psi_n(m_1x) - m_1\chi_n(m_2x)\psi_n'(m_1x)}. \tag{6}$$

A_n and B_n have forms similar to the coefficients produced from light scattering by spherical particles. a_n and b_n are very complicated and derived by matching the boundary conditions on the interfaces. When $z \ll 1$ is satisfied, we can use the approximations for spherical Bessel (first and second kinds) and Hankel (second kind) functions:

$$h_n^{(2)}(z) \approx \frac{z^n}{(2n+1)!!} + j\frac{(2n-1)!!}{z^{n+1}}, \tag{7a}$$

$$j_n(z) \approx \frac{z^n}{(2n+1)!!}, \tag{7b}$$

$$y_n(z) \approx -\frac{(2n-1)!!}{z^{n+1}}. \tag{7c}$$

As z becomes smaller, the approximations will be more accurate. Taking these approximations into Eqn. (5) and Eqn. (6), we can find

$$A_n \approx \frac{(1+n)(xm_2)^{2n+1}(m_2^2 - m_1^2)}{(2n+1)!!(2n-1)!!((n+1)m_2^2 + nm_1^2)}, \tag{8a}$$

$$B_n \approx 0. \tag{8b}$$

The approximation of a_n is very complicated and will not be shown here.

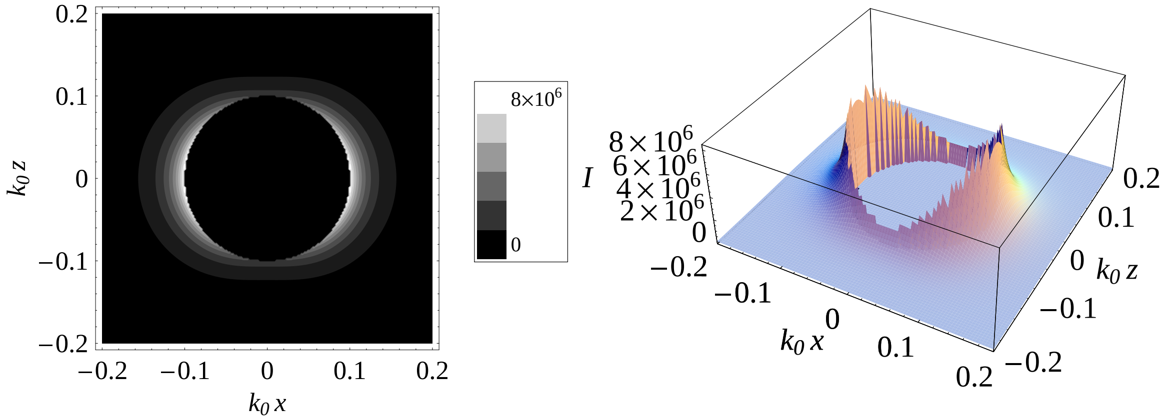

Figure 2: Near-field energy intensity distribution in the xoz plane for $n = 1$, $\xi = 1.67\ldots$, $y = 0.1$, $x = 0.05$, $m = 1$ and $m_1 = 1$

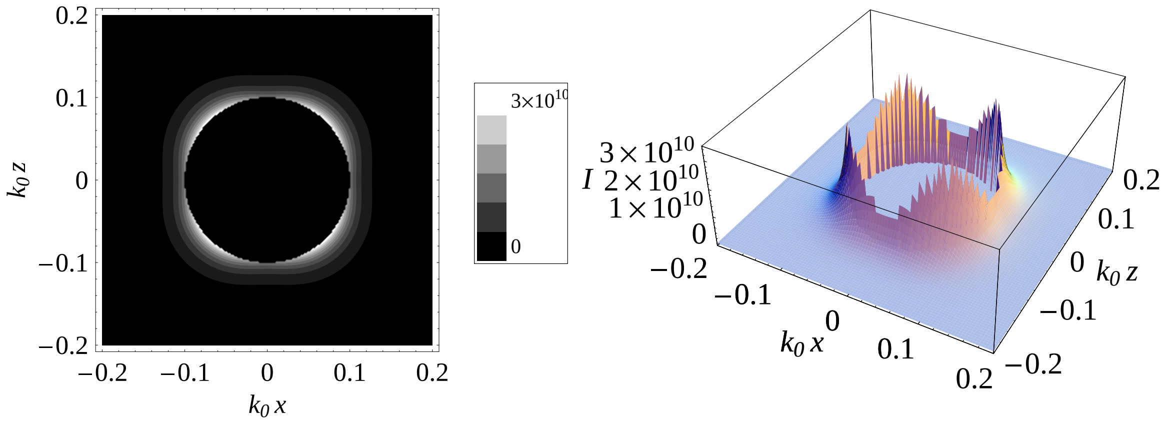

Figure 3: Near-field energy intensity distribution in the xoz plane for $n = 2$, $\xi = 1.31\ldots$, $y = 0.1$, $x = 0.05$, $m = 1$ and $m_1 = 1$

3. Near-Field Energy Distribution around the Coated Sphere

Here we assume $\xi = m_2 j$ to plot the energy distribution. We plot the energy intensity which is given as $I = \boldsymbol{E} \cdot \boldsymbol{E}^{\star}$, where $\boldsymbol{E} = \hat{r}E_r + \hat{\theta}E_\theta + \hat{\phi}E_\phi$ near the resonances. The incident wave is plane wave propagating along z-axis. The amplitude of it is $E_0 = 1$. One can see that near the resonance, the near-field energy can be increased a lot. The first resonance is $\xi = 1.67\ldots$ and the second is $\xi = 1.31\ldots$. They are shown in Fig. 2 and Fig. 3, respectively. We can see that the near-field energy distributions for the two cases are different but both can reach a very great value. For the higher orders, the situations are similar and will not be shown here.

We have discussed the multiple plasmonic resonances of a coated nanosphere with refractive index near resonances and shown the near-field energy distribution. It can be seen that the energy intensity can be increased greatly. There are also some other cases which will not be addressed here because of the limited space.

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