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Approximate Spectral Expression for Plane-Wave Scattering by A Thick Half-Plane

Mayumi NAKAGAWA [1], Hironori FUJII [1] and Kazunori UCHIDA [1]
[1] Intelligent Information System Engineering, Fukuoka Institute of Technology
3-30-1 Wajiro-Higashi, Higashi-ku, Fukuoka, 811-0295 JAPAN
k-uchida@fit.ac.jp

1. Introduction

The two-dimensional (2D) plane wave scattering by a thick conducting half-plane is one of the fundamental electromagnetic field problems. It includes reflection at the faces of the half-plane as well as multiple diffractions at the two wedges. Although this problem can be solved rigorously based on the Wiener-Hopf (WH) technique in spectral domain [1]- [3], its Fourier inverse transformation cannot be performed analytically because the rigorous WH solutions in the spectral domain include coefficients to be solved by infinite simultaneous equations [4]- [5].

When we analyze a complicated problem of the electromagnetic wave propagation in urban areas, it is important to have some analytical solutions to the fundamental electromagnetic problems such as the one treated here. In this case, the scattered fields should be expressed analytically in a compact form, and the results should exhibit versatility, accuracy and stability. In this context, we attempt to introduce an approximate analytical solution from the WH solution in order to apply it to more complicated electromagnetic problems.

In this paper, we first investigate the behavior of the WH solution numerically in the spectral domain focusing on the effect of the incident angle (θ) and the thickness of the half-plate. Second, based on the Kirhihoff-Huygens principle, we introduce an approximate spectral expression for the far field in case of $\theta > 90°$. Third, we compare the approximation with the WH solution in the spectral domain numerically. It is shown that the results of the approximate expression are in good agreement with those of the WH solution.

2. Wiener-Hopf solutions

Fig.1 shows the geometry of the 2D plane wave scattering by a conducting half-plate. The thickness of the half-plate is $2b$ and the incident angle of the plane-wave is θ ($0 < \theta < \pi$). We define the total (t), incident (i) and scattered (s) fields as follows:

$$(\boldsymbol{E}^t, \boldsymbol{H}^t) = (\boldsymbol{E}^i, \boldsymbol{H}^i) + (\boldsymbol{E}^s, \boldsymbol{H}^s) \tag{1}$$

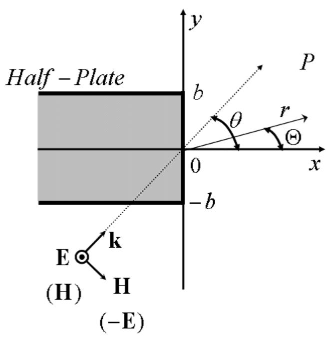


Figure 1: Geometry of the problem for a half-plate.

ISBN: 978-4-88552-223-9 C3055©IEICE

where the incident wave (E-wave) is given by

$$E_z^i = e^{-j\kappa x\cos\theta - j\kappa y\sin\theta} \tag{2}$$

The Maxwell equations (E-wave) are rewritten as follows:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \kappa^2\right)E_z^s(x,y) = 0,\ \ H_x^s = \frac{-1}{j\omega\mu_0}\cdot\frac{\partial E_z^s}{\partial y},\ \ H_y^s = \frac{1}{j\omega\mu_0}\cdot\frac{\partial E_z^s}{\partial x} \tag{3}$$

where the time dependence $e^{j\omega t}$ is assumed and $\kappa = \omega\sqrt{\epsilon_0\mu_0}$ is the wave number in the free space.

In the WH technique, we have used the following Fourier transformation pair defined by

$$F(\zeta) = \int_{-\infty}^{\infty} f(x)e^{j\zeta x}dx, \qquad f(x) = \frac{1}{2\pi}\int_c F(\zeta)e^{-j\zeta x}d\zeta. \tag{4}$$

The rigorous solutions of the scattered field in the spectral domain can be obtained by use of the WH technique; first we derive the spectral expressions for the scattered field by applying the complex Fourier transformation to the wave equation in eq.(3), and second we decompose the spectral expressions into two parts, regular in the upper or lower half plane in the spectral domain. Neglecting detailed discussions, the final results are summarized as follows [4]- [5]:

$$\begin{aligned} E_z^s(\zeta,y) =& \frac{je^{-jk(y-b)}}{2G_c^+(\zeta)}\cdot\frac{U_c^+(\zeta_\theta)G_c^+(\zeta_\theta)}{(\zeta-\zeta_\theta)} + \frac{je^{-jk(y-b)}}{2G_c^+(\zeta)}\sum_{n=1}^{\infty}\frac{U_c^+(-k_{cn})b_{cn}^2G_c^-(k_{cn})}{(k_{cn}+\zeta_\theta)(\zeta-k_{cn})k_{cn}} \\ &+ \frac{je^{-jk(y-b)}}{2G_s^+(\zeta)}\cdot\frac{U_s^+(\zeta_\theta)G_s^+(\zeta_\theta)}{(\zeta-\zeta_\theta)} + \frac{je^{-jk(y-b)}}{2G_s^+(\zeta)}\sum_{n=1}^{\infty}\frac{U_s^+(-k_{sn})b_{sn}^2G_s^-(k_{sn})}{(k_{sn}+\zeta_\theta)(\zeta-k_{sn})k_{sn}} \\ =&\Lambda_p(\zeta)\frac{j\sqrt{\kappa-\zeta_\theta}}{(\zeta-\zeta_\theta)\sqrt{\kappa-\zeta}}e^{-jk(y-b)} + \Omega_p(\zeta)\frac{j\sqrt{\kappa}}{\sqrt{\kappa-\zeta}}e^{-jk(y-b)} \qquad for \quad y > b \end{aligned} \tag{5}$$

$$\begin{aligned} E_z^s(\zeta,y) =& \frac{je^{jk(y+b)}}{2G_c^+(\zeta)}\cdot\frac{U_c^+(\zeta_\theta)G_c^+(\zeta_\theta)}{(\zeta-\zeta_\theta)} + \frac{je^{jk(y+b)}}{2G_c^+(\zeta)}\sum_{n=1}^{\infty}\frac{U_c^+(-k_{cn})b_{cn}^2G_c^-(k_{cn})}{(k_{cn}+\zeta_\theta)(\zeta-k_{cn})k_{cn}} \\ &- \frac{je^{jk(y+b)}}{2G_s^+(\zeta)}\cdot\frac{U_s^+(\zeta_\theta)G_s^+(\zeta_\theta)}{(\zeta-\zeta_\theta)} - \frac{je^{jk(y+b)}}{2G_s^+(\zeta)}\sum_{n=1}^{\infty}\frac{U_s^+(-k_{sn})b_{sn}^2G_s^-(k_{sn})}{(k_{sn}+\zeta_\theta)(\zeta-k_{sn})k_{sn}} \\ =&\Lambda_m(\zeta)\frac{j\sqrt{\kappa-\zeta_\theta}}{(\zeta-\zeta_\theta)\sqrt{\kappa-\zeta}}e^{jk(y+b)} + \Omega_m(\zeta)\frac{j\sqrt{\kappa}}{\sqrt{\kappa-\zeta}}e^{jk(y+b)} \qquad for \quad y < -b \end{aligned} \tag{6}$$

where

$$k = \sqrt{\kappa^2-\zeta^2}, \tag{7}$$

$$\epsilon_{s,n} = \begin{cases} 1/2 & for \quad n = 0 \\ 1 & for \quad n = 1,2,3,\cdots \end{cases} \tag{8}$$

In the above expressions, we have omitted the scattered field in the region $-b < y < b$, since the present discussions are restricted only to the far field. It should be noted that the infinite number of unknown coefficients must be determined by the related infinite set of algebraic equations [1].

In order to obtain the scattered field in the space domain in a compact form, we rearrange the spectral expressions in eqs. (5) and (6) into two parts depending on whether it includes infinite number of unknown coefficients or not. They are defined by use of the functions $\Lambda_{p,m}(\zeta)$ and $\Omega_{p,m}(\zeta)$ as follows:

$$\Lambda_{p,m}(\zeta) = \frac{\sqrt{\kappa-\zeta}G_c^+(\zeta_\theta)U_c^+(\zeta_\theta)}{2\sqrt{\kappa-\zeta_\theta}G_c^+(\zeta)} \pm \frac{\sqrt{\kappa-\zeta}G_s^+(\zeta_\theta)U_s^+(\zeta_\theta)}{2\sqrt{\kappa-\zeta_\theta}G_s^+(\zeta)} \tag{9}$$

$$\Omega_{p,m}(\zeta) = \frac{\sqrt{\kappa-\zeta}}{2\sqrt{\kappa}G_c^+(\zeta)}\sum_{n=1}^{\infty}\frac{U_c^+(-k_{cn})b_{cn}^2G_c^-(k_{cn})}{(k_{cn}+\zeta_\theta)(\zeta-k_{cn})k_{cn}} \pm \frac{\sqrt{\kappa-\zeta}}{2\sqrt{\kappa}G_s^+(\zeta)}\sum_{n=1}^{\infty}\frac{U_s^+(-k_{sn})b_{sn}^2G_s^-(k_{sn})}{(k_{sn}+\zeta_\theta)(\zeta-k_{sn})k_{sn}} \tag{10}$$

where $\Lambda_{p,m}(\zeta)$ and $\Omega_{p,m}(\zeta)$ are associated with the spectral functions of the scattered fields at the two half-planes($y = \pm b$, $x < 0$) [6] and at the thickness part of the half-plate ($|y| < b$, $x = 0$), respectively.

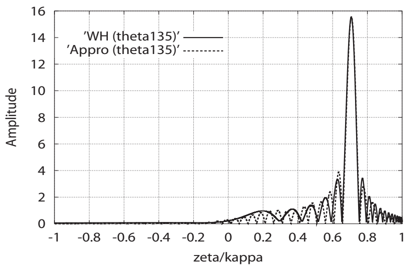


Figure 2: WH and approximate solutions (f = 300MHz, θ = 135°, $b = 10.1\lambda$).

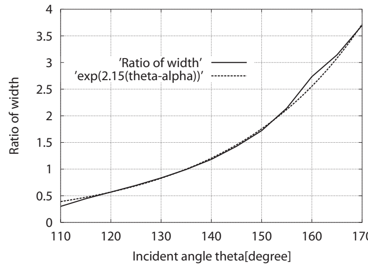


Figure 3: Width of main lobe $W^w(\theta)$.

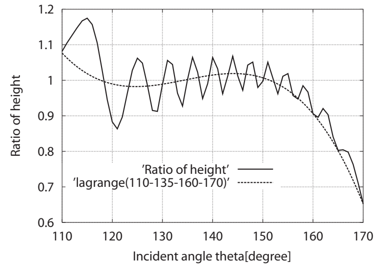


Figure 4: Height of main lobe $W^h(\theta)$.

3. Approximation

The functions $\Lambda_{p,m}(\zeta)$ and $\Omega_{p,m}(\zeta)$ are dependent on the incident angle (θ) and thickness of the half-plate(b) as shown in eqs.(9) and (10). Here, however, we consider only the function $\Omega_p(\zeta)$ in case of $\theta > 90°$. In order to derive an approximate field expression, we apply the Kirhihoff-Huygens principle to the boundary condition ($E^s = -E^i$) at the thickness of the half-plate ($-b < y < b$). Using sampling and Fresnel functions, we introduce an approximation as follows:

$$\Omega_p(\zeta) \approx \frac{2\sin[(k_\theta - \zeta)b]}{k_\theta - \zeta}\sqrt{\frac{2(\kappa-\zeta)}{\kappa}}F[\sqrt{2}\zeta\cos\theta] \quad (11)$$

$$k_\theta = \sqrt{\kappa - \zeta_\theta}. \quad (12)$$

The above approximate solution agrees quite well with the WH solution in case of $\theta = 135°$ as shown in fig.2. In other cases, however, eq.(11) exhibits some discrepancies in comparison with the WH solutions, and so we modify the approximation by adding two terms; one is related to the width of the main lobe and the other is associated with its height.

Fig.3 shows the ratio of the width of the main lobes of the WH and approximate solutions by using the weighting function $W^w(\theta)$. Fig.4 shows the ratio of the height of the main lobes of the WH and approximate solutions by using the weighting function $W^h(\theta)$. As a result, we can modify eq.(11) as follows:

$$\Omega_p(\zeta) \approx \frac{2\sin[(k_\theta - \zeta)bW^w(\theta)]}{k_\theta - \zeta}\sqrt{\frac{2(\kappa-\zeta)}{\kappa}}F[\sqrt{2}\zeta\cos\theta]\cdot\frac{1}{W^h(\theta)} \quad (13)$$

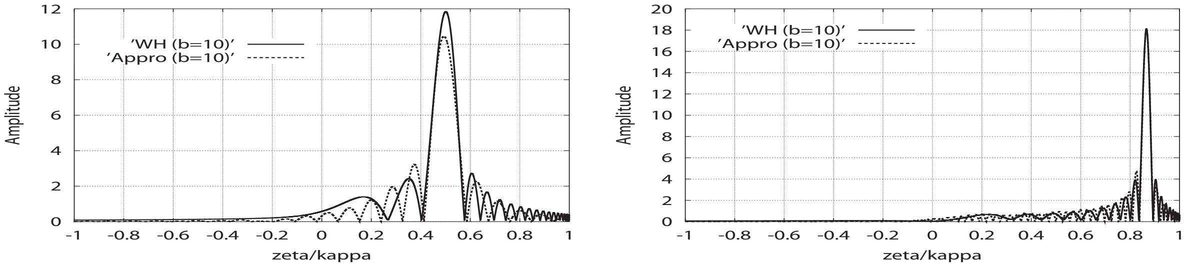
$$W^w(\theta) = \exp[2.15(\theta-\alpha)], \quad \alpha = 3\pi/4 \quad (14)$$

$$W^h(\theta) = \sum_{j=0}^{3} f(x_j)\prod_{i=0,i\neq j}^{3}\frac{(x-x_i)}{(x_j-x_i)}, \quad x = \theta[rad]. \quad (15)$$

In the above expressions, $W^h(\theta)$ is given by the Lagrange interpolation where numerical parameters are chosen as $x_0 = 1.920$ ($\theta = 110°$), $x_1 = 2.356$ ($\theta = 135°$), $x_2 = 2.793$ ($\theta = 160°$) and $x_3 = 2.967$ ($\theta = 170°$), $f(x_0) \cong 0.843$, $f(x_1) \cong 1.001$, $f(x_2) \cong 0.906$ and $f(x_3) \cong 0.652$.

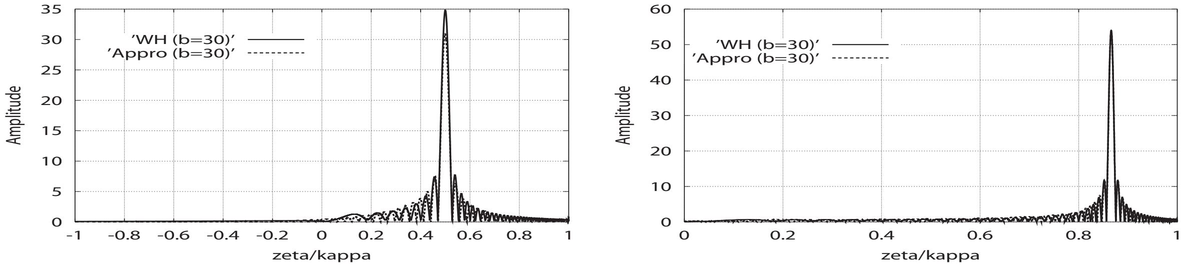
4. Numerical examples

In this section, we check the accuracy of the proposed method by comparing the approximations with the WH solutions in figs.5 and 6 for the function $\Omega_p(\zeta)$ in case of $\theta > 90°$. The parameters used are chosen as f = 300MHz, (a) $\theta = 120°$, (b) $\theta = 150°$ and $b = 10.1\lambda$, $b = 30.1\lambda$ when ζ is changed in the region $[-\kappa, \kappa]$. The numerical results show that the presented approximations are in good agreement with the WH solutions even if θ and b are varied.



(a) $\theta = 120°$ (b) $\theta = 150°$

Figure 5: Comparison between WH and approximate solutions ($\theta = 120°, 150°$ and $b = 10.1\lambda$).



(a) $\theta = 120°$ (b) $\theta = 150°$

Figure 6: Comparison between WH and approximate solutions ($\theta = 120°, 150°$ and $b = 30.1\lambda$).

5. Conclusions

In this paper, we have derived a simplified analytical expression for the plane wave scattering by a thick half-plate by using the sampling and Fresnel functions. We have also modified the expression so that it can show acceptable accuracy for a wide range of incident angle. Numerical examples for the function $\Omega_p(\zeta)$ show that the approximations are in good agreement with the WH solutions in case of $\theta > 90°$.

As a future work, we need to derive analytical expressions for other functions and to apply these results to the electromagnetic wave propagation problems in urban areas.

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