

MICROWAVE SCATTERING AND EMISSION FROM MULTILAYERED MEDIUM WITH ROUGH SURFACE

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**Introduction**

Theoretical consideration of thermal emission and scattering of electromagnetic waves by rough surface attracts growing interest nowadays. Detailed study of these problems has been performed for the cases of well conducting medium or homogeneous dielectric halfspace with irregular boundary [1-3]. We employ more sophisticated model of a medium characterized by inhomogeneous permittivity profile. For such model we derive analytical solution of the above-mentioned problems in the case of small roughness. The equilibrium thermal fluctuation theory [4] is applied to reduce the emission problem to the diffraction one for plane monochromatic wave. The multiple scattering theory and the small perturbation method are used to calculate coherent reflection coefficients [5] and bistatic scattering coefficients respectively. As the two-layered medium model has been used successfully in the studies of sea ice, lake ice and vegetation canopies [6,7] we present here detailed analysis of how both spatial inhomogeneity and small roughness influence on emissivity and backscattering of such structure.

**Model**

The starting point is the model of a regular layered medium occupying the lower halfspace  $z < 0$  (terrain) and characterized by the permittivity profile  $\epsilon(z)$ , where  $\epsilon(z)$  is a piecewise-continuous complex-value function of the vertical coordinate. The upper halfspace  $z > 0$  is free (air). Magnetic permeability takes everywhere constant value 1. When the irregularities are present the real air-terrain interface is given in each realization by the equation  $z = \zeta(\vec{r})$ , where  $\vec{r} = (x, y, 0)$ ;  $\zeta$  is a random function with zero mean value and correlation function  $\langle \zeta(\vec{r}) \zeta(\vec{r}') \rangle = B(\vec{r} - \vec{r}')$ . We assume that the rms roughness height  $\sigma = \langle \zeta^2 \rangle^{1/2}$  is much smaller than the wavelength in free space and characteristic horizontal size of the irregularities.

Suppose that a plane wave of arbitrary polarization is coming from free space in the direction of the unit vector  $\vec{l}_i = (\cos\phi_i \sin\alpha_i, \sin\phi_i \sin\alpha_i, -\cos\alpha_i)$ :

$$\vec{E}^{(i)}(\vec{R}) = \vec{e}_i \exp(ik\vec{l}_i \cdot \vec{R}), \quad \vec{H}^{(i)}(\vec{R}) = \vec{h}_i \exp(ik\vec{l}_i \cdot \vec{R}), \quad (1)$$

where the time factor  $\exp(-i\omega t)$  is omitted,  $k = \omega/c$ ,  $c$  is the speed of light in free space;  $\alpha_i, \phi_i$  - angle of incidence and azimuth angle of propagation;  $\vec{e}_i, \vec{h}_i$  are vector's amplitudes defined by the relations  $\vec{e}_i \cdot \vec{l}_i = 0$ ,  $\vec{h}_i = \vec{l}_i \times \vec{e}_i$ . Let's represent the diffracted field  $\vec{E}^{(d)}(\vec{R}), \vec{H}^{(d)}(\vec{R})$  in the region  $z > 0$  as a sum of the statistically mean (coherent) and fluctuating components:

$$\vec{E}^{(d)}(\vec{R}) = \langle \vec{E}^{(d)}(\vec{R}) \rangle + \vec{E}^{(f)}(\vec{R}), \quad \vec{H}^{(d)}(\vec{R}) = \langle \vec{H}^{(d)}(\vec{R}) \rangle + \vec{H}^{(f)}(\vec{R}). \quad (2)$$

The coherent reflected field in free space is given by the expressions:

$$\langle \vec{E}^{(d)}(\vec{R}) \rangle = \vec{e}_r \exp(ik\vec{l}_r \cdot \vec{R}) \quad \langle \vec{H}^{(d)}(\vec{R}) \rangle = \vec{h}_r \exp(ik\vec{l}_r \cdot \vec{R}); \quad (3)$$

$$\vec{e}_r = \vec{z}_0 \times \vec{n}_i (A_{HH} R_{HH} + A_{HV} R_{HV}) - \vec{\theta}_r (A_{VH} R_{HV} + A_{VH} R_{VH}),$$

$$\vec{h}_r = \vec{z}_0 \times \vec{n}_i (A_{VV} R_{VV} + A_{VH} R_{VH}) + \vec{\theta}_r (A_{HH} R_{HH} + A_{HV} R_{HV}).$$

Here  $\vec{z}_r$  and  $\vec{z}_0$  are unit vectors along the reflectance direction and along Oz axis correspondingly:  $\vec{z}_r = \vec{n}_i \sin \alpha_i + \vec{z}_0 \cos \alpha_i$ ,  $\vec{n}_i = (\cos \phi_i, \sin \phi_i, 0)$ ;  $\vec{\theta}_r = -\vec{n}_i \cos \alpha_i + \vec{z}_0 \sin \alpha_i$ ;  $A_\nu$  - scalar amplitude of vertically ( $\nu=V$ ) or horizontally ( $\nu=H$ ) polarized components of the incident wave:  $A_V = \vec{z}_0 \times \vec{n}_i \cdot \vec{E}_i$ ,  $A_H = \vec{z}_0 \times \vec{n}_i \cdot \vec{E}_i$ ;  $R_{\mu\nu}$  - coherent reflection coefficient of a  $\nu$ -polarized plane wave into a plane wave of  $\mu$ -polarization ( $\mu, \nu=V, H$ ). The coherent reflection coefficients are found in [5]. Henceforth we shall assume that the relevant quantities are known.

The energy flux density of the fluctuating component  $\vec{P}_f(\vec{R}) \equiv (c/8\pi) \text{Re} \langle \vec{E}^{(f)*}(\vec{R}) \times \vec{H}^{(f)}(\vec{R}) \rangle$  (\* denotes complex conjugate) at the observation point located in free space at a sufficient distance from the rough boundary we have obtained using the small perturbation method. As a result we arrive at the following expression for the backscattering cross section of the rough interface:

$$\sigma(\alpha, \phi, \vec{d}) = |\epsilon - 1|^2 k^4 \tilde{B}(\vec{p}) \left[ |\vec{d}^* \cdot \vec{z}_0 \times \vec{n}_i|^4 |1 + R_H(\alpha)|^4 + |\vec{d}^* \cdot \vec{\theta}_i|^4 [(1 + R_V(\alpha))^2 \sin^2 \alpha / \epsilon + (1 - R_V(\alpha))^2 \cos^2 \alpha] \right] / 4. \quad (5)$$

We use the following notations:  $\vec{d}$  is a unit vector defined by polarization of the receiving antenna:  $\vec{d}^* \cdot \vec{d} = 1$ ;  $\epsilon \equiv \epsilon(-0)$ ,  $\alpha$  and  $\phi$  - angles associated with observation point:  $0 \leq \alpha < \pi/2$ ,  $0 \leq \phi < 2\pi$ . E.g. for vertical direction we have  $\alpha=0$  and  $\phi$  arbitrary. The symbol  $\tilde{B}$  denotes spatial spectrum of roughness:  $\tilde{B}(\vec{p}) \equiv (2\pi)^{-2} \int dx dy B(\vec{r}) \exp(i\vec{p} \cdot \vec{r})$ ,  $\vec{p} = 2k\vec{n}_i \sin \alpha$ ,  $\vec{n}_i = (\cos \phi, \sin \phi, 0)$ ,  $\vec{\theta}_i = \vec{n}_i \cos \alpha + \vec{z}_0 \sin \alpha$ ;  $R_j$  is unperturbed reflection coefficients from the smooth boundary  $z=0$  of layered halfspace for  $j$ -polarized plane wave ( $j=V, H$ ).

We assume that the thermodynamic temperature  $T$  is a constant throughout the medium. Then the brightness temperature  $T_b$  of the terrain associated with polarization  $\vec{d}$  and observation angles  $\alpha, \phi$  is given by the following expressions [8]:

$$T_b(\alpha, \phi, \vec{d}) \equiv e(\alpha, \phi, \vec{d}) T; \quad e(\alpha, \phi, \vec{d}) = 1 - |\vec{d}^* \cdot (\vec{\theta}_i R_{VV}(\alpha, \phi) - \vec{z}_0 \times \vec{n}_i R_{VH}(\alpha, \phi))|^2 - |\vec{d}^* \cdot (\vec{z}_0 \times \vec{n}_i R_{HH}(\alpha, \phi) + \vec{\theta}_i R_{HV}(\alpha, \phi))|^2 - (1/4) \sec \alpha |\epsilon - 1|^2 k^4 \int_0^{2\pi} d\phi' \int_0^{\pi/2} d\alpha' \sin \alpha' \tilde{B}(\vec{q}) \left[ |(1 + R_H(\alpha))(1 + R_H(\alpha')) \cos(\phi' - \phi) (\vec{d}^* \cdot \vec{z}_0 \times \vec{n}_i) - (1 - R_V(\alpha)) \sin(\phi' - \phi) \langle \vec{d}^* \cdot \vec{\theta}_i \rangle|^2 + |((1 + R_V(\alpha))(1 + R_V(\alpha')) \sin \alpha \sin \alpha' / \epsilon - (1 - R_V(\alpha))(1 - R_V(\alpha')) \cos \alpha \cos \alpha' \cdot \cos(\phi' - \phi) \langle \vec{d}^* \cdot \vec{\theta}_i \rangle - (1 + R_H(\alpha))(1 - R_V(\alpha')) \cos \alpha' \sin(\phi' - \phi) (\vec{d}^* \cdot \vec{z}_0 \times \vec{n}_i))^2 \right]. \quad (6)$$

Here  $e(\alpha, \phi, \vec{d})$  is the surface emissivity for polarization  $\vec{d}$ ,  $\vec{\theta}_i = -\vec{n}_i \cos \alpha + \vec{z}_0 \sin \alpha$ ,  $\vec{q} = (q_x, q_y, 0)$ ,  $q_x = k(\cos \phi \sin \alpha - \cos \phi' \sin \alpha')$ ,  $q_y = k(\sin \phi \sin \alpha - \sin \phi' \sin \alpha')$ ;  $\vec{n}_i = (-\cos \phi, -\sin \phi, 0)$ .

These expressions are valid in case of arbitrary layered medium and anisotropic roughness and can be used to obtain the Stokes parameters.

#### Numerical simulation and discussion

We payed special attention to the model of two-layered halfspace (layer of finite thickness  $b$  placed on semi-infinite substrate) with isotropic

roughness on its outer boundary. Thus function  $\varepsilon(z)$  equals  $\varepsilon_1$  in the layer  $-b < z < 0$  and  $\varepsilon_2$  in the substrate  $-\infty < z < -b$ , where  $\varepsilon_{1,2}$  - some complex constants. We have investigated the influence of both surface roughness and background inhomogeneity on backscattering and emissivity of the halfspace  $z < 0$ .

In the backscattering the effect of the layer presence can be characterized by the ratio  $G_j(\alpha) = \sigma_j(\alpha, \phi) / \sigma_{j\infty}(\alpha, \phi)$ , where  $\sigma_j, \sigma_{j\infty}$  are backscattering cross sections on  $j=H, V$  polarization for the above mentioned model ( $\sigma_j$ ) and the model with the layer of infinite thickness ( $\sigma_{j\infty}: b = \infty$ ). It can be easily shown that this ratio is independent of azimuth angle  $\phi$  and roughness spectrum  $\tilde{B}$ . Angular behaviour of  $G_V$  (solid line) and  $G_H$  (dotted line) for the structure with  $kb=0.3$  (curve 1) and  $kb=2.4$  (curve 2) are shown on Fig. 1. Other parameter values are chosen as  $\varepsilon_1 = 2 + 0.02i, \varepsilon_2 = 20 + 0.2i$ .

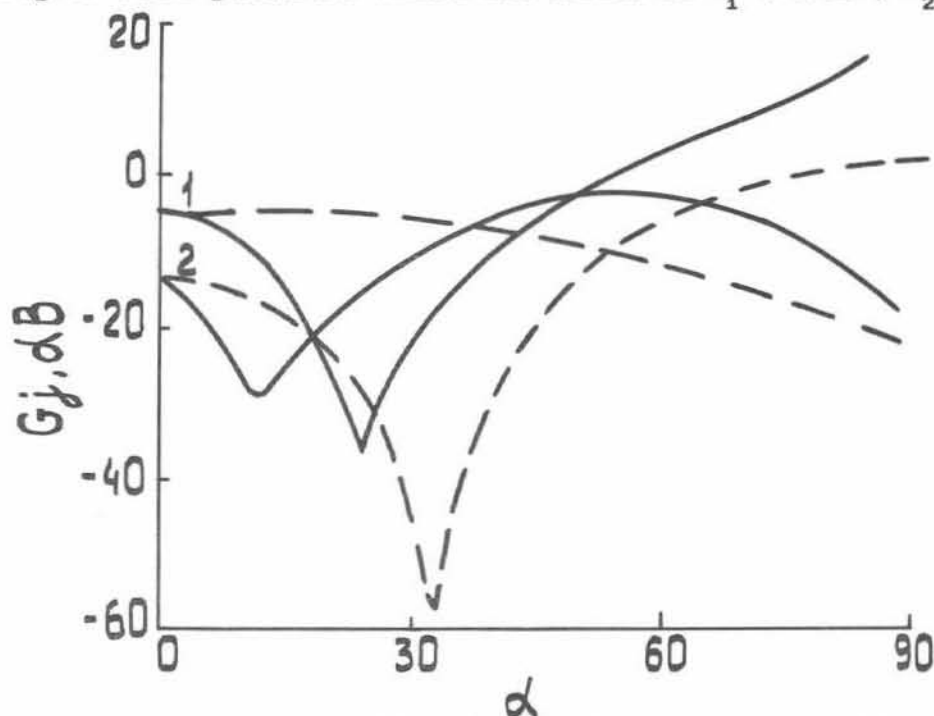


Figure 1. The normalized backscattering cross-sections  $G_j, dB$ .

As for the problem of thermal radioemission we calculated emissivities of the two-layered structure mentioned above for the case when the outer boundary  $z=0$  is smooth ( $e_0$ ) and rough ( $e$ ), and investigated the difference  $\Delta = e - e_0$  due to roughness. Thermal emission of regular multilayered structures has been studied e.g. in [3]. We treated the case of isotropic roughness with Gaussian correlation function:  $B(\vec{r}) = \sigma^2 \exp(-r^2/l^2)$ , where  $l$  is a horizontal scale of irregularities.

The angular dependences of  $\Delta_V$  (solid line) and  $\Delta_H$  (dotted line) for different values of  $tg \delta_1 = \text{Im} \varepsilon_1 / \text{Re} \varepsilon_1$  ( $1 - tg \delta_1 = 0.001; 2 - tg \delta_1 = 0.12$ ) are shown in Fig. 2. Here  $kr = 0.09, kl = 1.79, kb = 20, \text{Re} \varepsilon_1 = 3, \varepsilon_2 = 14 + 3i$ . Surface roughness causes decrease in emissivity for horizontal polarization in the vicinity of  $\alpha_n$ , where  $\alpha_n$  is defined by the equation  $kb \text{Re}(\varepsilon_1 - \sin^2 \alpha_n)^{1/2} = \pi(n-1/2), n=1, 2, 3, \dots$ . Note, that for the same angles ( $\alpha = \alpha_n$ ) the maxima of emissivity  $e_0$  for regular structure occurs. Thus surface roughness reduces interference phenomena in the layer.

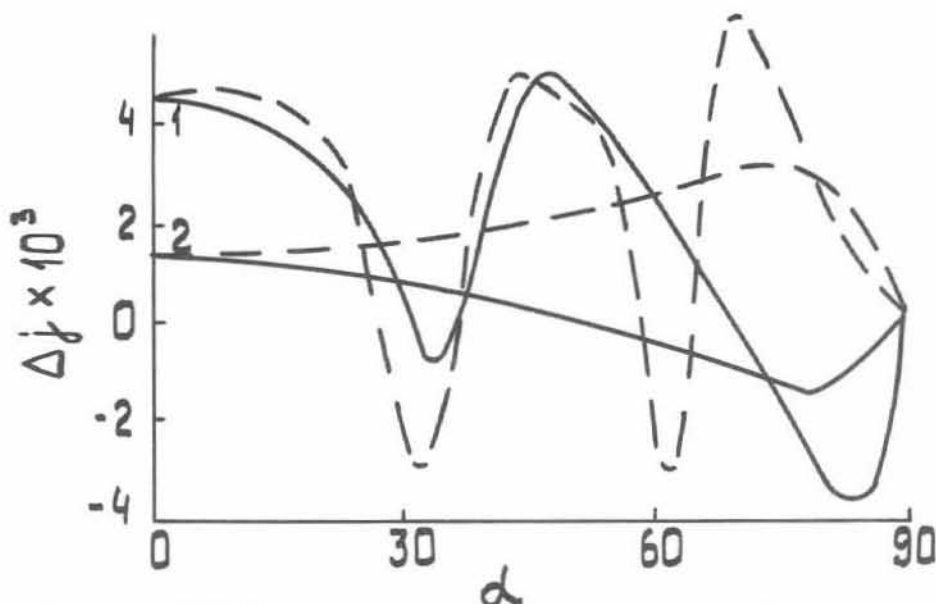


Figure 2. The additional (due to roughness) emissivity  $\Delta_j$ , as a function of observation angle.

#### Conclusion

The analytical expressions for emissivity and backscattering cross-sections of multilayered medium with rough surface are obtained. Numerical analysis of microwave scattering and emission of two-layered structure with small roughness of the upper boundary is carried out. It is shown that inhomogeneous background leads to essential changes in the character of microwave backscattering and emission from rough surface as compared with the case when the structure is electrically homogeneous. The results obtained extend possibilities in the interpretation of remote sensing data.

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