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Theoretical Foundation of The Equivalent Transmission-Path Model for Assessing Wideband Digital Transmission Characteristics in Nakagami-Rice Fading Environments

Hisato IWAI* and Yoshio KARASAWA**

* KDD Research & Development Laboratories
2-1-15 Ohara, Kamifukuoka, Saitama, 356, Japan

** ATR Optical and Radio Communications Research Labs.
2-2 Hikaridai, Seika-cho, Soraku-gun, Kyoto 619-02, Japan

1. Introduction

Nakagami-Rice fading seems to appear dominantly in wideband and high-capacity mobile systems such as micro-cellular or indoor wireless communication systems. The expression of Nakagami-Rice fading is more general compared with that of Rayleigh fading because the former includes the latter as an extreme case. Based on a series of numerical analyses using computer simulations and theoretical considerations, we have so far proposed an approximated evaluation scheme for wideband digital transmission characteristics in frequency-selective Nakagami-Rice fading environments [1]. We called the scheme 'Equivalent Transmission-Path (ETP) model'. In this paper, the foundation on which the ETP model is based is theoretically identified. While the model was originally aimed at estimation of statistical characteristics of transmission quality, we develop a new theoretical formula of the ETP model capable of expression for an instantaneous fading environment varying with time.

2. The ETP model for statistical assessment [1]

The ETP model was originally developed based on the results of computer simulation analysis [1] for the purpose of evaluating statistical wideband digital transmission characteristics in Nakagami-Rice fading environments. In the analysis, we made a few assumptions as below.

(i) The effect of thermal noise is omitted.
(ii) The speed of temporal fluctuation of fading is sufficiently small compared with duration of information symbol T_s.
(iii) Delay spread of a transmission path is relatively small compared with duration of information symbol.

From a series of simulation analyses for the BER floor characteristics assuming several types of delay profiles under the assumed conditions stated above, we arrived at a basic foundation concerning statistical wideband digital transmission characteristics described below [1].

The same BER floor characteristics will be obtained in an arbitrary fading environment having the same values of the parameters stated below, irrespective of differences in delay profiles of multipath waves (see Fig.1). We call the parameters 'key parameters' for Nakagami-Rice fading.

(i) The ratio of the mean multipath power (P_R) to the direct wave power (P_D) : s^2 ($\equiv P_R/P_D$)
(ii) The mean delay of the multipath waves relative to the delay of the direct wave : τ_m
(iii) The delay spread of the multipath waves (excluding the direct wave) : $\sigma_{\tau,R}$

Based on the above foundation of the key parameters, we can transform an arbitrary shape of delay profile of Nakagami-Rice environments to a very simple equivalent delay profile, such as a two-wave model (namely, a preceding wave and a delayed wave, see Fig.2) [1]. In this two-wave model [the ETP model], the preceding component consists of a direct wave and a Rayleigh-distributed wave, and the delayed component whose delay is $\Delta\tau$ is Rayleigh-distributed. The two Rayleigh-distributed waves fluctuate independently each other. The statistical parameters of the two-wave ETP model (P_{R1}, P_{R2}, $\Delta\tau$: average powers of the two Rayleigh-distributed waves and delay separation) are expressed using the three key parameters as below.

$$P_{R1} = \frac{s^2\sigma_{\tau,R}^2}{\tau_m^2+\sigma_{\tau,R}^2}P_D \ , \ P_{R1} = \frac{s^2\tau_m^2}{\tau_m^2+\sigma_{\tau,R}^2}P_D \ , \ \Delta\tau = \frac{\sigma_{\tau,R}^2+\tau_m^2}{\tau_m} \ . \quad (1)$$

Once the two-wave model is obtained, we can have a simple prediction scheme for assessing statistical wideband digital transmission characteristics. The scheme for predicting the floor value of BER due to intersymbol interference of multipath environments developed is given in Ref.[1]. The description in the following chapter examines the model theoretically.

3. The theoretical foundation of the ETP model

In this chapter, we discuss equivalent expression of a general impulse response of an actual transmission path on a snapshot basis. Here we assume the expression by N delayed waves located with every $\Delta\tau$ delay separation (Fig.3(b)).

We consider the instantaneous expression of the ETP model to provide the same digital transmission characteristics as those obtained from an arbitrary instantaneous impulse response of actual Nakagami-Rice fading channel (Fig.3(a)). An impulse response of an actual transmission path $h(\tau,t)$ and frequency transfer function $g(f,t)$ are expressed as below.

$$h(\tau,t) = a_0\delta(\tau)+\sum_{i=1}^{\infty} \dot{a}_i(t)\delta\{\tau-\tau_i(t)\} \quad (2)$$

$$g(f,t) = a_0+\sum_{i=1}^{\infty} \dot{a}_i(t)\, e^{-j2\pi(f-f_c)\tau_i(t)} \quad (3)$$

where τ, t and f indicate delay, time and frequency, respectively. Next, impulse response and frequency transfer function of the N-wave equivalent expression, $h_{e,N}(\tau,t)$ and $g_{e,N}(f,t)$, are also obtained by the following formulas similar to Eq.(2) and (3), respectively.

$$h_{e,N}(\tau,t) = a_0\delta(\tau)+\sum_{n=1}^{N} \dot{a}_{eN,n}(t)\,\delta\{\tau-(n-1)\Delta\tau\}, \quad (4)$$

$$g_{e,N}(f,t) = a_0+\sum_{n=1}^{N} \dot{a}_{eN,n}(t)\,e^{-j2\pi(f-f_c)(n-1)\Delta\tau} \quad . \quad (5)$$

Here, we suppose that the values of transfer function itself and their derivatives of the i-th order (i=1, 2, ..., N-1) at f=fc (fc : carrier frequency) of the N-wave equivalent expression are set equal to those of the actual transmission path. By that procedure, it is expected that accuracy of approximation will be improved as the value of N becomes greater. This process can be represented by the following expression.

$$\frac{\partial^m g_{e,N}(f,t)}{\partial f^m}\Big|_{f_c} = \frac{\partial^m g(f,t)}{\partial f^m}\Big|_{f_c} \quad \text{(for m=0, 1,....., N-1)}. \quad (6)$$

Eq.(6) can be expressed by the following formulas.

$$\sum_{n=1}^{N} \dot{a}_{eN,n}(t) = \sum_{i=1}^{\infty} \dot{a}_i(t) \quad (m=0), \quad (7)$$

$$\sum_{n=2}^{N} \dot{a}_{eN,n}\{-j2\pi(n-1)\Delta\tau\}^m = \sum_{i=1}^{\infty} \dot{a}_i(-j2\pi\tau_i)^m (1\le m\le N-1).$$

N unknown variables ($\dot{a}_{eN,1}$, $\dot{a}_{eN,2}$, ..., $\dot{a}_{eN,N}$) can be obtained by solving the above system of simultaneous linear equations of order N. Through this procedure, an equivalent instantaneous expression with N waves is provided.

In particular, the cases where N=1, 2 and 3 are expressed as follows.

N=1 (frequency-flat fading)

$$\dot{a}_{e1,1}(t) = \sum_{i=1}^{\infty} \dot{a}_i(t)\,. \quad (8)$$

N=2 (frequency-selective fading)

$$\dot{a}_{e2,1}(t) = \sum_{i=1}^{\infty} \dot{a}_i(t) - \frac{\sum_{i=1}^{\infty} \dot{a}_i(t)\,\tau_i(t)}{\Delta\tau},$$

$$\dot{a}_{e2,2}(t) = \frac{\sum_{i=1}^{\infty} \dot{a}_i(t)\,\tau_i(t)}{\Delta\tau}\,. \quad (9)$$

N=3 (frequency-selective fading)

$$\dot{a}_{e3,1}(t) = \sum_{i=1}^{\infty} \dot{a}_i(t) - \frac{3\sum_{i=1}^{\infty} \dot{a}_i(t)\,\tau_i(t)}{2\Delta\tau} + \frac{\sum_{i=1}^{\infty} \dot{a}_i(t)\,\tau_i^2(t)}{\Delta\tau^2},$$

$$\dot{a}_{e3,2}(t) = \frac{2\sum_{i=1}^{\infty} \dot{a}_i(t)\,\tau_i(t)}{\Delta\tau} - \frac{\sum_{i=1}^{\infty} \dot{a}_i(t)\,\tau_i^2(t)}{\Delta\tau^2},$$

$$\dot{a}_{e3,3}(t) = \frac{-\sum_{i=1}^{\infty} \dot{a}_i(t)\,\tau_i(t)}{2\Delta\tau} + \frac{\sum_{i=1}^{\infty} \dot{a}_i(t)\,\tau_i^2(t)}{2\Delta\tau^2}\,. \quad (10)$$

In any case of N, the value of $\Delta\tau$ is not uniquely determined by the above equations. But it seems desirable to select an appropriate value to express statistical fading environments. Further, in order to use the model as a statistical model, it should be required for $\Delta\tau$ to be constant with time. As a method to obtain a suitable value of $\Delta\tau$ for these requirements, we consider the following scheme.

In a general case, significant correlation values remains between each time-variant fluctuation of N component waves in the equivalent expression, since N solutions of Eq.(7) vary in cooperation each other to achieve closer approximation. Here we try to set the value of $\Delta\tau$ to cancel the correlation to zero, in order to make the model useful to use as a statistical model.

In the case of N=2, cross-correlation between the preceding wave and the delayed wave of the equivalent expression is represented as follows.

$$\rho_{a_1a_2} \equiv \frac{\langle \dot{a}_{e2,1}\dot{a}_{e2,2}\rangle}{\sqrt{P_{e2,1}P_{e2,2}}} = \left(\frac{\tau_m}{\Delta\tau} - \frac{\tau_m^2+\sigma_{\tau,R}^2}{\Delta\tau^2}\right)\frac{P_R}{\sqrt{P_{e2,1}P_{e2,2}}} \quad , \quad (11)$$

where $P_{e2,1}$ and $P_{e2,2}$ are average powers of the preceding wave and the delayed wave, respectively. From the above equation, the value of $\Delta\tau$ by which correlation between fluctuations is nullified is shown by the following formula.

$$\Delta\tau = (\tau_m^2 + \sigma_{\tau R}^2)\,/\,\tau_m. \quad (12)$$

On the other hand, in the case of N≥3, a general formula to provide a value of $\Delta\tau$ to cancel the correlation for an arbitrary delay profile seems not to exist, because we have only one tunable variable $\Delta\tau$ while more than three correlations must be zero. Therefore, in the case of N≥3, the ETP model is not useful as a statistical model, however, for instantaneous expression, it can provide better approximation results than N=2. The value which is given by Eq.(12) is available even when N≥3.

The average powers of the preceding wave $P_{e2,1}$ and the delayed wave $P_{e2,2}$ are obtained as follows from Eq.(9) :

$$P_{e2,1} = \left\langle \sum_{i=1}^{\infty} \dot{a}_i(t)\,\dot{a}_i^*(t)\right\rangle - \frac{2}{\Delta\tau}\left\langle \sum_{i=1}^{\infty} \dot{a}_i(t)\,\dot{a}_i^*(t)\,\tau_i(t)\right\rangle + \frac{1}{\Delta\tau^2}\left\langle \sum_{i=1}^{\infty} \dot{a}_i(t)\,\dot{a}_i^*(t)\,\tau_i^2(t)\right\rangle = s^2\left(1 - \frac{2\tau_m}{\Delta\tau} + \frac{\tau_m^2+\sigma_{\tau,R}^2}{\Delta\tau^2}\right)P_D$$

$$P_{e2,2} = \frac{1}{\Delta\tau^2}\left\langle \sum_{i=1}^{\infty} \dot{a}_i(t)\,\dot{a}_i^*(t)\,\tau_i^2(t)\right\rangle = s^2\,\frac{\tau_m^2+\sigma_{\tau,R}^2}{\Delta\tau^2}\,P_D \quad (13)$$

Finally, $P_{e2,1}$ and $P_{e2,2}$ can be expressed by the three key parameters by substituting Eq.(12) into Eq.(13), as follows :

$$P_{e2,1} = \frac{s^2\sigma_{\tau,R}^2}{\tau_m^2+\sigma_{\tau R}^2}\,P_D\,,\quad P_{e2,2} = \frac{s^2\tau_m^2}{\tau_m^2+\sigma_{\tau,R}^2}\,P_D\,. \quad (14)$$

The above expression and Eq.(12) agree with parameters in Eq.(1) which are equivalently transformed by the ETP model so as to keep the key parameters' values unchanged constant.

We discussed a theoretical foundation of the ETP model for N=2 through the analysis of more general time-variant transmission path model. The ETP model represented by Fig.2 (N=2) which is obtained based on the key parameter concept can be derived from the model designed to achieve agreement of zero-th and first order derivatives of the frequency transfer function of a transmission path at the carrier frequency.

From the analysis, we theoretically confirmed that parameters, s^2, τ_m, $\sigma_{\tau,R}$, act as 'key parameters' for expressing statistical properties of Nakagami-Rice fading environments.

4. Accuracy of the ETP model as an instantaneous expression

In this chapter, we examine the accuracy of the instantaneous expression of the ETP model by means of simulations of BER characteristics due to intersymbol interference of multipath waves.

The configuration of the simulation assumed here is roughly described in Fig.4 (details are given in Ref.[1]). As modulation and demodulation schemes, π/4-shift QPSK and differential detection (π/4-DQPSK) are assumed. 1000 samples of impulse responses without correlation between each sample are generated. On each generated impulse response, information signals of 10000 bits are transmitted. Temporal variation of impulse response due to fading is not added during the 10000 bits transmission. Because thermal noise is not involved in the transmission path, all bit errors appearing in this simulation are purely due to intersymbol interference of the multipath component.

Here, we compare BER characteristics obtained from three types of propagation paths described below.

(i) A reference model proposed by Ref.[2].
(ii) Equivalent two-wave model (ETP model for N=2).
(iii) Equivalent three-wave model (ETP model for N=3).

As a reference model which approximates actual mobile propagation environments, we use here the wideband propagation model producing exponential delay profile on statistical basis [2]. Here we compare the simulated results obtained from the reference model with those obtained from the two-wave equivalent model (one direct wave + two multipath waves, same environment as Fig.2), or with those obtained from the three-wave equivalent model. As the value of $\Delta\tau$, the expression Eq.(12) (or Eq.(15)) is used. Further, the two key parameters become inevitably the same value as $\sigma_{\tau,R}=\tau_m$ since we adopt the model based on an exponential type of delay profile.

Figure 5 is a scatter diagram which shows the relationship between the simulated BER according to the reference model (i) and the two- (ii) or three-wave ETP model (iii). Whereas sufficiently accurate estimation results seem to be obtained using either the two- or the three-wave model, it is seen from Fig.6 that the three-wave ETP model provides a more precise estimation than the two-wave model.

We now discuss more quantitative results for evaluation accuracy of the two- and the three-wave ETP model. As an indicator of estimation accuracy, we here adopt cross-correlation of the logarithm value of BER (with a very small offset for BER=0). Figure 6 shows the variation of the correlation related to $\sigma_{\tau,R}=\tau_m$, changing the value of the parameter s^2. Irrespective of the value of N or s^2, the correlation decreases as $\sigma_{\tau,R}=\tau_m$ increases. It can be understood why, as the delay spread of the multipath component increases, the frequency-selectivity of the transmission path grows, the frequency bandwidth where the ETP model provides a good approximation becomes relatively small and, as a result, the estimation accuracy of the ETP model decreases. It also should be noted from the figure that a greater correlation value can be obtained by the equivalent model of N=3 compared with that of N=2. Namely, the transformation accuracy is improved by means of taking a derivative of the more higher order into consideration in the approximation process. No significant dependence corresponding to s^2 can be observed in the figure.

Figure 7 shows the average BER characteristics according to 1000 samples of impulse responses. It can easily be expected that average BER characteristics of the equivalent models agree well with those of the reference model because correspondence between instantaneous characteristics has already been shown. Thus it is clearly shown that the statistical characteristics such as average BER can also be estimated with precise accuracy, based on the instantaneous expression of the ETP model.

5. Conclusion

In this paper, the foundation of the ETP model for Nakagami-Rice fading is theoretically examined. Using equivalent transformation of an impulse response in a transmission path, a new formula of the model for instantaneous expression is represented. Through the theoretical investigation, the appropriateness of the model for both instantaneous and statistical expression is justified in an analytical way. Also computer simulations are made for the purpose of quantitative examination of the theory.

Particularly in the simplest case of N=2, since cross-correlation between the two fluctuating waves can be canceled, a very simple and useful configuration for evaluation of statistical characteristics can be obtained. Therefore the two-wave ETP model seems to be effective in the viewpoint of an actual application where evaluation of average BER characteristics is needed.

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[References]

[1] Y. Karasawa, T. Kuroda and H.Iwai : "The equivalent transmission-path model - a tool for analyzing error floor characteristics due to inter-symbol interference in Nakagami-Rice fading environments," IEEE Trans. VT (in press).

[2] H.Iwai and Y. Karasawa : "Wideband propagation model for the analysis of the effect of the multipath fading on the near-far problem in CDMA mobile radio systems," IEICE Trans. Commun., Vol.E76-B, 2, pp.103-112, (1993).

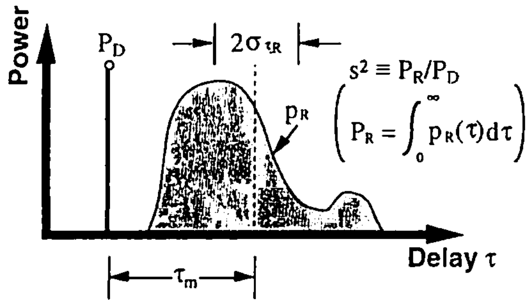


Fig.1 Conceptual view of delay profile in Nakagami-Rice fading.

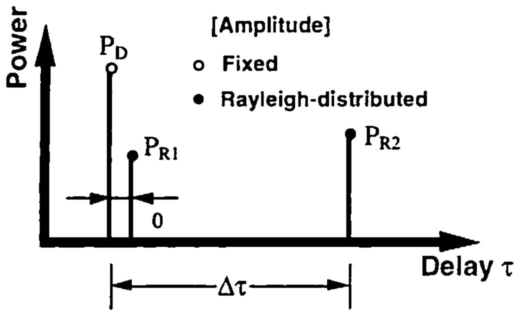
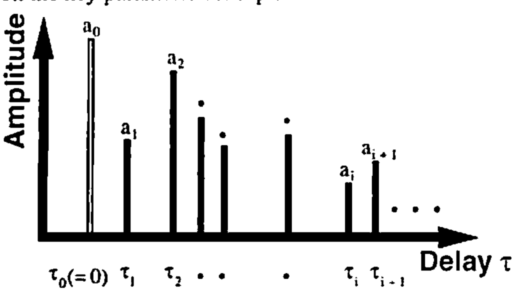
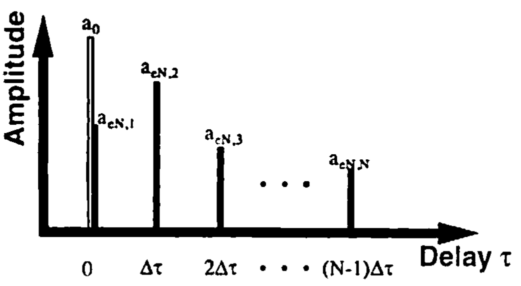


Fig.2 Equivalent transmission path model derived based on the key parameter concept.



(a) A sample of impulse response at time t.



(b) Approximated equivalent expression with N waves.

Fig.3 Impulse response of Nakagami-Rice fading channel and approximated equivalent expression assumed in this paper.

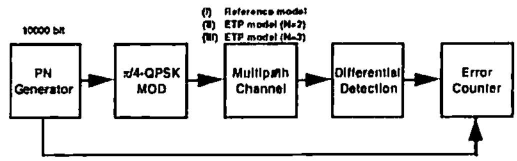


Fig.4 Configuration of BER simulation.

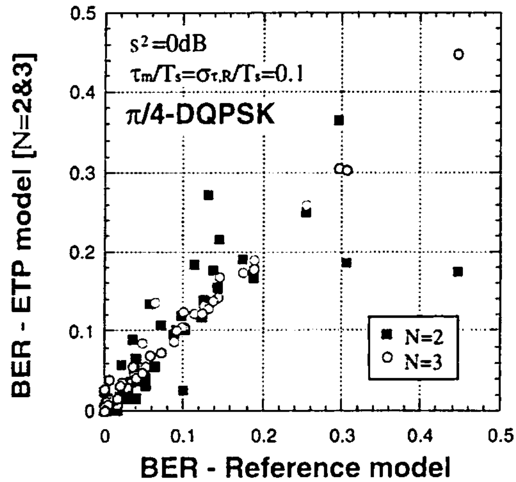


Fig.5 Scatter diagram of simulated results.

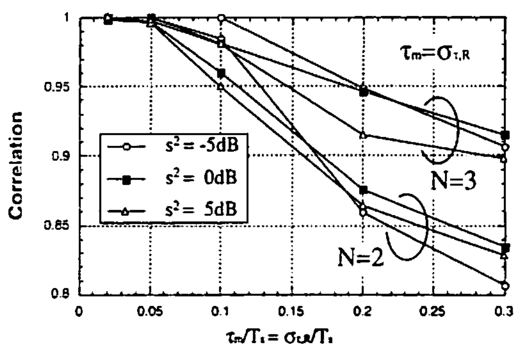


Fig.6 Correlation between reference model and two- and three-wave equivalent models.

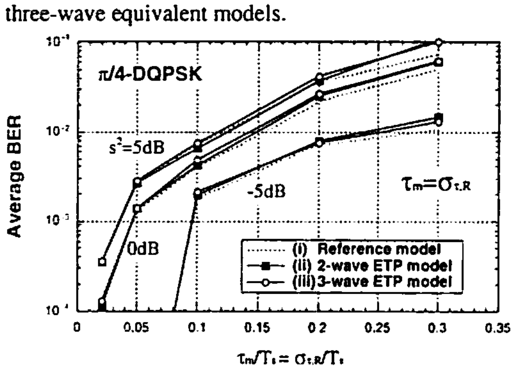


Fig.7 Average BER characteristics.