The Scattering of Whistler Wave from Anisotropic Irregularities in Anisotropic Ionosphere

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1.Summary

Whistler waves have been studied for a few decades. Through studies of whistler waves, the ionosphere and the magnetosphere are better understood. However many problems about whistler wave propagation still remain .[1] It is believed by many people that whistlers are trapped in geomagnetic field-aligned ducts in the ionosphere and the magnetosphere. One problem is when the whistler penetrates through the ionosphere and arriving at a receiver of another hemisphere, the observation shows the arriving direction of the whistler waves have an arriving direction different from the traping cone of the whistlers in the ionosphere. For understanding this problem, the detailed physical processes of propagation of whistlers must be studied. In this study of whistler wave propagation, we assumed that the structure of the ionosphere is constant and it gives a constant pattern of received whistler amplitude.

2.The power spectrum of the amplitude of fluctuation of whistler waves

Throughout this paper it is assumed that the earth is flat and the scattering ionospheric layer is horizontal. A Cartesian coordinate with z axis vertically downwards is used. The x-z plane is the magnetic meridian plane. In north hemisphere the magnetic field $\mathbf{B}_0$ point downwards with an angle Θ with z axis. Hereafter we use letters in bold face denote vectors except with other explanation. Now suppose that a plane wave is obliquely incident on the layer, with its wave normal in a direction given by polar angles θ, ϕ:

$$\mathbf{E}_i = E_0 \mathbf{a}_i \exp\{-ik_i(z\cos\theta + x\sin\theta\cos\phi + y\sin\theta\sin\phi)\} \tag{1}$$

where $\mathbf{a}_i$ the normalized characteristic polarization of incident wave. The scattered wave field $\mathbf{E}_s$ is

$$\nabla\times\nabla\times(\mathbf{E}_i+\mathbf{E}_s) - k_0^2\underline{\mathbf{K}}(\mathbf{E}_i+\mathbf{E}_s) = 0 \tag{2}$$

$$\underline{\mathbf{K}} = \underline{\mathbf{I}} - X\underline{\mathbf{M}} - \Delta X\underline{\mathbf{M}} \tag{3}$$

$\underline{\mathbf{I}}$ is the unit tensor, $k_0 = \frac{\omega}{c}$,

$$\underline{\mathbf{M}} = \begin{pmatrix} 1-\mathbf{Y}_x^2 & -\mathbf{Y}_x\mathbf{Y}_y + i\mathbf{Y}_z & -\mathbf{Y}_x\mathbf{Y}_z - i\mathbf{Y}_x \\ -\mathbf{Y}_x\mathbf{Y}_y - i\mathbf{Y}_z & 1-\mathbf{Y}_y^2 & -\mathbf{Y}_y\mathbf{Y}_z + i\mathbf{Y}_x \\ -\mathbf{Y}_x\mathbf{Y}_y + i\mathbf{Y}_y & -\mathbf{Y}_y\mathbf{Y}_z - i\mathbf{Y}_x & 1-\mathbf{Y}_z^2 \end{pmatrix} \tag{4}$$

$\mathbf{Y} = \frac{e\mathbf{B}_0}{m\omega}$, $X = \frac{Ne^2}{m\epsilon_0}$, N is the number density, ΔX is due to the fluctuation of electron density and ion density. Since the fluctuations are weak, the scattered field E_s is much smaller than the incident wave field. We have approximately

$$\nabla\times\nabla\times\mathbf{E}_s - k_0^2(\underline{\mathbf{I}} - X\underline{\mathbf{M}})\mathbf{E}_s = -k_0^2\Delta X\underline{\mathbf{M}}\mathbf{E}_i \tag{5}$$

Following D.M. Simonich and K.C. Yeh[2], we have the scattered field at $\mathbf{r}_2$ as

$$\mathbf{E}_{rs}(\mathbf{r}_2) = -k_0^2\int_V \Delta X(\mathbf{r})\underline{\Gamma}\cdot\underline{\mathbf{M}}\cdot\mathbf{E}_i d\mathbf{r}_0 \tag{6}$$

where V is the volume containing scatterers, $\mathbf{r}_2 = \mathbf{r}_0 - \mathbf{r}$, $\underline{\Gamma}$ is the Green dyadic tensor. According to M. J. Lighthill[3], the asymptotitc expression of Green dyadic tensor is

$$\underline{\Gamma} = \frac{1}{4\pi}\sum_s \frac{d_s\mathbf{a}_s\mathbf{a}_s^*}{k_s\sec\alpha_s\sqrt{|C_s|}}\frac{e^{i\mathbf{k}_s(\mathbf{r}_0-\mathbf{r})}}{|\mathbf{r}_0-\mathbf{r}|} \tag{7}$$

where $\mathbf{a}_s$ is the normalized characteristic polarization corresponding the the wave mode s with wave vector $\mathbf{k}_s$, $\cos\alpha_s = \hat{\mathbf{k}_s}\cdot\hat{\mathbf{r}_2}$, $\hat{\mathbf{k}_s}$ and $\hat{\mathbf{r}_2}$ are unit vectors, C_s is the Gaussian curvature of the surface evaluated at the stationary point and d_s is a constant that is (a) $\pm i$ if $C_s < 0$ and $\nabla_k(k^2 - k_s^2)$ is in the direction $\pm\mathbf{r2}$, (b)± 1 if the $C_s > 0$ and the surface is convex to $\pm\nabla_k(k^2 - k_s^2)$. If $\mathbf{k}_i$ is the incident wave vector, the incident wave normal through the origin meets the ground at the point $z\tan\theta\cos\phi$, $z\tan\theta\sin\phi$ and z. We consider the field on the ground near this point. Thus let

$$X_1 = x - z\tan\theta\cos\phi \quad Y_1 = y - z\tan\theta\sin\phi \tag{8}$$

$$X_0 = x_0 - z_0\tan\theta\cos\phi \quad Y_0 = y_0 - z_0\tan\theta\sin\phi \tag{9}$$

The X_0 and Y_0 are small horizontal distance of the scattering point from the wave normal through the origin. The incident wave at the scattering point and the received wave field are as follows

$$\mathbf{E}_{is} = E_0\mathbf{a}_i \exp\left(-ik_i(z_0\sec\theta + X_0\sin\theta\cos\phi + Y_0\sin\theta\sin\phi\right) \tag{10}$$

$$\mathbf{E}_{ir} = E_0\mathbf{a}_i \exp[-ik_i(z\sec\theta + X_1\sin\theta\cos\phi + Y_1\sin\theta\sin\phi)] \tag{11}$$

Therefore we have the received field $\mathbf{E}_{rs}$ due to scattering as

$$\mathbf{E}_{rs}(\mathbf{r}) = -k_0^2\frac{1}{4\pi}\sum_s \frac{d_s\mathbf{a}_s\mathbf{a}_s^* E_i}{k_s\sec\alpha_s\sqrt{C_s}}\underline{\mathbf{M}}\mathbf{a}_i$$

$$\times\int_V \exp[-ik_i(z_0\sec\theta + X_0\sin\theta\cos\phi + Y_0\sin\theta\sin\phi)]\frac{e^{i\mathbf{k}_s\cdot\mathbf{r}_2}}{r_2}\Delta X d\mathbf{r}_0 \tag{12}$$

The integral in equation (12) can be evaluated approximately. We have

$$\mathbf{E}_{rs} = -\frac{k_0^2}{4\pi}\sum_s \frac{d_s\mathbf{a}_s\mathbf{a}_s^*}{k_s\sec\alpha_s\sqrt{C_s}}\underline{\mathbf{M}}\cdot\mathbf{E}_{ir}$$

$$\times\int_V \exp[-i(k_i - k_s\cos\alpha_s)(z_0\sec\theta + X_0\sin\theta\cos\phi + Y_0\sin\theta\sin\phi)]\frac{\Delta X(\mathbf{r}_0)}{(z - z_0)\sec\theta}e^{ik_sH\cos\alpha_s}dX_0dY_0dz_0 \tag{13}$$

where

$$H = \frac{(X_1 - X_0)^2 + (Y_1 - Y_0)^2 - \sin^2\theta[(X_1 - X_0)\cos\phi + (Y_1 - Y_0)\sin\phi]}{2(z - z_0)\sec\theta} \tag{14}$$

Corresponding spectrum of the received electric field is

$$F(\kappa_1, \kappa_2) = -\pi k_0^2\sum_s \frac{d_s\mathbf{a}_s\mathbf{a}_s^*}{k_s\sec\alpha_s\sqrt{C_s}}\underline{\mathbf{M}}\cdot\mathbf{E}_{ir}$$

$$\times\int_V e^{[-i(k_i - k_s\cos\alpha_s)(z_0\sec\theta + X_0\sin\theta\cos\phi + Y_0\sin\theta\sin\phi) + i(\kappa_1X_0 + \kappa_2Y_0)]}\Delta X(\mathbf{r}_0)\sin[\alpha(z - z_0)]dx_0dy_0dz_0 \tag{15}$$

where

$$\alpha = \frac{\sec\theta[\kappa_1^2 + \kappa_2^2 + \tan^2\theta(\kappa_1\cos\phi + \kappa_2\sin\phi)^2]}{2k_s\cos\alpha_s} \tag{16}$$

By using of (15), the autocorrelation function of E_{rs} is

$$|F(\kappa_1, \kappa_2)|^2 = \pi^2 k_0^4\left(\sum_s \frac{d_s\mathbf{a}_s\mathbf{a}_s^*}{k_s\sec\alpha_s\sqrt{C_s}}\underline{\mathbf{M}}\cdot\mathbf{E}_i\right)^2$$

$$\times\int_V \exp\{i(k_i - k_s\cos\alpha_s)[(x_0 - x_0')\sin\theta\cos\phi + (y_0 - y_0')\sin\theta\sin\phi\}$$

$$\times\exp\{(z_0 - z_0')[\sec\theta - \tan\theta\sin\theta\sin\phi(\cos\phi - \sin\phi)]]\}\Delta X(\mathbf{r}_0)\Delta X(\mathbf{r}_0')\sin[\alpha(z - z_0)]\sin[\alpha(z - z_0')]d\mathbf{r}_0d\mathbf{r}_0' \tag{17}$$

where the scattered wave vector k_s and k_s' are assumed equal. For calculating the spectrum of the fluctuated scattered wave field, we have to know the statistical characteristic property of the fluctuated medium.

3.The Power Spectrum of Scattered waves by elongated Irregularities

Equation (17) gives the power spectrum for single scattering layer. We assumed the deviations of the ΔX are given by

$$\Delta X = X(z_0)X_f(x_0, y_0, z_0) \tag{18}$$

where X_f is the stochastic function which is stochastically homogeneous and stationary with respect x_0, y_0, z_0, and has a mean value zero and unity variance. The autocorrelation function ρ is defined by

$$\rho(\xi,\eta,\varsigma) = < X_f(x_0,y_0,z_0)X_f(x_0+\xi, y_0+\eta, z_0+\varsigma) > \tag{19}$$

$$x'_0 = x_0+\xi, \quad y'_0 = y_0+\eta, \quad z'_0 = z_0+\varsigma \tag{20}$$

And the Fourier transformation of ρ is

$$\rho(\xi,\eta,\varsigma) = \iiint P(\sigma_1,\sigma_2,\sigma_3)\exp[i(\sigma_1\xi+\sigma_2\eta+\sigma_3\varsigma)]d\sigma_1 d\sigma_2 d\sigma_3 \tag{21}$$

$$\frac{1}{4\pi^2}\iint \rho(\xi,\eta,\varsigma)e^{-i(\sigma_1\xi+\sigma_2\eta)}d\xi d\eta = \int P(\sigma_1,\sigma_2,\sigma_3)e^{i\sigma_3\varsigma}d\sigma_3 \tag{22}$$

Equation (17) becomes

$$S(\kappa_1,\kappa_2) = 4W^2\pi^4k_0^4\left(\sum_s \frac{d_s\mathbf{a}_s\mathbf{a}_s^*}{k_s\sec\alpha_s\sqrt{C_s}}\underline{\mathbf{M}}\cdot\mathbf{E}_{ir}\right)^2 \iiint X(z_0)X(z'_0)P(\kappa'_1,\kappa'_2,\sigma_3)$$

$$\times\exp\{-i\varsigma[(k_i-k_s\cos\alpha_s)(\sec\theta-\tan\theta\sin\theta\sin\phi(\cos\phi-\sin\phi)-\sigma_3-\tan\theta(\kappa_1\cos\phi+\kappa_2\sin\phi)]\}$$

$$\times\sin[\alpha(z-z_0)]\sin[\alpha(z-z'_0)]dz_0dz'_0d\sigma_3 \tag{23}$$

where the limits of integration of x_0, y_0 have been set at $\pm\frac{W}{2}$, W a constant and

$$\kappa'_1 = \kappa_1+\sin\theta\cos\phi(k_i-k_s\cos\alpha_s) \tag{24}$$

$$\kappa'_2 = \kappa_2+\sin\theta\sin\phi(k_i-k_s\cos\alpha_s) \tag{25}$$

If we make the assumption about the profile function $X(z_0)$ as a Gaussian function, we therefore have

$$X(z_0) = X_m\exp(-\frac{z_0^2}{L^2}) \tag{26}$$

where L is a measure of the thickness of the scattering layer. If we insert (26) into (23), the integration with respect to z_0 and z'_0 can be done. It leads to

$$S(\kappa_1,\kappa_2) = 4W^2\pi^4k_0^4\left(\sum_s \frac{d_s\mathbf{a}_s\mathbf{a}_s^*}{k_s\sec\alpha_s\sqrt{C_s}}\underline{\mathbf{M}}\cdot\mathbf{E}_{ir}\right)^2 X_m^2$$

$$\int P(\kappa'_1,\kappa'_2,\sigma_3)\{\cosh[(\sigma_3+\beta)\alpha L^2]-\cos(2\alpha z)\}\exp\{-\frac{1}{2}[(\sigma_3+\beta)^2+\alpha^2]L^2\}d\sigma_3 \tag{27}$$

where

$$\beta = \tan\theta(\kappa_1\cos\phi+\kappa_2\sin\phi)-(k_i-k_s\cos\alpha_s)(\sec\theta-\tan\theta\sin\phi\sin\theta(\cos\phi-\sin\phi)) \tag{28}$$

It is believed that the irregularities in the ionosphere are elongated in the direction of the earth's magnetic field[4]. For studying the scattered waves, we use the description introduced by K.G. Budden[5]. It is assumed the geomagnetic field is in the x-z plane at an angle Θ to the vertical. We assumed that the autocorrelation function $\rho(\xi,\eta,\varsigma)$ is a function only of u, where the surface u=constant are ellipsoids of revolution about the direction of the earth's magnetic field, with an axis ratio b. The displacements ξ,η,ς may be treated as a column matrix or vector **l**. Then u is the length of a vector **u** given by

$$\mathbf{u} = \mathbf{Q}\cdot\mathbf{l} \tag{29}$$

where $\mathbf{Q}$ is the matrix

$$\begin{pmatrix} \cos\Theta/b & 0 & -\sin\Theta/b \\ 0 & 1/b & 0 \\ \sin\Theta & 0 & \cos\Theta \end{pmatrix} \tag{30}$$

The three quantities $\sigma_1, \sigma_2, \sigma_3$ may also be regarded as a vector σ. The inverse Fourier transform of (22) is

$$P(\sigma_1, \sigma_2, \sigma_3) = \frac{b^2}{8\pi^3} \iiint \rho(u) \exp(-i\sigma \mathbf{Q}^{-1}\mathbf{u}) d^3\mathbf{u} \tag{31}$$

The factor b^2 is the Jacobian of the transformation from ξ, η, ς to the components of $\mathbf{u}$, as variables of the integration. It is easy to show that $P(\sigma_1, \sigma_2, \sigma_3)$ depends only on the length s of the vector

$$s = \sigma \mathbf{Q}^{-1}$$

and that

$$s^2 = \kappa_1'^2(b^2\cos^2\Theta + \sin^2\Theta) + b^2\kappa_2'^2 + \sigma_3^2(b^2\sin^2\Theta + \cos^2\Theta) + 2\kappa_1'\sigma_3\sin\Theta\cos\Theta(1-b^2) \tag{32}$$

Next we assume following forms for the function P and ρ,

$$P = 8\pi^{\frac{3}{2}} R^3 b^2 \exp(-\frac{1}{4}R^2 s^2) \tag{33}$$

$$\rho = \exp(-\frac{\mathbf{u}^2}{R^2}) \tag{34}$$

Thus R is the measure of the size of the irregularities for the directions parallel to the earth's magnetic field and bR is the measure for the two directions perpendicular to it. If (33) and (34) are inserted into (27), the integration may be done. Hence, we have following result.

$$S(\kappa_1, \kappa_2) = 32W^2\pi^{\frac{11}{4}} k_0^4 \left(\sum_s \frac{d_s \mathbf{a}_s \mathbf{a}_s^*}{k_s \sec\alpha_s \sqrt{C_s}} \underline{\mathbf{M}} \cdot \mathbf{E}_{ir} \right)^2 X_m^2$$

$$\left[\exp\left(\frac{\alpha^2 L^4}{B^2R^2 + 2L^2} \right) \cosh\left(\frac{L^2B^2R^2\alpha(\beta-\gamma)}{B^2R^2+2L^2} \right) - \cos(2\alpha z) \right]$$

$$\times \exp\{ -\frac{1}{4}R^2b^2(\kappa_2'^2 + \frac{\kappa_1'^2}{B^2}) - \frac{1}{2}\alpha^2L^2 - \frac{B^2R^2L^2(\beta-\gamma)^2}{2(B^2R^2+2L^2)} \} \tag{35}$$

where

$$B^2 = \cos^2\Theta + b^2\sin^2\Theta \tag{36}$$

and

$$\gamma = \frac{\kappa_1' \sin\Theta\cos\Theta(1-b^2)}{B^2} \tag{37}$$

This is the power spectrum of the wave scattered by the elongated irregularities in an anisotropis ionoshere. It can be used for studying the propagation of the whistler waves in the ionosphere.For multiple scattered wave in random medium, much work has been done[6]. However, for the multiple scattering of vector waves, problems still remain . It is also the future work we are going to pursue.

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