GTD inaccuracies in 3D edge diffraction

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1. Introduction

The Geometrical Theory of Diffraction (GTD) was established by Keller as an extension of Geometrical Optics (GO). In presence of structures with local radius of curvature close to or smaller than the wavelength, the GO rays alone fail to predict accurately the electromagnetic field distribution, especially in the shadow regions. The GTD simply adds to the GO field its diffracted rays whose possible directions of propagation follow simple laws, similar to the GO ones. In the case of reflection and refraction the field amplitude and phase changes at an interface are uniquely and exactly described by the Fresnel coefficients. On the contrary, diffraction coefficients depend on the geometry of the structure. The wedge diffraction coefficients are based on the analytical solution obtained by Sommerfeld for the perfectly conducting half plane. In this paper, we will first clarify and correct this attractive and simple GTD approach for arbitrary plane wave illumination; then we show how inaccurate it can be in the prediction of some components of the fields.

2. Full 3D solution of the PEC half plane illuminated by a plane wave

Early 1896, Sommerfeld solved exactly the 2D problem of a perfectly conducting half plane illuminated by a TE or TM plane wave [1]. This great result was later generalized by Copson [2] for arbitrary values of β. We summarize below the results, based on the axis conventions chosen in [3].

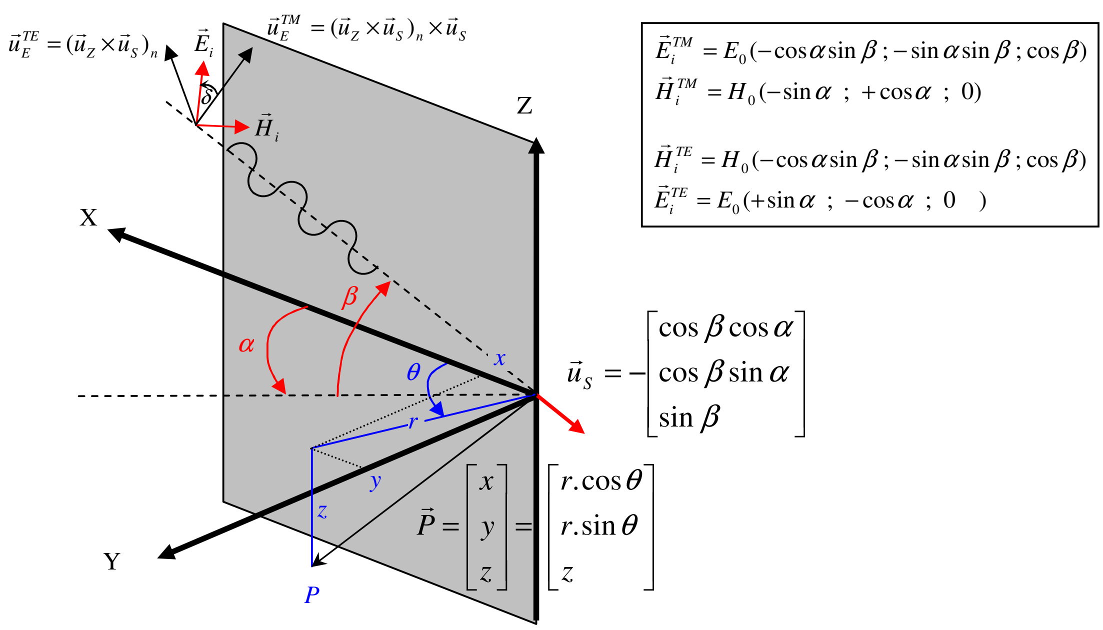

Figure 1 : Geometry for the 3D plane wave illumination of the half plane

The incident plane wave $\boldsymbol{E_i} \perp \boldsymbol{H_i}$ propagates along $\boldsymbol{u_S}$. Its polarisation is defined by the angle δ. It has a complex amplitude $E_0 = \eta_0 H_0$. Choosing the phase reference at *(0,0,0)* the phase factor of $\boldsymbol{E_i},\boldsymbol{H_i}$ at the observation point *P(x,y,z)* is given by $e^{ik.\vec{P}.\vec{u}_S} = e^{ik.(x\cos\alpha\cos\beta + y\sin\alpha\cos\beta + z\sin\beta)}$. In the presence of the

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infinitely thin and perfectly conducting half plane located at y = 0, x > 0, the field in P can be expressed as $\boldsymbol{TM}.cos\delta + \boldsymbol{TE}.sin\delta$, where the TM and TE modes are given by :

Table 1 : TM and TE modes for the 3D diffraction of a plane wave on a half plane

TM mode ($\vec{H}_i$ is $\perp$ to the edge, δ=0°)	*TE mode ($\vec{E}_i$ is $\perp$ to the edge, δ=90°)*
$H_x = H_0[-u_2 \sin\alpha - K.\sin(\alpha/2)\cos(\theta/2)]$	$E_x = E_0[+u_1 \sin\alpha - K.\cos(\alpha/2)\sin(\theta/2)]$
$H_y = H_0[+u_1 \cos\alpha - K.\sin(\alpha/2)\sin(\theta/2)]$	$E_y = E_0[-u_2 \cos\alpha + K.\cos(\alpha/2)\cos(\theta/2)]$
$H_z = 0$	$E_z = 0$
$E_x = [-H_y \sin\beta].\eta_0$	$H_x = +E_y \sin\beta/\eta_0$
$E_y = [+H_x \sin\beta].\eta_0$	$H_y = -E_x \sin\beta/\eta_0$
$E_z = [+u_1 \cos\beta].E_0$	$H_z = [+u_2 \cos\beta].H_0$

with $\quad K = \sqrt{\dfrac{2i}{\pi kr\cos\beta}}.e^{iks_d} \qquad s_d = r\cos\beta - z\sin\beta$

$$u_1 = e^{iks_i}F(p) - e^{iks_r}F(q) \qquad p = \sqrt{2kr\cos\beta}\cos((\theta-\alpha)/2) \qquad s_i = -r\cos(\theta-\alpha)\cos\beta - z\sin\beta$$

$$u_2 = e^{iks_i}F(p) + e^{iks_r}F(q) \qquad q = \sqrt{2kr\cos\beta}\cos(\theta+\alpha)/2) \qquad s_r = -r\cos(\theta+\alpha)\cos\beta - z\sin\beta$$

and the Fresnel integral $F[\xi] = \dfrac{1}{\sqrt{\pi i}}\displaystyle\int_{-\infty}^{\xi} e^{is^2}.ds = \dfrac{1}{\sqrt{\pi i}}\left[\int_{-\infty}^{0} e^{is^2}.ds + \int_{0}^{\xi} e^{is^2}.ds\right] = \dfrac{1}{2} + \dfrac{1}{\sqrt{\pi i}}\int_{0}^{\xi} e^{is^2}.ds$

On Fig. 2 we present two examples of field distribution in a 10λ x 10λ square area around the edge.

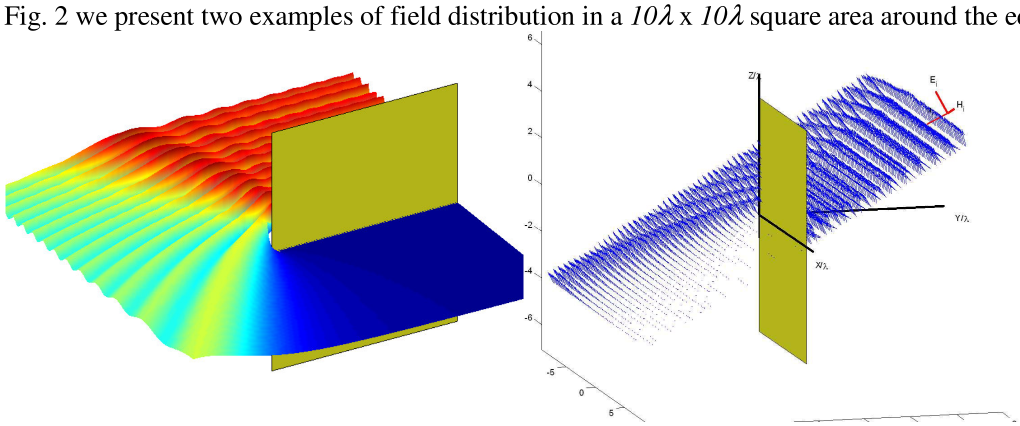

Figure 2 : Left = |Ez| for (α,β,δ) = (60°, 0°, 0°) Right = $\vec{E}$ for (α,β,δ) = (60°, 30°, 0°)

3. GTD coefficents for the full 3D case

In his famous paper [4], Keller presents a formula for two diffraction coefficients applicable to wedges. This formula will be shown to be slightly incorrect in the 3D case ($\beta \neq 0$). Also the way how to apply these only two D^+ and D^- coefficients on any incident $\boldsymbol{E_i}$ and $\boldsymbol{H_i}$ fields is not revealed. Finally, we will show that the GTD for the edge problems, based on these two diffraction coefficients only, is accurate for the Z component of the fields (aligned with the edge), but not at all for the X and Y components, that contain the K terms responsible for infinite values at $r = 0$.

3.1 The correct expressions for the 3D diffraction coefficients

As shown in [5] the expressions of D^+ and D^- are derived from u_1 and u_2, after the Fresnel integral has been replaced by the following non asymptotic expansion, valid only if |ξ| > 1 :

$$F[\xi] = \chi[\xi] - i\,\frac{e^{i\xi^2}}{2\pi}\sum_{n=0}^{\infty}\frac{\Gamma(n+1/2)}{(i\xi^2)^{n+1/2}} \overset{(\xi >> 0)}{\approx} \chi[\xi] - i\,\frac{e^{i\xi^2}}{2\pi}\frac{\Gamma(1/2)}{(i\xi^2)^{1/2}} = \chi[\xi] - \frac{e^{i\xi^2}}{2\xi}\sqrt{\frac{i}{\pi}}$$

where $\chi[\xi] = 1$ if $\xi > 0$ and $\chi[\xi] = 0$ if $\xi < 0$.
Applying the same scheme to the 3D expressions of u_1 and u_2, we obtain :

$$u_2^1 \approx \underbrace{e^{iks_i}.\chi[\theta - \pi - \alpha]}_{incident} - \underbrace{e^{iks_r}.\chi[\theta - \pi + \alpha]}_{reflected} - \underbrace{\frac{e^{iks_d}}{\sqrt{r}}\overbrace{\left[\sec\left(\frac{\theta-\alpha}{2}\right) \pm \sec\left(\frac{\theta+\alpha}{2}\right)\right]\frac{1}{2}\sqrt{\frac{i}{2\pi k\cos\beta}}}^{D_2^1}}_{diffracted}$$

Note that the factor $cos\beta$ (= $sin\beta_0$ with Keller's angles definition) enters *into* the square root.

3.2 The 3D parallel and perpendicular decomposition for the diffracted field

In the 2D case, the $\boldsymbol{u}^{//}$ direction corresponds naturally to the direction along the edge ($\boldsymbol{u_Z}$) and $\boldsymbol{u}^{\perp} = \boldsymbol{u_S} \times \boldsymbol{u_Z}$. Now if we remember that the diffraction coefficients are obtained from u_1 and u_2 , if we examine carefully the 3D solution, ampute it from the K terms, $\boldsymbol{F_u} = \boldsymbol{F} - \boldsymbol{F_K}$, we can condense it for any polarisation (any combination of the TM and TE mode) into the vector form :

$$\vec{E}_u = \left(\vec{E}_{ix} + \vec{E}_{iz}\right)u_1 + \left(\vec{E}_{iy}\right)u_2 = \left(\vec{E}_i^{//}\right)u_1 + \left(\vec{E}_i^{\perp}\right)u_2$$
$$\vec{H}_u = \left(\vec{H}_{ix} + \vec{H}_{iz}\right)u_2 + \left(\vec{H}_{iy}\right)u_1 = \left(\vec{H}_i^{//}\right)u_2 + \left(\vec{H}_i^{\perp}\right)u_1$$

From these expressions, it becomes clear how to use the diffraction coefficients D_1 and D_2 to evaluate the fields $\boldsymbol{E_u}$ and $\boldsymbol{H_u}$: the field incident on the edge must be decomposed into a component $\boldsymbol{F_{iy}}$ perpendicular to the half plane, and a component lying into the half plane. Note in passing that the classical TM/TE decomposition used to derive the Fresnel coefficients for the reflected ray can be equivalently replaced by the same new decomposition imposed to the diffracted rays. Not surprisingly : the reflection coefficients are obtained by enforcing the boundary conditions at the interface, and these naturally separate the components normal and tangential to the interface ! Fig. 3 shows how closely this corrected 3D GTD model allows to predict $\boldsymbol{E_u}$ and $\boldsymbol{H_u}$, except along the GO transition zones ($\theta = \pi \pm \alpha$) where the arguments p and q of the Fresnel integrals are too close to 0 to allow the use of the non asymptotic expansion.

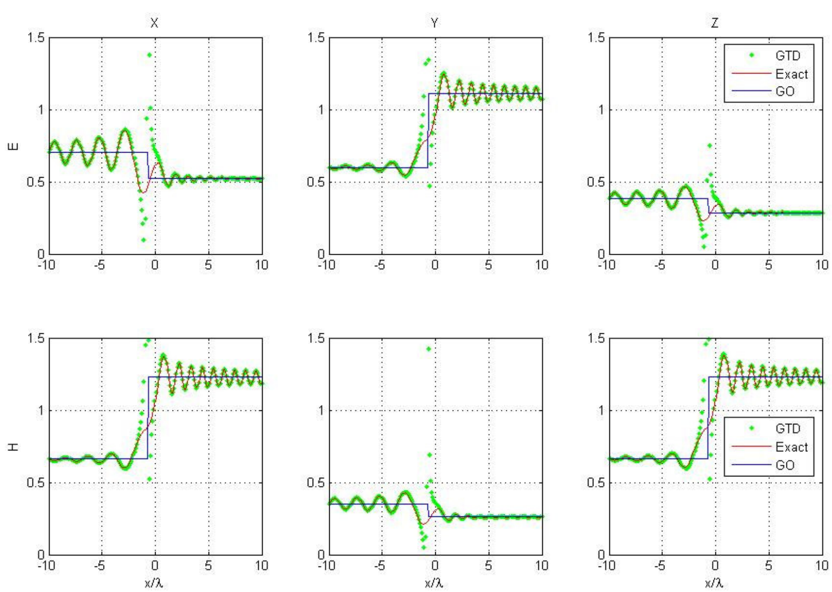

Figure 3 : 3D edge diffraction for z = 0 ; y = +2λ and (α , β , δ) = (70° , 40° , 60°)

3.3 The K terms

Unfortunately, the neglected K terms are not negligible at all, neither close to the edge nor at infinite distance from it : the K term and the GTD diffraction term are possibly of the same order of magnitude for many combinations of α and θ, independantly of the distance r to the edge ! We illustrate this with only one of the X,Y components, but it is a simple matter to verify that the same is true for all of them.

$$\frac{E_x^{TM}}{E_0} \approx GOfield + \sin\beta . \frac{e^{iks_d}}{\sqrt{r}} \sqrt{\frac{2i}{\pi k \cos\beta}} . \sin\frac{\alpha}{2} \sin\frac{\theta}{2} \left(\underbrace{\frac{-\cos\alpha}{\cos\alpha + \cos\theta}}_{GTD \quad term} + \underbrace{1}_{K \quad term} \right)$$

The example on Fig. 4 shows how wrong the GTD (u terms only, in blue) can be in predicting the exact X and Y components (full expression, in red) : as well the amplitude as the phase of the diffracted field, to be added to the GO field, can be highly over- or under-estimated.

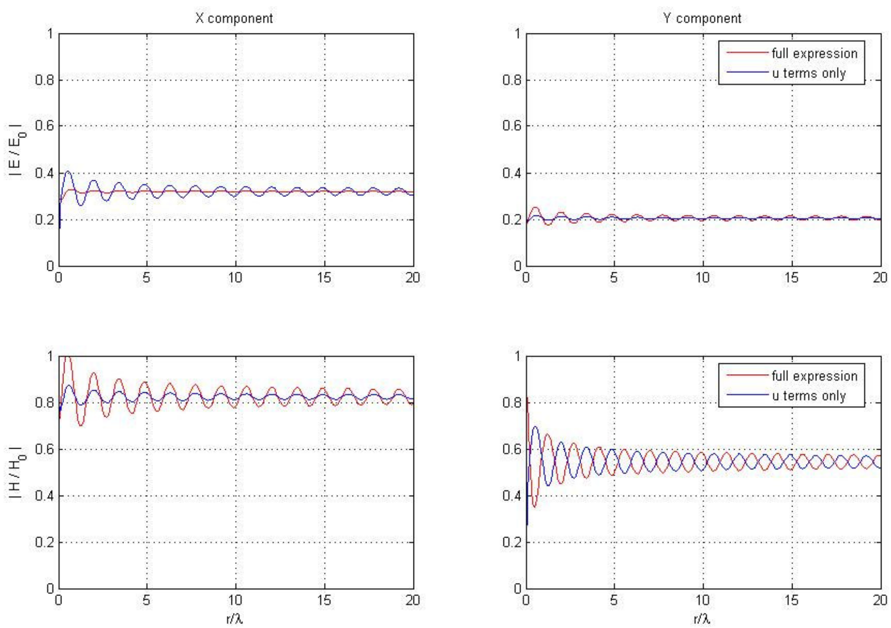

Figure 4 : θ = 225° and (α , β , δ) = (120°, 20°, 10°)

4. Conclusion

The GTD based on two diffractions coefficients only cannot model accurately the field components perpendicular to the diffracting edge, as these coefficients don't account for non negligible terms present in the analytical solution. Any attempt to incorporate these terms into the GTD diffraction coefficients is deemed to fail as they do not enter into a nice vector form like the *u* terms do.

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